# Optimal Policy for Material Requirements Planning and Procurements.

**K.J. Bassey, M.Sc.**

Department of Mathematical Sciences
Federal University of Technology, Akure, Nigeria.

*E-mail: simybas@yahoo.com

## ABSTRACT

Material requirements planning (MRP) uses the relationship between the demand for the final product and the materials used in the production to create production schedule that leads to inventory reduction. This paper discusses an extension of MRP to procurements policy (MRPP) that will minimize the cost of purchasing orders when there is material stock out. Sufficient conditions are presented for MRPP to be optimal.

(Keywords: material requirements planning, MRPP, MRP, procurements, probability, cost control, inventory)

## INTRODUCTION

Material requirements planning (MRP) is the consensus for a material management system in a dependent demand environment (Edgar 1983). Under the conditions of uncertain demand, the decision environment becomes more complex. The stochastic nature of demand always produces an added dimension to the decision environment which can result in material stockout for planned production.

Several authors have expressed concern about the state of MRP in inventory control. Gotzel and Inderfurth (2001) examined the performance of an extended MRP approach for a hybrid single–stage production/manufacturing system with external return flows. Edgar (1983) determined the status of MRP utilization within the defense industry.

While MRP accuracy depends on accuracy of stock status, Fleischmann and Kuik (1998) wrote on optimal inventory control with stochastic item returns. Other commentators include, Inderfurth (1998), Khang and Fujiwara (2000), Indefurth and Jensen (1999), Homen-de-Mello et al. (1992).

We can produce anything if we have the materials at the right time. But in a manufacturing set-up, material capacity is sometimes a constraint due to wear-out, damage, or demand uncertainties. The main aim of any method employed in manufacturing set-up is for cost minimization and product maximization. This paper presents a policy that sustains this aim as an extension of MRP approach for inventory control.

## MRP SYSTEMS - A REVIEW OF ITS USE IN PRACTICE

MRP systems became a prominent approach to managing the flow of raw material and components on the floor of industry in the late 20th century. In 1985, it was estimated that between 2000 and 5000 American companies used MRP in production schedules (see Winston, 1994).

A practical example of MRP could be illustrated by considering a company that produces two final products: A and B. Each unit of product A uses some units of component X (plus other subcomponents). Each unit of component X requires some units of subcomponent Y (plus other subcomponents). Each unit of product B uses some components of component X (plus other subcomponents). If it takes, two months to produce a unit of X and one month for a unit of Y, given the demands for A and B within a certain period of t months, the initial stock level of X from previously planned manufacturing or purchasing orders, and the units of X scheduled for receipts at the beginning of month t to avoid shortages with assumption of zero assembling time of A and B is given by:

$$NR_t(X) = GR_t(X) - SR_t(X) - OHI_{t-1}(X) \quad \textbf{(1)}$$

and,

$OHI_t (X) = SR_t (X) + OHI_{t-1} (X) – GR_t (x)$ **(2)**

where,

$NR_t$ (X) = Net requirements of X for period t
$GR_t$ (X) = Gross requirements of X during period t
$SR_t$ (X) = Scheduled receipts of X during period t
$OHI_t$ (X) = On-hand inventory of X at the end of period t.

For more information see Winston (1994) and Ehrhardt (1997).

## DESIGNING PROCUREMENT POLICY FOR MRP

One of the basic assumptions of MRP is based on the time taken to implement the MRP recommendations and update the transactional data. Planning, we know, requires explicit consideration of all possible factors related to change (i.e., planning is a function of change). In manufacturing industries, one of the major factors that must be taken into consideration when planning is the cost function.

When there is material stockout either caused by the stochastic nature of demand, wear-out, or damage, the expected number of stockout incorporated into the MRP policy will make MRP design more appropriate. When this is done, the procurement policy that will minimize cost of purchasing orders will make MRP optimal and could be called "Material Requirements Planning and Procurement Policy" (MRPP).

Now, let us consider the following:

$\gamma$ = the total number of items in the system from previously planned or purchasing orders

S(t) = Schedule receipt of item during time t (this quantity is equivalent to number of stockout items)

$\theta_t$ = expected number of stockout items at the end of time t.

P = probability of each item being out of stock before the end of first period of production.

q = probability of each item being out of stock before the end of the second period (say, month) (q = 1 = p).

T = total cost of purchasing orders until the end of time t.

α = unit cost of purchasing orders if all materials are to be procured at the .

β = unit cost of purchasing orders if every item that is out of stock is to be procured after time t.

E = life expectancy of any item

P(t) = probability of any item being out of stock at time t.

At the beginning, all items are in stock either from previously planned or purchasing orders. We assume that all items have equal probability of extinction, and denoted it as p, before the end of the first month of production. If we also assume a period of two month,

Let,

$$\theta_t = \theta_0 = \theta \text{ at } t=0 \quad \textbf{(3)}$$

$$\theta_1 = \theta_0 p = \theta p = \theta(1-q) \quad \textbf{(4)}$$

$$\theta_2 = \theta_0 q + \theta_1 p = \theta (1 – q + q^2) \quad \textbf{(5)}$$

In general,

$$\theta_k = \theta \{1- q + q^2 – q_3 + \ldots\ldots + (-q)^k\} \quad \textbf{(6)}$$

and

$$\begin{aligned}\theta_{KH} &= \theta_{k-1} q + \theta_k p \\ &= \theta\{1-q+q^2+\ldots\ldots+(-q)^{k-1}\}q \\ &+ \theta\{1-q+q^2+\ldots\ldots+(-q)^{k}\}(1-q) \\ &= \theta\{1-q+q^2+\ldots\ldots\ldots+(-q)^{k+1}\}\end{aligned} \quad \textbf{(7)}$$

Let $\theta_r$ be the expected number of items that are out of stock at the end of month r (r < t).
Then,

$$\begin{aligned}\theta_r &= \theta\{1-q+q^2+\ldots\ldots + (-q)^r\} \\ &= \frac{\theta\{1-(-q)^{r+1}\}}{1+q}\end{aligned} \quad \textbf{(8)}$$

and

$$\lim_{r \to \infty} \theta_r = \lim_{r \to \infty} \frac{\theta\{1-(-q)^{r+1}\}}{1+q}$$
$$= \begin{cases} \theta(1+q)^{-1}, & for \quad q < 1 \\ 0 \quad , & otherwise \end{cases} \quad \textbf{(9)}$$

(see also Olkin et al., 1978 for more details on mathematical induction and probability).

The cost of purchasing orders for stock out items after time t is given by:

$$T(t) = \gamma\alpha + \beta\sum_{r-1}^{t-1} s(r) \quad \textbf{(10)}$$

For time t, the MRPP is given by:

$$\text{MRPP} = \text{Min}\left\{ ave \quad \left[T(t) = \gamma\alpha + \beta\sum_{r-1}^{t-1} s(r) \ \right] \right\}$$
**(11)**

Subject to:

$$T(t)\delta^{-(t-1)} < O < T(t)\delta^{-t} \quad \textbf{(12)}$$

When,

$$T(t)\delta^{-t} > O,$$
$$\frac{T(t+1)}{t+1} - \frac{T(t)}{t} > O \quad \textbf{(13)}$$

Where,

$$\frac{T(t+1)}{t+1} - \frac{T(t)}{t} = \gamma\alpha\left[\frac{1}{tH} - \frac{1}{t}\ \right] + \beta\sum_{r-1}^{t-1} S(r)\left[\frac{1}{t+1} - \frac{1}{t}\right] + \frac{\beta.S(t)}{t+1}$$
$$= \left\{-\gamma\alpha - \beta\sum_{r-1}^{t-1} S(r) + \beta S(t)\ \right\} / t(t+1)$$
**(14)**

When,

$$T(t-1)\delta^{-(t-1)} > O,$$
$$\frac{T(t-1)}{t-1} - \frac{\beta}{t} > 0 \quad \textbf{(15)}$$

and equation 13 holds *iff*

(i) $\beta S(t) > \left\{\gamma\alpha + \beta\sum_{r-1}^{t-1} S(r)\right\} \Big/ t$ **(16)**

and

(ii) $\beta S(t-1) < \left\{\gamma\alpha + \beta\sum_{r-1}^{t-2} S(r)\right\} \Big/ t-1$ **(17)**

Therefore, the average cost of purchasing orders in each period where all items are procured at the end of the time t is given by:

$$Ave(T) = \gamma\alpha + \beta\sum_{r-1}^{t-1} S(r) \ \ /t \quad \textbf{(18)}$$

And the optimal policy for MRPP is:

$$MRPP = OPT \quad iff \begin{cases} \gamma\alpha + \beta\gamma p < \gamma\alpha/1+q : \quad Purchase\ all \quad item \\ \quad at \quad the\ end \quad of \quad time \quad t. \\ \alpha < \frac{\beta(1-p)^2}{1+q}\ or \quad \beta > \frac{\alpha(1+q)}{(1-p)^2} : Purchase\ each\ item\ wherever \\ \quad there\ is \quad a \quad stockout \end{cases}$$

**(19)**

## NUMERICAL EXAMPLE

Let $X_1$, $X_2$,………. $X_n$ be independent items required for production planning of a certain product with the assumptions of fixed lead time and stochastic demand. Suppose that the cost of purchasing orders of $X_i$'s at the same time will attract a certain discount per item (let the unit cost be N300 with discount), and on procurement of every $X_i$'s that is out of stock before the end of the production period, the unit cost is N500. If we have initial stock level (γ) of 1000, and from past experience, we note that the average percentage stock out at the end of every period t is distributed as follows:

| t | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| % Stock out | 5 | 20 | 30 | 55 | 100 |

Then, we can obtain the extinction probability of Xi at every t and then the expected number of stockout during t. This will assist us in our next planning endeavor. Thus we would have the following:

| t | P(x) | θt | E(t) | β | α(N) | Ave (T) (N) |
|---|---|---|---|---|---|---|
| 1 | 0.05 | 50 | 0.05 | 25,000 | 325,000 | 325,000 |
| 2 | 0.15 | 153 | 0.30 | 76,500 | 376,500 | 188,250 |
| 3 | 0.10 | 115 | 0.30 | 57,500 | 357,500 | 119,166.67* |
| 4 | 0.25 | 284 | 1.00 | 142,000* | 442,000 | 110,500 |
| 5 | 0.45 | 509 | 2.25 | 254,500 | 555,500 | 110,900 |

The expected number of stockout items during t is given as $\frac{1000}{3.9} = 256$ and the expected cost of

Procuring this is 256 x 500 = ₦128,000.

From the above result, it could be seen that the cost of procuring each items is higher in the fourth month than the average cost for three months. So the optimal policy is: purchasing orders should be placed on all items every three months.

## CONCLUSIONS

This paper argues that stock status is not accurate at every given point of time due to some few manageable or non manageable reasons. In any case, the main proposal is that MRP policies should be extended to some of these manageable reasons. The paper presents a policy that will help in minimizing cost of purchasing orders when there is a stockout. To this end we conclude with the following proposition:

**PROPOSITION 1:** When the average cost of purchasing orders exceeds the purchasing cost of each item at stockout despite the discounting rates, procurement of each item at stockout can be optimal (numerical example).

## REFERENCES


1. Edgar, D.W. 1983. "Material Requirements Planning (MRP) Within the Defense Industry". Master's Thesis. http://handle.dtic.mil/100.2/ADA134973.

2. Ehrhardt, R. 1997. "A Model of JIT Mark-To-Stock Inventory with Stochastic Demand". *Operations Research Society*. 48:1013-1021.

3. Fleischmann, M. and Kuik, R. 1998. "On Optimal Inventory Control with Stochastic Item Returns". *Management Science Report*. (21). Erasmus University, Rotterdam.

4. Gotzel, C. and Inderfurth, K. 2001. "Performance of MRP in Product Recovery Systems with Demand, Return, and Lead-time Uncertainties". Reprint 6 (FWW). University of Magdeburg, Germany.

5. Homem-De-Mello, T., Shapiro, A., and Spearman, L. 1999. "Finding Optimal Material Release Times Using Simulation-Based Optimization". *Management Science*. 45(1):86-102.

6. Inderfurth, K. 1998. "The Performance of Simple MRP Driven Policies For Stochastic Manufacturing/Remanufacturing Problems". Reprint 32 (FWW), University of Magdeburg, Germany.

7. Inderfurth, K. and Jensen, T. 1998. "Analysis of MRP Policies with Recovery Options". In: *Modeling and Decisions in Economics*. Leopold, Wildburger, Feichtinger, and Kistner (eds.). Verlag Heidelberg: New York, NY.

8. Khang, D.B. and Fujiwara, O. 2000. "Optimality of Myopic Ordering Policies for Inventory Model with Stochastic Supply". *Operations Research*. 48(1): 181-184.

9. Olkin, I., Gleser, L.J., and Derman, C. 1978. *Probability Models and Applications*. Macmillan Publishing Company: New York, NY.

10. Vollman, T., Berry, W., and Whybark, C. 1992. *Manufacturing Planning and Control Systems*. Homewood: Irwin, CA.

11. Winston, W. 1994. *Operations Research: Applications and Algorithms (3rd Ed)*. Wadsworth, Los Angeles, CA.


## ABOUT THE AUTHOR


**Kufre John Bassey,** is a Lecturer II at the Federal University of Technology, Akure, Dept. of Mathematical Sciences. He holds an M.Sc. in Operations research from the University of Nigeria, Nsukka and is currently a perusing doctoral studies at the same institution. His area of interest is in operations research.


## SUGGESTED CITATION


Bassey, K.J. 2008. "Optimal Policy for Material Requirements Planning and Procurements". *Pacific Journal of Science and Technology.* 9(2):432-435.

Pacific Journal of Science and Technology