Theoretical Analysis of a Relationship between Master/Wheel Cylinder Diameter Ratio and Brake Efficiency.

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ABSTRACT

In any brake system of an automobile or passenger vehicle, an increase in Master/Wheel cylinder diameter ratio leads to an increase in brake efficiency. This paper focuses on Master/Wheel cylinder diameter effect on the brake efficiency. A VISUAL BASIC computer code was developed for computation of Master/Wheel cylinder diameter ratio corresponding to break efficiency. Aided by the developed software, the effect of increasing the Master/Wheel cylinder ratio on the brake was investigated. Results obtained for case studies examined, at varying Force on the Control pedal, showed that brake efficiency increases with increase with Master/Wheel cylinder diameter ratio. The numerical results were further processed with the EXCELL package, which yields a relationship of form:

$$\eta = a\left(\frac{M_c}{W_c}\right)^2 + b\frac{M_c}{W_c} + c$$

where a, b, and c are real constants; implying that the Master/Wheel cylinder diameter ratio has a non-linear relationship with the brake efficiency of a motor vehicle.

(Keywords: master cylinder diameter, wheel cylinder diameter, brake efficiency, force on control pedal, force on actuating mechanism, radii on pedal)

INTRODUCTION

Road accidents as a major causes of misery, morbidity, and mortality in Africa, particularly in Nigeria, have been of considerable concern to the general public; however, the interrelationship of the host (road users), agent (motor vehicles), and environment (road conditions) has not been well examined (Igoche and Kwamina, 1992), hence, the need for further studies.

Vehicles as a major agent of road transportation contribute immensely to the loss of invaluable life, goods, and properties. It worth noting that motor vehicle conditions are to a great degree, the determinant of accident occurrence. It is established that vehicles defects can made them the agent of demise on the road. These defects can include bad tires, incompatibility of vehicle parts, brake failure, absence of front lights, and malfunctioning brake lights, just to mention few (Igoche and Kwamina, 1992).

Motor vehicles are complex machines that contain various system including ignition, lubrication, brakes, steering, and suspension systems. Brake systems may be mechanically, pneumatically, or hydraulically operated. It is necessary to retard or stop the vehicle during operations. The hydraulic brake is noted for small actuating time, simple design, small overall dimensions, small mass, and low cost of procurement and maintenance. Among the major components of hydraulic brake are master cylinder, wheel cylinder, brake shoes and their mechanism, brake lines, brake pedal, and brake fluid (Salami, 1986; Dolan,1982; and Hillier and Pittuck, 1972).

Modern braking systems employ either a rotating drum or disc. The necessary friction is obtained by passing a stationary shoe or pad against the rotating member. The force felt at each shoe is governed by the piston area in the wheel cylinder; the larger the area the greater the thrust exerted on the drum or disc, hence the higher the efficiency of the brake (Hillier and Pittuck, 1972).

The purpose of wheel cylinder is to transmit the master cylinder pressure to the brake shoe and force them outward against the drum. The wheel cylinder assembly is simple in construction; it consists of cast iron housing, two aluminum pistons, two-rubber cup expander, two push rods and two rubber dust boots. In the wheel cylinder, the hydraulic pressure applied between the two piston cups forces the piston out, thus, the brake shoe actuating pins force the brake shoes into contact the brake drum.

The efficient braking of vehicle is one of the principal factors in securing their safe operation. Studies have shown that the typical brake efficiency percentage related to the condition of the brakes are as follows (Table 1).

Table 1: Percentage Efficiency Corresponding to Brake Conditions (Igoche and Kwamina, 1992).

EFFICIENCY $^0/_0$	BRAKES CONDITION
30	Minimum allowable for any vehicle.
50	Four wheel brakes in good condition.
60	Four wheel brake in very good condition.
70	Four wheel brakes in excellent condition.
80	Safest degree of efficiency.
100	Theoretical limit for brakes on all wheels.

According to Salami, 1996 and Hillier and Pittuck (1972) the stopping distance of a brake (efficiency) depends on:

1) coefficient of friction between the tire and the road,
2) coefficient of friction between braking surfaces,
3) the foot pedal pressure,
4) the piston area of the wheel cylinder,
5) and the methods of actuating the brakes.

It has been reported (Abdrahim) that the present condition of most auto assembly plants and general economy of Africa, in particular Nigeria, has made them dumping grounds for used vehicles commonly called TOKUNBO in Nigeria. Tokunbo vehicle have associated problems of various type that can cause great loss to African nations as a result of related accidents. Such problems are associated with mechanical failures (including brake failures) and overheating. At times, needs do arise for the interchange of parts, especially brake parts, to suit African conditions. This is usually done without any consideration for brake efficiency, thereby leading to mechanical failure in such vehicles.

The ratio of master cylinder to wheel cylinder diameter which has great impact on the brake efficiency, thereby determines the stopping distance of the vehicle. The impact of master cylinder ratio to wheel cylinder diameter on the brake efficiency is of paramount importance, hence the need to establish the degree of their influence(s). This will no doubt be a useful tool in generating the necessary database which could aid perfect changeability of wheel cylinder, master cylinder, and other brake parts.

The effect of the master cylinder ratio to wheel cylinder on brake efficiency is determined in this research by integrating the master cylinder/wheel cylinder diameter ratio into the general design equation for force on the control pedal of a brake (Lukin et al., 1989). The resulting expression is developed in the VISUAL BASIC computer code to generate precision values of brake efficiency corresponding to series of master cylinder/wheel cylinder diameter.

MATERIAL AND METHODS

The relationship between the wheel cylinder diameter and developed driving force is expressed as (Lukin et al., 1989):

$$W_c = \sqrt{\frac{P}{0.25\pi\, p_K}} \quad ..1$$

where,

W_c = Wheel cylinder diameter (m)

P = Driving force (N)

p_K = Fluid pressure in the cylinder with due consideration to the action of the braking force regulator.

The force on the control pedal is given as:

$$F_p = \left[\frac{r_1}{r_2}\right]\frac{\pi M_c^2 P_k}{4\eta} \quad ..2$$

where,

$F_p =$ Force on the control pedal

$\left[\frac{r_1}{r_2}\right] =$ Radii on the pedal

$M_c =$ Master cylinder diameter (m)

$\eta =$ Hydraulic brake efficiency

For hydraulic drive, the driving force at the shoes is equal to the sum of the driving force at leading shoe and at trailing shoe:

$$P = P_l + P_T$$

The driving force at leading and trailing shoes according to Banga and Sing (1986) and Champion (1986) is :

$$P_l = \frac{F_a m}{0.5m + \mu R} \quad ..3$$

and

$$P_l = \frac{F_a m}{0.5m - \mu R} \quad ..4$$

Therefore,

$$P = \frac{F_a m^2}{0.25 m^2 - \mu^2 R^2} \quad ..5$$

where,

$F_a =$ Force of actuating mechanism (N)

$m =$ Distance between anchor pin and actuating force (N)

$\mu =$ Coefficient of friction material

$R =$ Brake drum radius.

From Equation 1:

$$P_k = \frac{P}{0.25 W_c^2 \pi} \quad ..6$$

Also from Equation 2:

$$\eta = \left[\frac{r_1}{r_2}\right]\frac{\pi M_c^2 P_k}{4 F_p} \quad ..7$$

Manipulating Equations 5, 6, and 7:

$$\eta = \left[\frac{r_1}{r_2}\right]\left[\frac{M_c}{W_c}\right]^2\left[\frac{F_a m^2}{0.25 m^2 - (\mu R)^2}\right]\left[\frac{1}{F_p}\right] \quad .8$$

The ratio of master cylinder (M_c) to wheel cylinder diameter (W_c).

Force of the actuating mechanism (F_a), coefficient of friction material (μ), and force on control pedal are features of the design condition for determining the efficiency of motor vehicle brakes.

Therefore, if force on control pedal (F_p), radii of the pedal ($\frac{r_1}{r_2}$), the distance from pivot and actuating mechanism (m), coefficient of friction material (μ), force on actuating mechanism (F_a), and radius of the brake drum (R) are held constant, the effect of the ratio of master cylinder to wheel cylinder on brake efficiency of a motor vehicle can be determined from Equation (8).

Computer simulation of Equation (8) was carried out in this study. The program was developed in FORTRAN 77® and was structured in interactive data input form. The software evaluates the relationship between the ratio of master cylinder to wheel cylinder and efficiency of a motor vehicle brake.

Data generated through the program was then processed with the EXCEL® package to obtain mathematic expressions, which describe the effect of the ratio of master cylinder to wheel cylinder on the brake efficiency.

Case Study

A typical case study (A) with the following parameter was investigated:

r_1 =0.007m,

r_2 =0.03 m,

F_a =890 N,

F_p =500 N,

μ =0.25,

R =0.21319 m,

and m =0.426 m.

Keeping all other parameter constant, the ratio of the master cylinder to wheel cylinder diameter was varied between (0.83 - 1.1) m generating different brake efficiency values.

Also for case B, C, D, and E using an increment in force on control pedal at a rate of 10 N and a ratio of master cylinder to wheel cylinder variation between (0.83 –1.1)m to generate different values for brake efficiency.

RESULTS AND DISCUSSION

The results obtained are presented in Tables 2-6. From the tables, the brake efficiency increases with increase in ratio of master cylinder to wheel cylinder diameter.

Accordingly, data from the output of the computer program (above tables) were processed with the EXCEL® package to obtain mathematical expression relating brake efficiency to ratio of Master cylinder diameter with wheel cylinder diameter in an hydraulic brake system.

As evident from Figures 1 and 2, the cases considered (Case A-E) gave quantitative expression in respective order as presented below with each having R^2 value equal to 1.

Table 2: Program Output for the Variations of Master/Wheel Cylinder Diameter Ratio with Brake Efficiency Results (Force on Control Pedal = 500N).

SN	MASTER/WHEEL CYLINDER DIAMETER RATIO	BRAKE EFFICIENCY
1	0.83	1.9745
2	0.857	2.105
3	0.884	2.2397
4	0.911	2.3786
5	0.938	2.5217
6	0.965	2,669
7	0.992	2.8204
8	1019	2.976
9	1.046	3.1358
10	1.073	3.2998
11	1.1	3.468

Table 3: Program Output for the Variations of Master/Wheel Cylinder Diameter Ratio with Brake Efficiency Results (Force on Control Pedal = 510N).

SN	MASTER/WHEEL CYLINDER DIAMETER RATIO	BRAKE EFFICIENCY
1	0.83	1.9357
2	0.857	2.0637
3	0.884	2.1958
4	0.911	2.332
5	0.938	2.4723
6	0.965	2,6166
7	0.992	2.7651
8	1019	2.9177
9	1.046	3.0743
10	1.073	3.2351
11	1.1	3.40

Table 4: Program Output for the Variations of Master/Wheel Cylinder Diameter Ratio with Brake Efficiency Results (Force on Control Pedal = 520N).

SN	MASTER/WHEEL CYLINDER DIAMETER RATIO	BRAKE EFFICIENCY
1	0.83	1.8985
2	0.857	2.024
3	0.884	2.1536
4	0.911	2.2871
5	0.938	2.4247
6	0.965	2,5663
7	0.992	2.7119
8	1019	2.8616
9	1.046	3.0152
10	1.073	3.1729
11	1.1	3.3346

Table 5: Program Output for the Variations of Master/Wheel Cylinder Diameter Ratio with Brake Efficiency Results (Force on Control Pedal = 530N).

SN	MASTER/WHEEL CYLINDER DIAMETER RATIO	BRAKE EFFICIENCY
1	0.83	1.8627
2	0.857	1.9858
3	0.884	2.1129
4	0.911	2.244
5	0.938	2.379
6	0.965	2,5179
7	0.992	2.6608
8	1019	2.8076
9	1.046	2.9583
10	1.073	3.113
11	1.1	3.2717

Table 6: Program Output for the Variations of Master/Wheel Cylinder Diameter Ratio with Brake Efficiency Results (Force on Control Pedal = 540N).

SN	MASTER/WHEEL CYLINDER DIAMETER RATIO	BRAKE EFFICIENCY
1	0.83	1.8282
2	0.857	1.9491
3	0.884	2.0738
4	0.911	2.2024
5	0.938	2.3349
6	0.965	2,4713
7	0.992	2.6115
8	1019	2.7556
9	1.046	2.9036
10	1.073	3.0554
11	1.1	3.2111

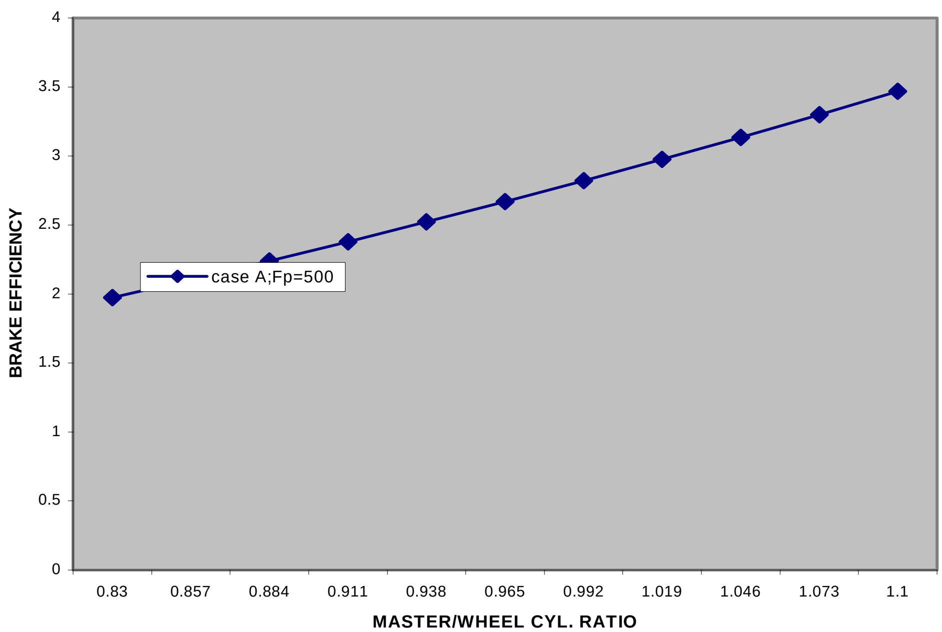

Figure 1: Master/Wheel Cylinder Diameter Ratio Variation with Brake Efficiency.

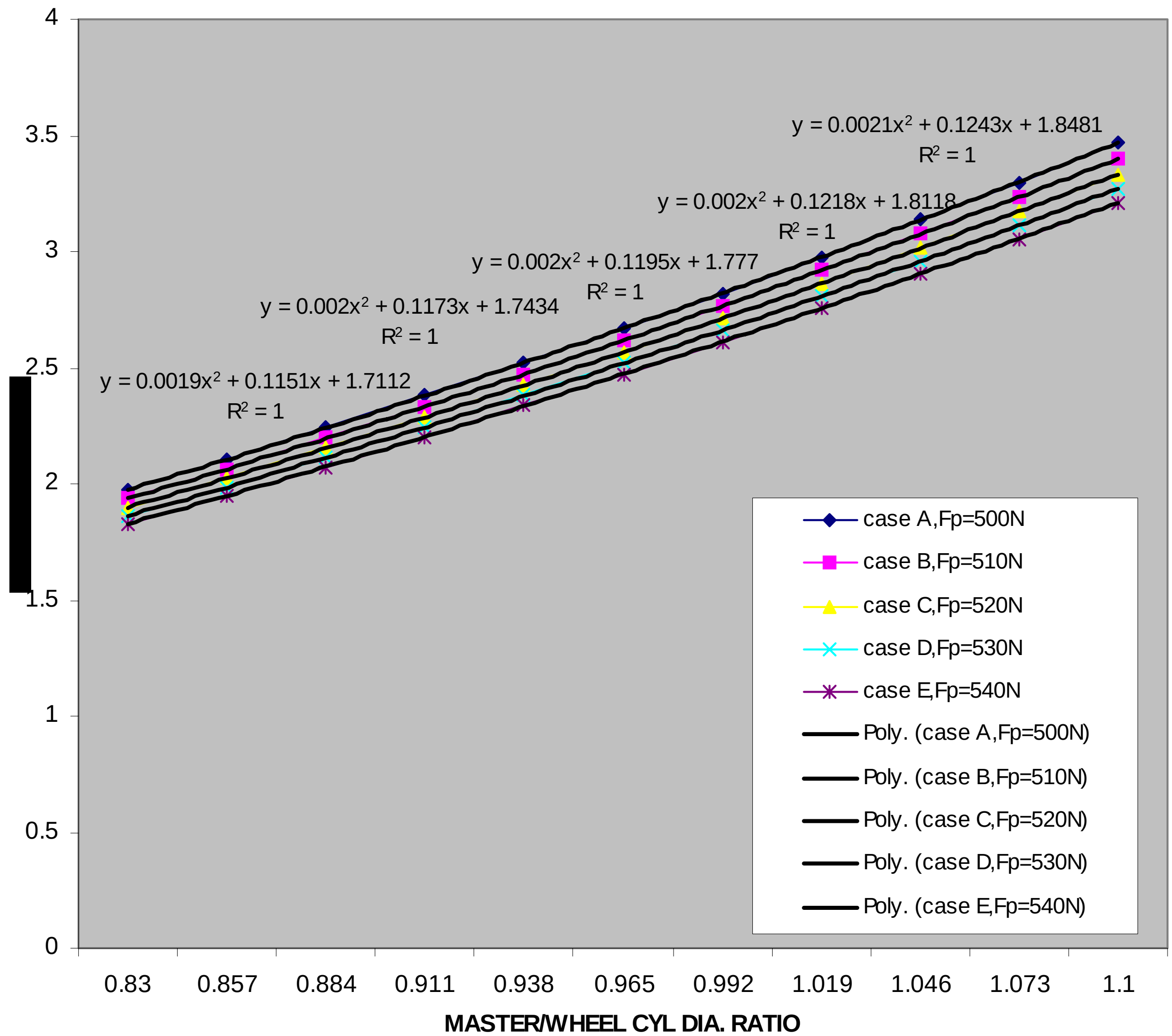

Figure 2: Master/Wheel Cylinder Diameter Ratio Variation with Brake Efficiency at Different Fp Values.

$$\eta = 0.0021\left(\frac{M_c}{W_c}\right)^2 + 0.1243\frac{M_c}{W_c} + 1.8481$$

$$\eta = 0.002\left(\frac{M_c}{W_c}\right)^2 + 0.1218\frac{M_c}{W_c} + 1.8118$$

$$\eta = 0.002\left(\frac{M_c}{W_c}\right)^2 + 0.1195\frac{M_c}{W_c} + 1.777$$

$$\eta = 0.002\left(\frac{M_c}{W_c}\right)^2 + 0.117\frac{M_c}{W_c} + 1.7434$$

$$\eta = 0.0019\left(\frac{M_c}{W_c}\right)^2 + 0.1151\chi + 1.7112$$

$$\eta = a\left(\frac{M_c}{W_c}\right)^2 + b\frac{M_c}{W_c} + c$$

Generally, these relationships are non-linear (quadratic) and of the form:

$$\eta = a\left(\frac{M_c}{W_c}\right)^2 + b\frac{M_c}{W_c} + c$$

where a, b and c are real characteristic values of the particular problem of hydraulic braking system of an automobile vehicle. It can thus be deduced that the larger the ratio of master/wheel cylinder diameter, the greater the brake efficiency, in agreement with the above relationship, and the faster the rate of retarding/stopping the vehicle.

The correlations above are for the specific cases considered. An attempt to obtain a single correlation to generalize the results for the relationship between the brake efficiency and ratio of master cylinder diameter to wheel cylinder diameter using line of best fit (Figure 3) generates an approximated quadratic function:

$$\eta = 2.7571\left(\frac{M_c}{W_c}\right)^2 - 0.0023\frac{M_c}{W_c} + 0.001$$

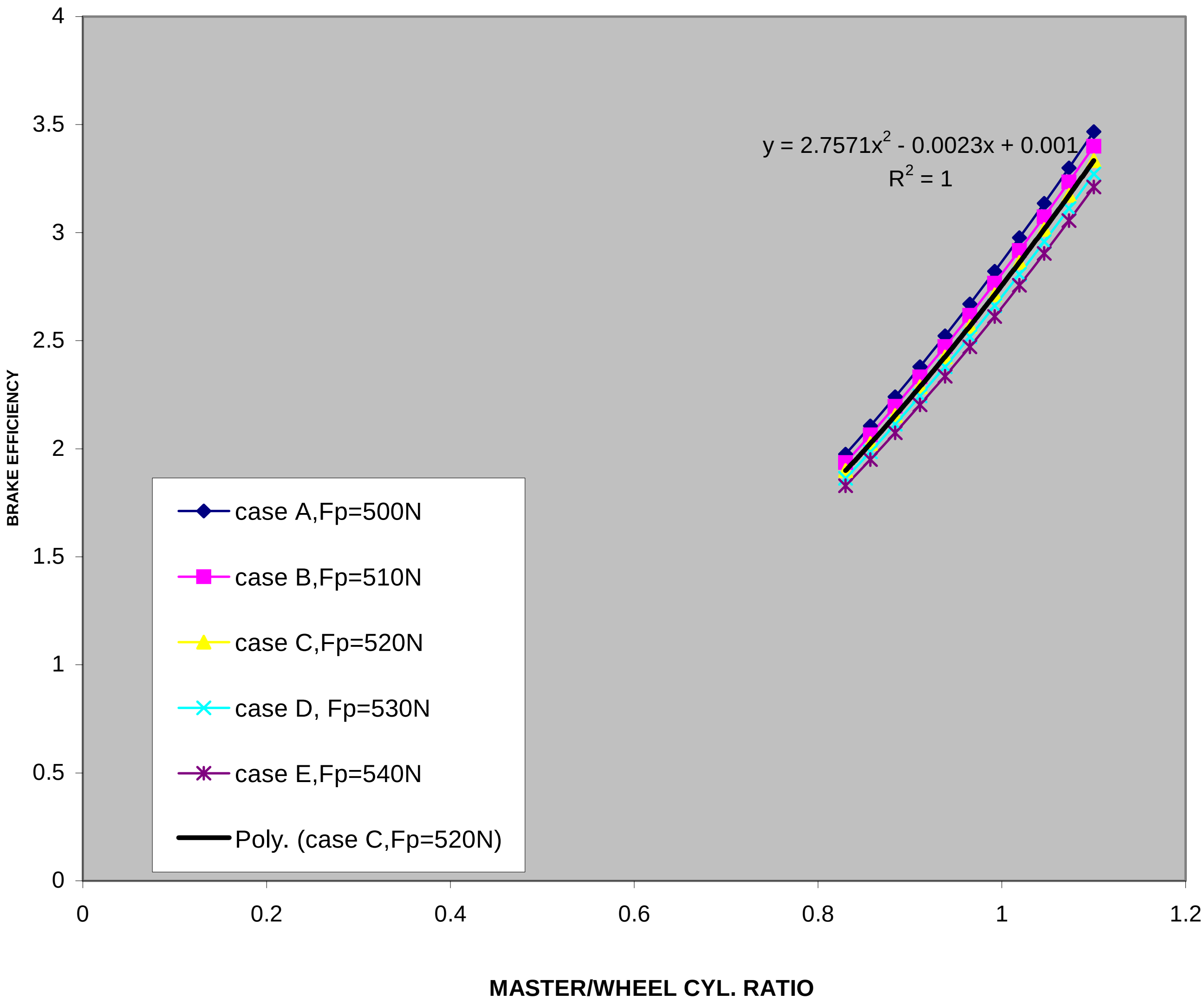

Figure 3: Scatter Diagram of a Master/Wheel Cylinder Ratio Variation against Brake Efficiency.

CONCLUSION

From the foregoing, it can be inferred that;

1) The brake efficiency increases non-linearly with an increase in the ratio of master cylinder/wheel cylinder diameter

2) The brake efficiency at constant ratio of master cylinder/wheel cylinder diameter value decreases with increases in force on the control pedal.

3) For the relationship between the ratio of the master cylinder/wheel cylinder diameter and brake efficiency:

$$\eta = a\left(\frac{M_c}{W_c}\right)^2 + b\frac{M_c}{W_c} + c.$$

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