Solving Electrical Circuits Transient Problems with MATLAB® and SIMPLORER®.

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ABSTRACT

In practical applications, the desired performance characteristics of systems are specified in terms of time-domain quantities; systems with energy storage elements cannot respond instantaneously and will exhibit transient response whenever they are suddenly subjected to inputs or disturbances. This paper adopted a procedural approach to circuit analysis in arriving at the state equations of a sample complex circuit. The state equations were programmed in MATLAB® to generate transient responses. The sample circuit was simulated using SIMPLORER® as a comparative alternative. The simulation results will be very useful to system engineers in studying the transient behaviors of electrical circuits.

(Keywords: A.C., alternating current, MATLAB®, SIMPLORER®, transient, circuit, modeling)

INTRODUCTION

Transient analysis of A.C. circuits is probably one of the most difficult areas of interest in electrical engineering and, by its nature, involves higher mathematical content when compared to other areas of electrical engineering. The transient analysis is used to describe the circuit behavior as a function of time before steady state is achieved. This paper is aimed at showing, with the aid of MATLAB® and SIMPLORER®, how the transient behaviors of typical electrical circuits can be obtained with relative ease.

MATLAB®, an acronym for 'Matrix Laboratory' is a product of Math Works, Inc. It is a scientific software package designed to provide integrated numeric computation and graphic visualization in high level programming language [1]. The combination of analysis capabilities, flexibility, reliability, and powerful graphics makes MATLAB® the premier software for engineers and scientists. MATLAB® is a high-level technical computing language and interactive environment for algorithm development, data visualization, data analysis, and numeric computation. Using the MATLAB® product, one can solve technical computing problems faster than with traditional programming languages, such as C, C++, and FORTRAN [2].

SIMPLORER®, a trademark of Ansoft Corporation, is a simulation package for electric circuit simulations that allows one to easily and quickly model all components of an application. Engineers can design and model with electric and electronic facts; control and mechanical components; discontinuous processes; and controls with electric circuits, block diagrams, and state graph components [3].

MODELING WITH MATLAB® AND SIMPLORER®

Modeling with MATLAB®

The task of modeling the transient behaviors of electrical circuits in MATLAB® entails developing the analysis equations in state space form. The electrical circuit to be analyzed is represented in differential equations formed with the aid of Kirchoff's Voltage and Current Laws as well as other associated relationships. State Space representation allows an n^{th} order, continuous system to be represented by a set of n-simultaneous, first–order differential equations [4, 5] as shown in Equation 1.

$$\dot{x} = Ax + Bu \qquad (1)$$

where:
x=State Variable Vector
u=Input Vector
A=Coefficient Matrix or System Matrix
B=Input or Control or Driving Matrix

Modeling with SIMPLORER®

The task of simulation using SIMPLORER® involves creation of project using SSC Commander, creation of model using the graphical input tool schematic or the SIMPLORER® text editor, evaluation and analysis of result using the simulator data and Day Post Processor applications respectively [3].

MATHEMATICAL DESCRIPTION

For the purpose of transient analysis using MATLAB®, a procedural approach is used to mathematically describe Figure 1. The resulting differential equations are presented in State Space form suitable for MATLAB® programming.

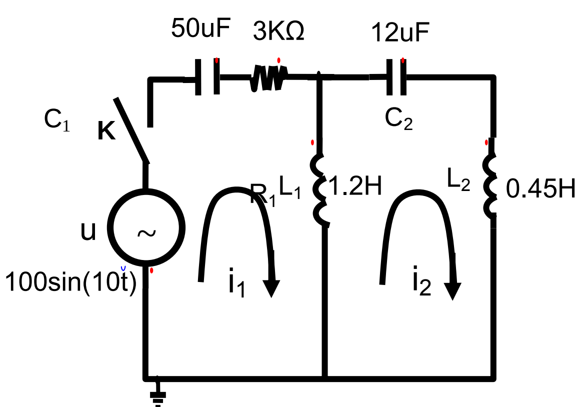

Figure 1: A Sample A.C. Circuit.

Taking KVL around loop 1 :

$$u - \frac{1}{C_1}\int i_1(t) - i_1 R_1 - L_1 \frac{d}{dt}(i_1 - i_2) = 0$$

$$u - \frac{1}{C_1}\int i_1(t) - i_1 R_1 - L_1 \frac{di_1}{dt} + L_1 \frac{di_2}{dt} = 0$$

Since, $Current(i) = \frac{Charge(Q)}{Time(t)}$

$$i = \frac{dQ}{dt};$$

$$\frac{di}{dt} = \frac{d^2Q}{dt^2};$$

$$Q = \int i\ (t)$$

We assumed that all currents and charges are zero at the instant of closure of switch, K. Therefore,

$$u - \frac{Q_1}{C_1} - i_1 R_1 - L_1 \frac{d^2Q_1}{dt^2} + L_1 \frac{d^2Q_2}{dt^2} = 0$$

Taking KVL around loop 2,

$$-L_1 \frac{d}{dt}(i_2 - i_1) - \frac{1}{C_2}\int i_2(t) - L_2 \frac{di_2}{dt} = 0 \qquad \textbf{(2)}$$

$$-L_1 \frac{di_2}{dt} + L_1 \frac{di_1}{dt} - \frac{1}{C_2}\int i_2(t) - L_2 \frac{di_2}{dt} = 0$$

$$-L_1 \frac{d^2Q_2}{dt^2} + L_1 \frac{d^2Q_1}{dt^2} - \frac{Q_2}{C_2} - L_2 \frac{d^2Q_2}{dt^2} = 0 \qquad \textbf{(3)}$$

Let

$$x_1 = Q_1$$

$$x_2 = \dot{Q}_1$$

$$\dot{x}_2 = \ddot{Q}_1$$

$$\dot{x}_1 = \dot{Q}_1 = x_2 \qquad \textbf{(4)}$$

Also, let

$$x_3 = Q_2$$

$$x_4 = \dot{Q}_2$$

$$\dot{x}_4 = \ddot{Q}_2$$

$$\dot{x}_3 = \dot{Q}_2 = x_4 \qquad \textbf{(5)}$$

Working in terms of x, Equations 2 and 3 become:

$$u - \frac{1}{C_1} x_1 - R_1 x_2 - L_1 \dot{x}_2 + L_1 \dot{x}_4 = 0 \qquad \textbf{(6)}$$

$$-L_1 \dot{x}_4 + L_1 \dot{x}_2 - \frac{1}{C_2} x_3 - L_2 \dot{x}_4 = 0 \qquad \textbf{(7)}$$

From Equation 6

$$\dot{x}_2 = \frac{u}{L_1} - \frac{1}{L_1 C_1} x_1 - \frac{R_1}{L_1} x_2 + \dot{x}_4 \qquad \textbf{(8)}$$

From Equation 7

$$\dot{x}_2 = \dot{x}_4 + \frac{1}{L_1 C_2} x_3 + \frac{L_2}{L_1} \dot{x}_4 \qquad \textbf{(9)}$$

Equating equation 8 and 9,

$$\frac{u}{L_1} - \frac{1}{L_1 C_1} x_1 - \frac{R_1}{L_1} x_2 + \dot{x}_4 = \dot{x}_4 + \frac{1}{L_1 C_2} x_3 + \frac{L_2}{L_1} \dot{x}_4$$

$$\frac{L_2}{L_1} \dot{x}_4 = \frac{u}{L_1} - \frac{1}{L_1 C_1} x_1 - \frac{R_1}{L_1} x_2 - \frac{1}{L_1 C_2} x_3$$

$$\dot{x}_4 = \frac{u}{L_2} - \frac{1}{L_2 C_1} x_1 - \frac{R_1}{L_2} x_2 - \frac{1}{L_2 C_2} x_3 \qquad \textbf{(10)}$$

Substituting Equation 10 into Equation 6, we have:

$$u - \frac{1}{C_1} x_1 - R_1 x_2 - L_1 \dot{x}_2 + L_1 \left[\frac{u}{L_2} - \frac{1}{L_2 C_1} x_1 - \frac{R_1}{L_2} x_2 - \frac{1}{L_2 C_2} x_3 \right] = 0$$

$$u - \frac{1}{C_1} x_1 - R_1 x_2 - L_1 \dot{x}_2 + \frac{L_1}{L_2} u - \frac{L_1}{L_2 C_1} x_1 - \frac{L_1 R_1}{L_2} x_2 - \frac{L_1}{L_2 C_2} x_3 = 0$$

$$\dot{x}_2 = \frac{1}{L_1} u - \frac{1}{L_1 C_1} x_1 - \frac{R_1}{L_1} x_2 + \frac{1}{L_2} u - \frac{1}{L_2 C_1} x_1 - \frac{R_1}{L_2} x_2 - \frac{1}{L_2 C_2} x_3 = 0$$

Collecting like terms the following:

$$\dot{x}_2 = \left[\frac{L_2 + L_1}{L_1 L_2} \right] u + \left[\frac{-L_2 - L_1}{L_1 L_2 C_!} \right] x_1 + \left[\frac{-R_1 L_2 - R_1 L_1}{L_1 L_2} \right] x_2 - \frac{1}{L_2 C_2} x_3 \qquad \textbf{(11)}$$

Finally, putting Equations 4, 5, 10, 11 in State Space form:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-L_2 - L_1}{L_1 L_2 C_1} & \frac{-R_1 L_2 - R_1 L_1}{L_1 L_2} & -\frac{1}{L_2 C_2} & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{1}{L_2 C_1} & \frac{-R_1}{L_2} & -\frac{1}{L_2 C_2} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{L_2 + L_1}{L_1 L_2} \\ 0 \\ \frac{1}{L_2} \end{bmatrix} u \qquad \textbf{(12)}$$

Table 1 shows the MATLAB® Function and Calling files for Equation 12.

Table 1: MATLAB® Function and Calling files for Equation 12.

FUNCTION FILE

```
%Function file that defines equation 12
%and save as transient_analysis.m
%and represent the function as xdot=Ax+BU
function[xdot]= transient_analysis(t,x)
xdot=zeros(4,1);
x1=[x(1);x(2);x(3);x(4)];
L1=1.2;
R1=3000;
L2=0.45;
C1=50e-6;
C2=12e-6;
u=100*sin(10*t);
A=[0 1 0 0;(-L2-L1)/(L1*L2*C1) (-R1*L2-R1*L1)/(L1*L2) -1/(L2*C2) 0;0 0 0 1;-1/(L2*C1) -R1/L2 -1/(L2*C2) 0];
B=[0;(L2+L1)/(L1*L2);0;1/L2];
U=u;
xdot=A*x1+B*U;
```

CALLING FILE

```
%Calling program that evaluates equation12
%and save as transient_analysiscall
to=0;
tf=2;
tinterval=0.001;
x0=[0;0;0;0];
tspan=to:tinterval:tf;
[T,x]=ode23('transient_analysis',tspan,x0);
%Graph of loop currents against time
figure(1)
plot(T,x(:,2),'k')
grid on
title('Loop 1 Current Against Time')
xlabel('Time [sec.]')
ylabel('Current [A]')
figure(2)
plot(T,x(:,4),'k')
grid on
title(' Loop 2 Current Against Time')
xlabel('Time [sec.]')
ylabel('Current [A]')
figure(3)
plot(T,(x(:,4)-x(:,2)),'k')
grid on
title('Current through the 1.2H inductor Against Time')
xlabel('Time [sec.]')
ylabel('Current [A]')
```

RESPONSE CURVES

MATLAB® Response Curves

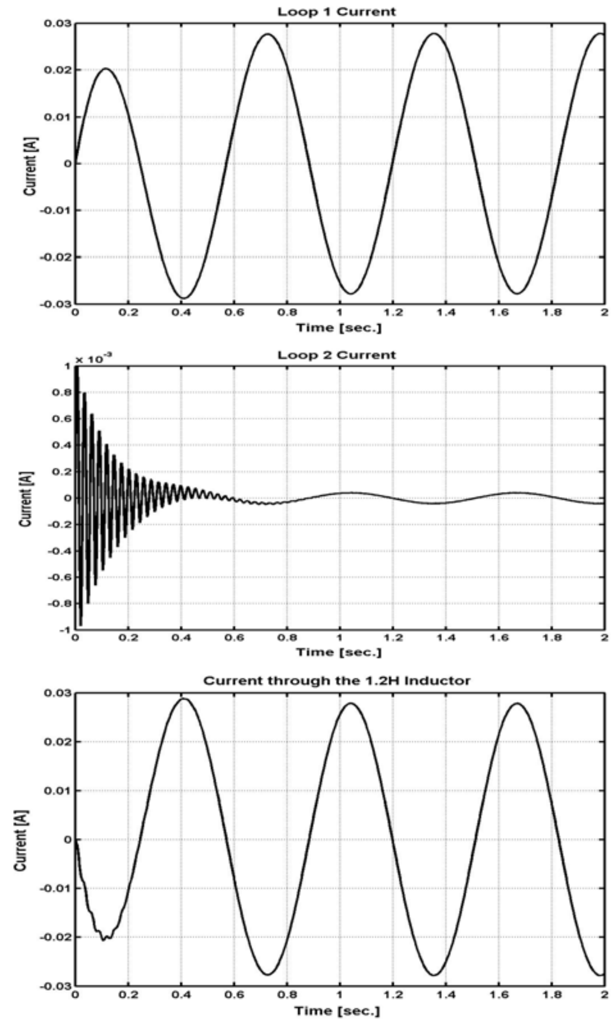

Figure 2: Circuit Response Curves using MATLAB®

SIMPLORER® Response Curves

A proper selection and initialization of the circuit components namely inductors L_1 and L_2, capacitors C_1 and C_2, resistor R_1, Alternating emf 100sin(10t) and ground is made. A firm connection is made, using the wire connector, to realize the circuit model as shown in Figure 1 (except for the switch K since simulation starts at time zero).

A 2-D Digital Graph is selected and used to initialize the relevant circuit currents through C_1 and C_2 and inductor L_1. The results are shown in Figure 3.

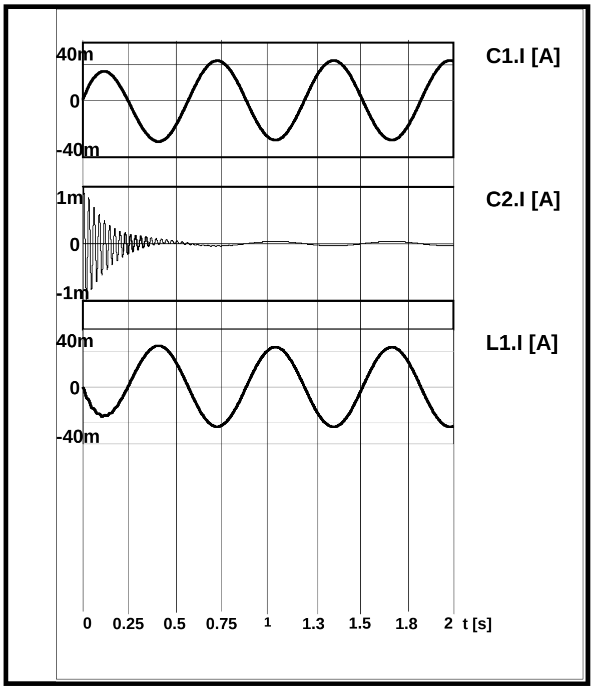

Figure 3: Circuit Response Curves using SIMPLORER®

CONCLUSION

The results have shown elegant ways of solving the problems associated with transients in electrical circuits. It is evident from the work that in addition to a good knowledge of MATLAB® programming, the MATLAB® approach requires a sound mathematical background and knowledge of the basic laws of circuit analysis.

The SIMPLORER® approach requires only a good understanding of the SIMPLORER® package, the ability to select and appropriately initialize circuit components and achieve good connections for successful simulation. Although a faster response time was achieved with the MATLAB® approach, the two approaches satisfactorily showed the transient behaviors of the A.C. circuit. It is, therefore, recommended that system engineers and students of electrical engineering who are confronted with problems involving transients in electrical circuits should utilize these modern tools in their analysis.

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