Modeling the Total Surface Area and Volume of Potholes on a Length of Road

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ABSTRACT

Potholes are essentials in minor road repairs. Information about their total surface area and volume are useful in predicting the time spent on road repairs, which involve light maintenance practices. In addition, cost information may be computed using these data. This paper presents a mathematical model for evaluating the total surface area and volume of potholes on a length of road. The problem of potholes is approached from different directions. One perspective relates to the wear caused by the activities of cars on the surface of roads whereby the tires of the cars are assumed to be rubbing the surface of the road as it moves along the road. The other direction is the effects of erosion by water of the bitumen in areas where cracks and depressions occur. A third direction is in form of probabilities of occurrence on the road with average (assume) radius for the potholes. Case studies are developed which illustrates the practical significance f the model formulated. It is envisaged that the work would benefit the stakeholders in road management.

(Keywords: area, volume, total area, surface of a 3-D object, three dimensional geometry, road engineering, maintenance, repair)

INTRODUCTION

Transportation modes are of three major categories: land, air, and water. However, road transport, which is a significant part of land transportation, is of interest to us in this paper. The road on which vehicles travel is of primary interest to road stakeholders, particularly the government. Many roads have potholes, requiring light maintenance practices to restore them to an acceptable condition. Unfortunately, scientific estimation of the total surface area and volume of potholes on a length of a road is a major problem. This has deprived road managers of adequate planning since they cannot ascertain the amount of materials and cost of the repairs for such potholes. This paper attempts to model the total volume, surface area and cost of refilling of potholes along roads in the Nigerian environment considering factors such as amount of rainfall, vehicular speed, mass of vehicles (Mabsout et al., 2004).

It is assumed that water is the only major natural source of pothole propagation on the road. The motion of vehicles on the roads especially the rubbing of the car tires on the surface of the road causes wear and the weight of vehicles cause depressions on the road especially if the speed of travel of such vehicles are not that much and if such vehicles are heavy vehicles. The depressions and cracks which are caused by such vehicles then allow for the gathering of water in them causing dissolution and wearing away of the bitumen creating a pothole (Singh, 2007). The paper attempts to incorporate most of these factors into this model. From the literature research by Mabsout (2004) and Singh (2007) provides a strong support for the study. Mabsout et al. (2004) presents a parametric study of the effect of potholes on load distribution in reinforced concrete slab bridges using finite-element analysis (FEA).

Consideration was given to the pothole location. It was reported that the presence of one or two potholes in the first two lanes created at the midspan of a bridge increased the maximum longitudinal bending moment by about 25% when compared with FEA results of slabs with no potholes, and by as much as 70% when compared with the AASHTO standard specifications. Singh (2007) collected data of 34 cases of pot-hole subsidence from Son-Mahanadi Master Coal Basin and analyzed from stand point of their causative factors, prediction, mechanism, and suggested mitigation measures. The results show that the risk of pothole subsidence is high when the pothole potential ratings of causative factors are between 80 and 100. The structure of the paper is as follows. The introduction provides a strong motivation for the study and its justification. Subsequent sections will present the methodology of the study; discuss case study application of the problem in real life, and also present a discussion of results. Concluding remarks are given in the final section of our paper.

METHODOLOGY

The problem of potholes will be approached from different directions one will be by wear caused by the activities of cars on its surface whereby the tires of the cars are assumed to be rubbing the surface of the road as it moves along the road. And the other will be the effects of erosion by water of the bitumen in areas where cracks and depressions occur. Another way will be in the form of probabilities of occurrence on the road with average (assume) radius for the potholes.

Definition of Terms

V_c	velocity of cars
M_c	mass of car
a	acceleration of car
F_b	the bonding force of bitumen
F_c	force from the cars tires
B.E	bonding energy, mc^2
K.E	kinetic energy, $\frac{1}{2}M_cV_c^2$
C	speed of light
m_p	mass of one particle
n_p	number of particles
n_p x	surface area of particle
m_c	mass of tire
r_c	radius of tire
ω	angular velocity of tire
m	total mass of particles displaced
v	critical velocity at which the particle
of	bitumen are detached
B	breadth of the tire
r	radius of the tire
X	distance traveled by the cars
	Since total surface worn
	SA in 1 rev. $\times$ number of revolutions
	No. of rev. total distance $\div$ 2π radius
F	force
A	area
l	length
δ	extension
E	modulus of elasticity/rigidity
ε	longitudinal strain
ν	Poisson ratio
$a_1b_1c_1$	dimensions of a representative cube
W_t	weight of trucks
f_t	frequency of trucks
n_t	number f trucks (total)
Σ	summation
$\overline{W}_c$	mean weight of cars
W_c	weight of cars
f_c	frequency of cars
n_c	total number of cars
V_t	velocity of trucks

Assuming that as a car moves along the road, as in Figure 1:

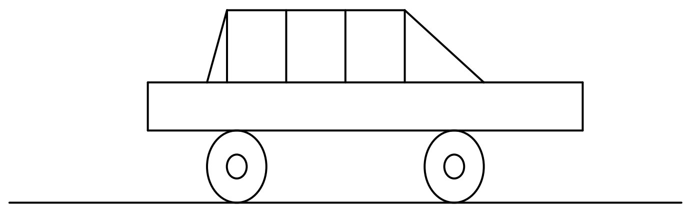

Figure 1: Car Movement Along the Road.

The tires not only rotate but also translates due only to a gain in momentum, as in Figure 2:

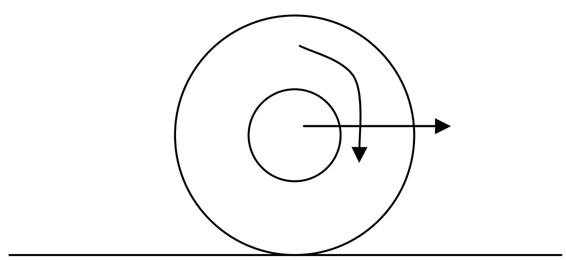

Figure 2: Interaction of the Wheel and Tire with the Surface of the Road.

As the car moves, the tire not only uses the friction between it and the road to move but also some of the friction is overcome and there is a corresponding sliding of the tire on the road.

$Fr = \mu R_N$: Sliding occurs when $M_c a_c > Fr$

Assuming also that for motion of the car there has to be friction between the tire and the road surface.

As the tire rotates we assume that some particles of bitumen are displaced or uprooted due to the motion of the tire (Figure 3).

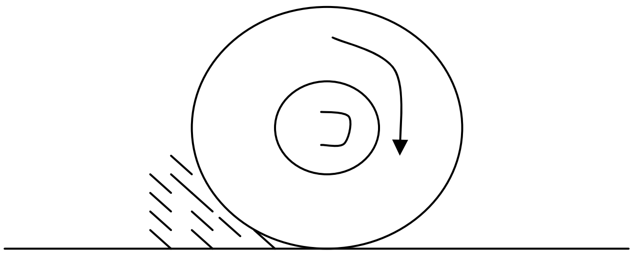

Figure 3: Interaction of the Tire/Wheel with the Road while in Motion.

That is a form of relation between the rotation of the tire and the displacement of the particles. To develop the first, taking a view at the particulate level (Figure 4):

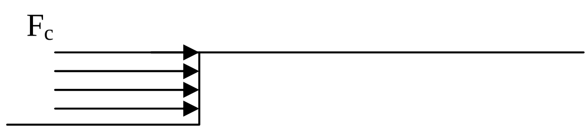

Figure 4: Force Analysis at the Road-Tire/Wheel Interaction.

The tiny layer of bitumen will detach itself when $F_c > F_b$ or more, appropriately when K.E > B.E, i.e. $\frac{1}{2}M_cV_c^2 = \Delta mc^2$, where M_c, V_c and C are constant, $\Delta m = \frac{\frac{1}{2}M_cV_c^2}{C^2}$.

For the second assumption that some particles are displaced as the tire rotates, we assume conservation of energy, i.e. $\frac{1}{2} I\omega^2 = \frac{1}{2} mV^2$, where

$I = m_c r_c^2$, then $\frac{m}{m_p} = n_p$.

The total surface area through which the energy is transmitted to the road is A = 2πrB × 4. For cars with different tire radii and breadth, the total surface area worn by these cars will come into play:

$$\sum S_A = 4 \sum_{i=1}^{b=B_{mas}} 2\pi b_1 r + 2\pi b_2 r + 2\pi b_3 r + \ldots + 2\pi b_n r$$

when r is constant,

$$\sum S_A = 4 \sum_{i=1}^{r=R_{mas}} 2\pi r_1 b + 2\pi r_2 b + 2\pi r_3 b + \ldots + 2\pi r_n b,$$

where b is constant.

This gives a double integral when merged because of the fact that there are two variables $S_A = 4\int_{B_{min}}^{B_{max}} \int_{R_{min}}^{R_{max}} 2\pi rb \, dr \, db$ in one rotation of the tires.

Multiplying the result of the integral by $X = \int^{X_{max}} dx$. Therefore total radius = $\int_{r_{min}}^{r_{max}} dr$ of the tires.

Total number of revolutions = $\dfrac{\int^{x_{max}} dx}{2\pi \int_{r_{min}}^{r_{max}} dr}$

$$= \frac{4\int_{B_{min}}^{B_{max}} \int_{R_{min}}^{R_{max}} 2\pi rb \, dr \, db \int^{x_{max}} dx}{2\pi \int_{r_{min}}^{r_{max}} dr}$$

$$= \frac{4\int_{B_{min}}^{B_{max}} \int_{R_{min}}^{R_{max}} rb \, dr \, db \int^{x_{max}} dx}{\int_{r_{min}}^{r_{max}} dr}$$

$$= \frac{4\left[\left(R_{max} - R_{min}\right)\left(B_{max} - B_{min}\right)\right] \times \left(x_{max}\right)}{\left(r_{max} - r_{min}\right)}$$

$= 4(B_{max} - B_{min}) \times x_{max}$

Considering the effects of the weight of a vehicle on the bitumen on the road, we examine the illustration of Figure 5.

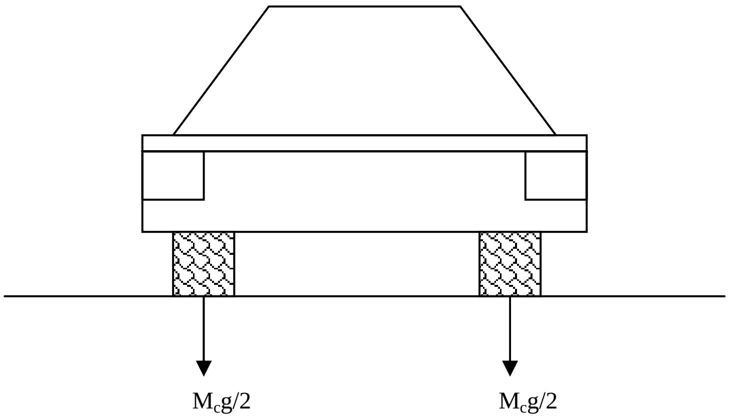

Figure 5: Force Analysis at of the Vehicle-Road Interaction.

From Figure 6, it can be seen that the bitumen experiences compressive stress in the form of Force (weight)/Area:

Since $$\frac{\text{stress}}{\text{strain}} = E = \frac{F}{A} \times \frac{l}{\delta} = E$$

because all solid bodies have a form of elasticity before fracture.

If the stress >>> E, there will be a relatively large strain which will cause a change in volume of the bitumen such that:

$V_o = a_1b_1c_1$

$V_f = a_1b_1c_1\,(1 + \varepsilon)\,(1 - \nu\varepsilon)\,(1 - \nu\varepsilon)$
$= a_1b_1c_1\,(1 - \nu\varepsilon + \varepsilon - \nu\varepsilon^2)\,(1 - \nu\varepsilon)$
$= a_1b_1c_1\,(1 - \nu\varepsilon - \nu\varepsilon + \nu^2\varepsilon^2 + \varepsilon - \nu\varepsilon^2 - \nu\varepsilon^2 + \nu^2\varepsilon^3)$

disregarding all multiples with powers: $a_1b_1c_1\,(1 - \nu\varepsilon - \nu\varepsilon + \varepsilon) = a_1b_1c_1\,(1 + \varepsilon - 2\nu\varepsilon)$. When the elastic limit of the bitumen has been exceeded cracks and faults emerge on the surface of the road. Assuming an exponential relationship between the weight of the vehicle and the speed of the vehicle, i.e. $W = W_{max}\,e^{-v/10}$ such that when $v = 0$, $W = W_{max}$ and when $V = V_{max}$, $W = W_m$

in

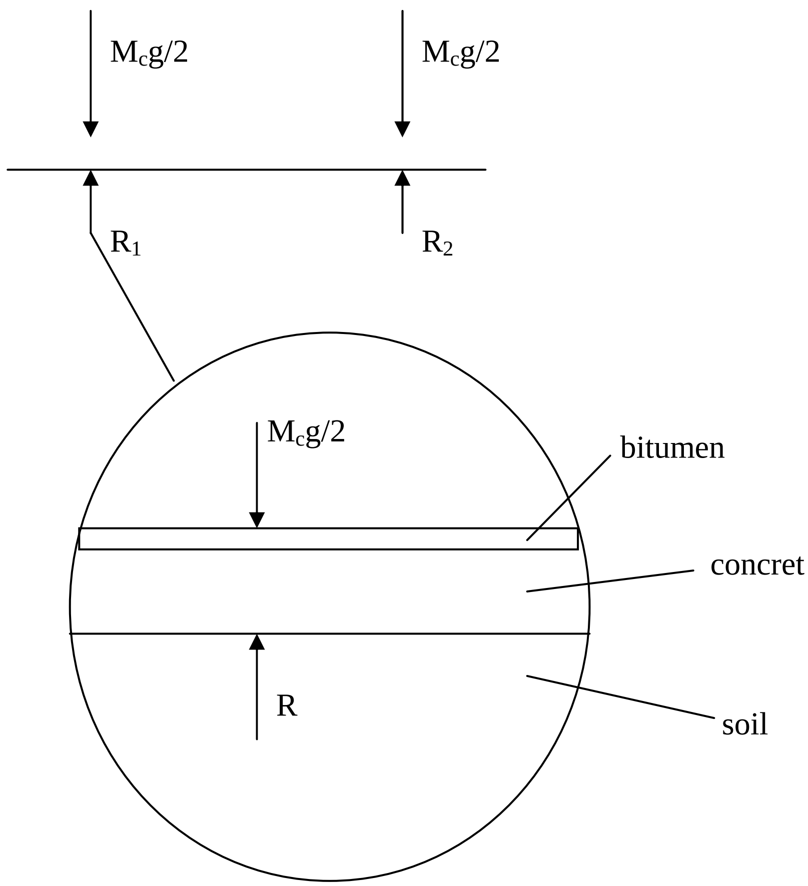

Figure 6: Further Analysis of the Vehicle-Road Interaction.

From this relationship, it can be seen that when fully loaded trucks play a road at very slow speeds there is the tendency that that road will get bad fast if they are not reinforced. Let us assume that a wide range of vehicles ply a certain road with a wide range of weights and at varying velocities with different frequencies on a continuous basis.

The average or mean weight of the trucks = $\frac{\sum W_t f}{n_t} = \frac{\sum W_t f_t}{\sum f_t}$, while the mean weight of cars, $\overline{W}_c = \frac{\sum W_c f_c}{n_c} = \frac{\sum W_c f_c}{\sum f_c}$, and the mean velocity of trucks, $\overline{V}_t = \frac{\sum V_t f_t}{n_t} = \frac{\sum V_t f_t}{\sum f_t}$. Also, the mean velocity of cars, $\overline{V}_c = \frac{\sum V_c f_c}{n_c} = \frac{\sum V_c f_c}{\sum f_c}$.

Getting these values and using them with the assumed relationship between velocity and effective weight of both the cars and the trucks the tendency of the road to develop potholes can be described (Nelkon and Parker, 1964). Using Fick's law of diffusion, the process of disintegration of bitumen can be explained: $\frac{dc}{dt}$ = k(a – c), C(0) = C_o

where C_o = concentration of the liquid initially

$C_o < a$

a = concentration of the diffusing body

so that a – c(t) = (a – c(0))e^{-kt}. For the dissolved particles of bitumen in moving water, to describe their relative movement from the environment, continuity equations for three-dimensional flow using Cartesian coordinates will be ideal (i.e. assuming a control volume ABCDEFGH) (Stroud, 2001; Stroud and Booth, 2003).

Figure 7 is taken in the form of a rectangular prism of sides δx, δy, δz in the x, y, z directions respectively. While the average values of the velocities in these directions as Vx, Vy, Vz. Mass inflow through ABCD in unit time = ρVxδyδz.

Taking a general case where mass density ρ and velocity Vx will change in the x direction, the following equations apply:

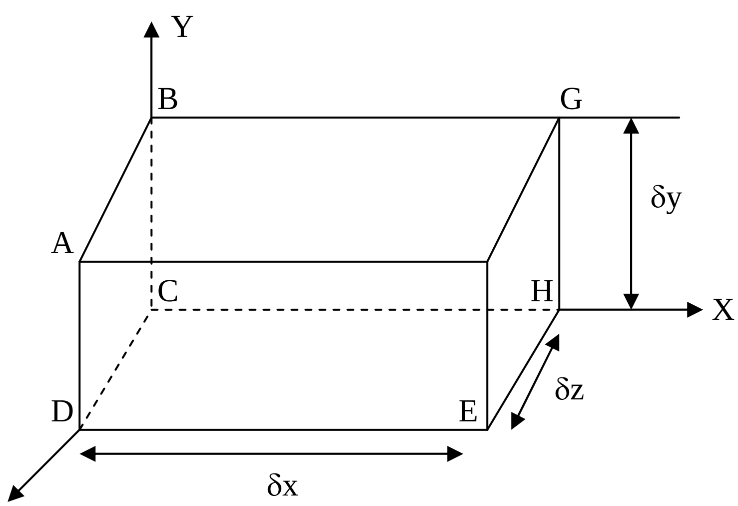

Figure 7: Control Volume.

Mass outflow through EFGH in unit time = $\left(\rho\mathrm{Vx}+\frac{\partial}{\partial \mathrm{x}}(\rho\mathrm{Vx})\delta\mathrm{x}\right)\delta\mathrm{y}\delta\mathrm{z}$

Thus, net outflow in unit time in x direction = $\frac{\partial}{\partial \mathrm{x}}(\rho\mathrm{Vx})\delta\mathrm{x}\delta\mathrm{y}\delta\mathrm{z}$

Similarly, net outflow in unit time in y direction = $\frac{\partial}{\partial \mathrm{y}}(\rho\mathrm{Vy})\delta\mathrm{x}\delta\mathrm{y}\delta\mathrm{z}$

Net outflow in unit time in z direction = $\frac{\partial}{\partial \mathrm{z}}(\rho\mathrm{Vz})\delta\mathrm{x}\delta\mathrm{y}\delta\mathrm{z}$

Therefore, total net outflow in unit time = $\left(\frac{\partial}{\partial \mathrm{x}}(\rho\mathrm{Vx})+\frac{\partial}{\partial \mathrm{y}}(\rho\mathrm{Vy})+\frac{\partial}{\partial \mathrm{z}}(\rho\mathrm{Vz})\right)\delta\mathrm{x}\delta\mathrm{y}\delta\mathrm{z}$

also, since $\frac{\partial \mathrm{p}}{\partial \mathrm{t}}$ is the change in mass density per unit time.

Change of mass in control volume in unit time = $-\frac{\partial \mathrm{p}}{\partial \mathrm{t}}\delta\mathrm{x}\delta\mathrm{y}\delta\mathrm{z}$

(the negative sign indicates that a net outflow has be assumed). Then, total net outflow in unit time = change of mass in control volume in unit time.

$$\left(\frac{\partial}{\partial \mathrm{x}}(\rho\mathrm{Vx})+\frac{\partial}{\partial \mathrm{y}}(\rho\mathrm{Vy})+\frac{\partial}{\partial \mathrm{z}}(\rho\mathrm{Vz})\right)\delta\mathrm{x}\delta\mathrm{y}\delta\mathrm{z} = -\frac{\partial \mathrm{p}}{\partial \mathrm{t}}\delta\mathrm{x}\delta\mathrm{y}\delta\mathrm{z}$$

$$\frac{\partial}{\partial \mathrm{x}}(\rho\mathrm{Vx})+\frac{\partial}{\partial \mathrm{y}}(\rho\mathrm{Vy})+\frac{\partial}{\partial \mathrm{z}}(\rho\mathrm{Vz}) = -\frac{\partial \mathrm{p}}{\partial \mathrm{t}}$$

For incompressible fluids like water:

$\frac{\partial(\mathrm{Vx})}{\partial \mathrm{x}}+\frac{\partial(\mathrm{Vy})}{\partial \mathrm{y}}+\frac{\partial(\mathrm{Vz})}{\partial \mathrm{z}} = 0$ (Douglas et al., 1995)

In this paper we have assumed potholes to be of a circular paraboloid shape with the formula $z = x^2 + y^2$.

Since the method of getting the surface area of a function is:

$$S = \int_s ds = \iint_R \sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\, dxdy$$

$\left(\frac{\partial z}{\partial x}\right)^2$ for a paraboloid = $(\partial x)^2$ and $\left(\frac{\partial z}{\partial y}\right)^2 = (\partial y)^2$

Therefore $\delta = \iint_R \sqrt{1+4x^2+4y^2}\, dxdy$

Converting the coordinates to polar coordinates, where:

$x = r\cos\theta$,
$y = r\sin\theta$

$x^2 + y^2 = r^2$, and $dxdy = rdrd\theta$.

Therefore $\delta = \iint_R \sqrt{1+4r^2}\, rdrd\theta$

Taking the boundary conditions of the paraboloid from $r = 0$ to $r = R$ and $\theta = 0$ to $\theta = 2\pi$,

$$\delta = \int_0^{2\pi}\int_0^{R} \sqrt{1+4r^2}\, rdrd\theta$$

$$= \int_0^{2\pi}\left[\frac{\cancel{2}}{3}\left(1+4r^2\right)^{3/2}\frac{1}{\cancel{8}_4}\right]_0^R d\theta = \frac{\pi}{6}\left(1+4R^2\right)^{3/2}$$

= surface area of a paraboloid

The approximate volume of a solid is found by having a multiple internal of:

$V = \int_{x_0}^{x_1}\int_{y_0}^{y_1}\int_{z_0}^{z_1} dz\, dy\, dx$ for Cartesian coordinates. However, $V = \int_{\theta_0}^{\theta_1}\int_{r_0}^{r_1}\int_{z_0}^{z_1} dz\, rdr\, d\theta$

for polar coordinates.

For a paraboloid $z = y^2 + x^2$, where:

$y = r \sin\theta$
$x = r \cos\theta$
$dxdy = rdr\, d\theta$
$z = r^2\sin^2\theta + r^2\cos^2\theta$
$r^2 (\sin^2\theta + \cos^2\theta) = r^2(1) = r^2$

Therefore volume of a paraboloid (circular)

$$V = \int_0^{2\pi}\int_0^{R}\int_0^{r_2} dz\, rdr\, d\theta$$

$$= \int_0^{2\pi}\int_0^{R} r^2 rdr\, d\theta$$

$$= \int_0^{2\pi}\int_0^{R} r^3 dr\, d\theta$$

$$= \int_0^{2\pi}\left[\frac{r^4}{4}\right]_0^R d\theta = \frac{R^4\pi}{2}$$

= volume of a paraboloid (circular)

Due to the many parameters or variables affecting the formation of potholes on a particular length of road, we would use probability to describe the occurrence of potholes on a road such that:

The expected number of potholes on the entire length of a road = total length of the road × probability of a pothole occurring on 1km of road, i.e. E = NX P(A),

where E is the expectation, N is the total probability space, while P(A) is the probability sample. Using an average radius for each pothole the total surface area and total volume for the potholes on the road can be deduced.

CASE STUDY AND DISCUSSION OF RESULTS

Case Study 1: On the Lagos-Ibadan expressway, it was noticed that for every 10km traveler there seemed to be between 1 to 2 potholes with an average radius of 0.62m. To find the total surface area and total volume first of all, we find the expectation of potholes on the road. Taking the length of the road as 130km a probability of a pothole as between 0.1 and 0.2. Using 0.1 the expected number of potholes = 13 potholes on the entire length of the road and = 26 potholes when using 0.2, all with an average radius of 0.62m.

Using this value we get a total surface area of $\frac{\pi}{6}\left[1+4(0.26)^2\right]^{3/2}$ = 0.537m^2 for one pothole, and $\frac{0.26^4\pi}{2}$ = 7.18 × 10^{-3}m^3 volume for one pothole.

From these figures and using the probability of 0.1, there will be a total pothole surface area of 13 × 0.537 = 6.981m^2 and a total volume of 13 × 7.18 × 10^3 = 0.093m^3.

Case Study 2: On a road of 1.5km an annual rainfall of about 100,000 litres of water falls on it and the mean speed of flow of the water off the road is 11.11ms^{-1}. On this road also is assumed that about 100,000 cars ply it annually with tires with a mean radius of 0.28m and a mass of 80kg (including the rim). The minimum tire breadth and maximum tire breadth are 0.09m and 0.12m respectively. The cars that ply this road are assumed to go at 100km/h. The road has the following properties: Its concentration is 10000kg/m^3 and a ready 900kg/m^3 dissolves annually when water arrives. The density of one particle of bitumen is about 1 × 10^{-3}kg/m^3, radius of 0.00025m and diffusion constant of 3 × 10^{-6}s^{-1}. Critical velocity of bitumen particle = 2 × 10^8.

Solution: Given that δ = 1.5km,
V_x = 100000 liters = 100m^3,
$\overline{v}$ = 11.11ms^{-1},
n_c = 100000.
$B_{max} + B_{min}$ = 0.12 + 0.09 = 0.21,
r_c = 0.28,
m_c = 80kg,
v_c = 100km/h,
w (of tires) = 99.2rad/s.
a = 10000kg/m^3,
C_o = 900kg/m^3,
g = 1 × 10^{-3}kg/m^3,
r_p = 0.00025m,
k = 3 × 10^{-6}s^{-1},
and v = 2 × 10^8.

$$\text{Surface area (total)} = \frac{80\times 0.28^2\times 99.2^2}{(2\times 10^8)^2} \times \frac{4\times(0.00025)^2}{3} + 4(0.21)\times 1500 = 1260\text{m}^2$$

Volume =

$$\frac{\frac{\left(80\times0.28^2\times99.2^2\right)}{\left(2\times10^8\right)^2}+100\times10000-\left(10000-900\right)e^{-3\times10^{-6}\times\frac{1300}{11.11}}}{1\times10^{-3}}$$

$= 90368312m^3$

Case Study 3: Total surface area worn = surface area worn by the motion of tires + surface of particles worn by erosion.

Surface area of particles = $n_p\times\frac{4}{3}\pi r_p^{\ 2}$ (using the surface area of a sphere as standard).

Therefore $\frac{m}{m_p}\times\frac{4}{3}\pi r_p^{\ 2}$, From ½ $I\omega^2$ = ½ mv^2, m = $\frac{I\omega^2}{v^2}$ and taking I as $m_c r_c^2$, m = $\frac{m_c r_c^{\ 2}\omega^2}{v^2}$.

Surface area worn by tires = $4(B_{max} - B_{min}) \times x_{max}$.

Therefore Total Surface Area = $\frac{m_c r_c^{\ 2}\omega^2}{v^2}\times\frac{4}{3}\pi r_p^{\ 2}$ $+ 4\,(B_{max} - B_{min}) \times x_{max}$

Volume of bitumen worn = $\frac{\text{total mass worn}}{\text{density of bitumen}}$

$$=\frac{\text{total mass due to wear ring} + \text{vol. of rain}\times\text{diffusion rate}}{\text{density of bitumen}}$$

$$m=\frac{I\omega^2}{v^2}=\frac{m_c r_c^{\ 2}\omega^2}{v^2}$$

Volume =

$$\frac{\frac{m_c r_c^{\ 2}\omega^2}{v^2}+\text{vol. of water}\times - a - \left(a - C_o\right)e^{-k\delta/\bar{v}}}{g}$$

Where:
n_p = number of particles,
r_p = radius of particles,
m = total mass of particles,
m_p = mass of one particle,
m_c = mass of tire,
r_c = radius of tire,
v = velocity (critical) for the removal of a particle,
B_{max} = maximum breadth of tires,
B_{min} = minimum breadth of tires,
a = concentration of diffusing bodies,
C_o = concentration of diffusing body in liquid,
K = diffusing coefficient,
δ = distance traveled,
and $\bar{v}$ = velocity of fluid (mean).

Case Study 4: The fact that potholes do not occur continuously but randomly on a stretch of road makes the problem of modeling the total surface area of potholes on a stretch of road probabilistic one. The portion on which potholes are found can be assumed to be characterized by a weak soil under layer with high seeping of groundwater into the concrete foundation of the road dissolving it. From this statement, it can be seen that the number of potholes that will occur on a road, the depth and the size in terms of radius are problems of both statistics and probability. Under this premise therefore, the modeling that will be developed will be on the dynamics of the formation of a pothole whose soil underlayer is being or has been washed away from under the roads foundation. The only causal factor for the formation of potholes that is assumed is that of water.

Assuming that the principle or steps that proceed before the appearance of a pothole on a road are these: (i) infiltration, (ii) dissolution, (iii) mass movement of soil, (iv) deflection of road under its own weight, (v) gathering of water on the surface of the road, (vi) dissolution and washing away of the bitumen.

Infiltration: Assuming that the process of infiltration is powered by both capillary action and soil water table pressure. The total height h reached by the water or head of the water is $H = H_c + H_w$,

H_c = capillary head,
H_w = water table head,

Where $H_c = 4\sigma\cos\theta/\rho gd$.

Note that:
σ = surface tension,
θ = angle of contact,
d = diameter of capillary
as $\theta \rightarrow 0$ and $d \rightarrow \delta d$,
$dH_i = 4\sigma/\rho g\delta d$,
$H_i = 4n\sigma/\rho gd$

While Hw = p/ρg; h = 4nσ/ρgd + p/ρg $= \frac{1}{\rho g}\left(\frac{n\sigma}{d} + p\right)$

The dissolution rate of any solute in a solvent is $\frac{kx(t)}{V}\left((x_o - c_o V) - x(t)\right)$, where V is the volume of the solute. Using the continuity equations, the movement of soils can be modeled. Assuming a control volume ABCDEFGH is taken in the form of a rectangular prism of sides δx, δy, δz in the x, y, z directions respectively. While the average values of the velocities in these directions are Vx, Vy, Vz.. Mass inflow through ABCD in unit time = ρVxδyδz.

Taking a general case where mass density ρ and velocity Vx will change in the α-direction. Mass outflow through EFGH in unit time = $\left(\rho V_x + \frac{\partial}{\partial x}(\rho V_x)\,\delta x\right)\delta y \delta z$.

Thus, net inflow in unit time in x-direction = $\frac{\partial}{\partial x}(\rho V_x)\,\delta x\,\delta y \delta z$. Similarly, net outflow in unit time in y-direction = $\frac{\partial}{\partial y}(\rho V_y)\,\delta x\,\delta y \delta z$. Net outflow in unit time in z-direction = $\frac{\partial}{\partial z}(\rho V_z)\,\delta x\,\delta y \delta z$.

Therefore, total net outflow in unit time = $\left(\frac{\partial}{\partial x}(\rho V_x) + \frac{\partial}{\partial y}(\rho V_y) + \frac{\partial}{\partial z}(\rho V_z)\right)\delta x\,\delta y \delta z$. Also, since $\frac{\partial \rho}{\partial t}$ is the change in mass density per unit time.

Change of mass in control volume in unit time = $-\frac{\partial \rho}{\partial t}\,\delta x\,\delta y \delta z$ (the negative sign indicates that a net outflow has been assumed). Then, total net outflow in unit time = Change mass in control volume in unit time

$$\left(\frac{\partial}{\partial x}(\rho V_x) + \frac{\partial}{\partial y}(\rho V_y) + \frac{\partial}{\partial z}(\rho V_z)\right)\delta x\,\delta y \delta z$$
$$= -\frac{\partial \rho}{\partial t}\,\delta x\,\delta y \delta z$$

Therefore $\frac{\partial}{\partial x}(\rho V_x) + \frac{\partial}{\partial y}(\rho V_y) + \frac{\partial}{\partial z}(\rho V_z) = -\frac{\partial \rho}{\partial t}$. Now for the movement of mass in water we assume a one-dimensional flow i.e. $\frac{\partial}{\partial x}(\rho V_x) = -\frac{\partial \rho}{\partial t}$.

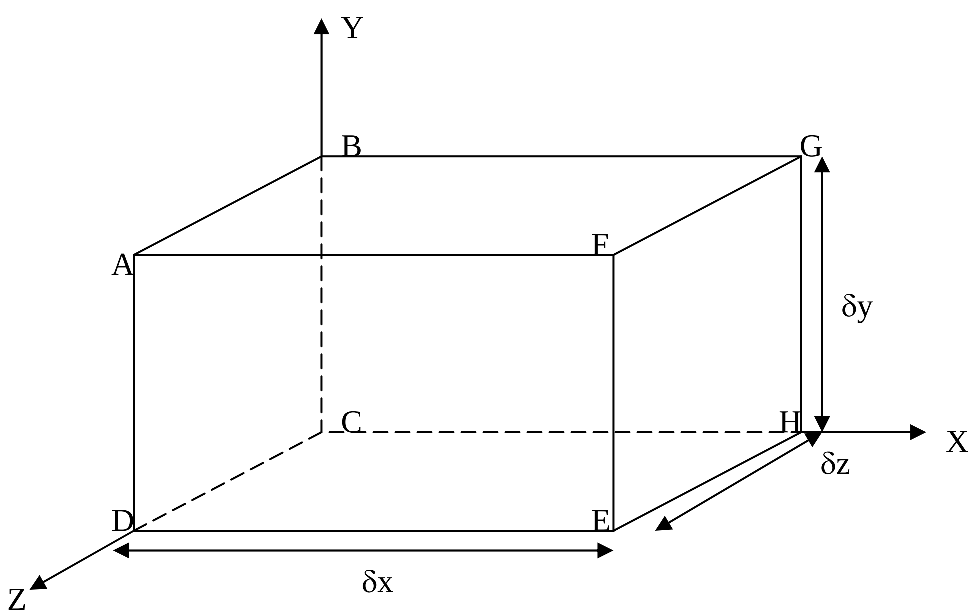

Figure 8: Further Analysis Control Volume.

Now assuming that $\frac{\partial \rho}{\partial t} = \frac{\partial n}{\partial t}$ = rate of dissolution:

$$Kn(t)\left(\frac{n_o - n(t)}{V} - c_o\right) \text{ as } \delta x \to 0$$

$$\frac{\partial (V_x)}{\partial x} \to \frac{d(V_x)}{dx}, \text{ therefore } \frac{d(V_x)}{dx} =$$

$$Kn(t)\left(\frac{n_o - n(t)}{V} - c_o\right).$$

Where:
n = mass per unit time of solute
c_o = maximum concentration of mass per unit time of solute
V = volume of solvent
K = coefficient of dissolution (or assuming diffusion)

$$\rho V_{o1} = \frac{x}{V_x} Kn(t)\left(cn_o - cn_o V\right) - n(t)$$

(integrating from 0 to x)

Assuming that each material has its own rate of dissolution, therefore $Kn(t)\left(cn_o - cn_o V\right) - n(t)$ can be represented by p (Timoshenko, 1953).

Therefore $mass = \frac{xp(t)}{V_x}$, wherex = length or radius of area, and t = time. Now due to the loss of the road under layer at a portion, that portion of road can be assumed to deflect under its own weight. Assuming the road to be a simply supported beam of span carrying a uniformly distributed load w per unit run over the whole span.

Therefore, the deflection at mid-span which is the maximum deflection available y = $\frac{5wl^4}{384EI}$, where E = the modulus of rigidity, and I = the moment of inertia of the beam/roads cross-section.

This y that has been got can be used as a value of head if the effect of pressure of a pool of water were to be considered. But since the effect of the deflection being considered is that of the cracking of the surface, it can be neglected.

Since only the effect of water in dissolving and washing away the bitumen is considered we can use the same formula derived previously for the surface removal of the bitumen i.e. $m = \frac{xpt}{V_x}$.

To get the volume obtained from the pothole, $v = \frac{m}{\rho}$, while for the total surface area.

Assuming that all the particles are spheres, therefore the surface area = $\frac{4}{3}\pi r^2 p$, where r_p = radius of particle.

Therefore total surface area = $n_p \times \frac{4}{3}\pi r^2 p$. But $n_p = \frac{m}{mp}$, where n_p = number of particles, and m_p = mass of one particle.

Assuming that different substances in whatever form have a constant diffusion rate, therefore assuming for bitumen p = 4.697 x 10^{-7}kg/sec and the radius of a particle is taken as 1mm while the mass of one particle is also taken as 0.0005g with a bitumen of density of 30,000kgm^{-s} assumed.

Therefore the formulae becomes:

$$m = \frac{x}{V_x} 4.697 \times 10^{-7}(t),$$

and $V = \frac{m}{30{,}000}$.

The surface area = $\frac{m}{0.0005} \times \frac{4}{3}\pi 0.0001^2$
= m x 8.378 x 10^{-3}.

Case Study 5: If a pothole were to occur on a portion of a road, it is desired to know the surface area and volume of bitumen that will be washed away in 30 years if the radius of the pothole would be about 0.5 m. The velocity of water that normally flow on the surface of that road due to the region it is in, is 11.11 ms^{-1}. Given that t = 30 years = 30 x 365 x 24 x 60 x 60 = 9460800005, x = 0.5m, V_x = 11.11ms^{-1}.

The mass that will be washed away is (m) = $\frac{0.5}{11.11} \times 4.697 \times 10^{-7} \times 946080000$ =19.999kg ≈ 20kg.

Now the volume is $V = \frac{20}{30{,}000} = 6.67 \times 10^{-4} m^3$ while the surface area is about 20 x 8.378 x 10^{-3} = 0.1676m^2.

With the case study just concluded, the total surface area and the total volume of potholes on a road can be effectively found, out after considering and analyzing the data for the different parameters that affect its formation.

CONCLUSION

With the increasing efforts by governments to repair roads, the issue of proper planning and budgeting has been a major challenge. Such a problem has limited the prompt repairs of potholes on roads, which require light maintenance practices. The motivation to solve this important problem has produced the current study. Consequently, a model is developed to analyze the total surface area and volume of potholes on roads. The model derived within this paper can effectively give an expected amount of potholes on any length of road.

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