Wave Propagation in Non-Homogenous Thin Films of Slowly Varying Refractive Index: W.K.B. Solution Model

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ABSTRACT

We present the study of wave propagation in non-homogeneous thin film medium with slowly varying refractive index. It is based on simple solution of the wave equation of a linearly polarized electromagnetic wave aligned in the direction using the correct boundary conditions considering both the transverse electric (T.E) mode and transverse magnetic (T.M.) mode.

Wentzel-Kramers-Brillouin (W.K.B.) solutions of incident wave of amplitude ϕ_{EO} generated at a source $0 < z < z$ was obtained and the Jeffries connection formula introduced which made possible the analysis of the reflection co-efficient of the film.

(Keywords: non-homogenous, refractive index, approximation, reflection co-efficient, amplitude, propagation, WKB, Wentzel Kramers Brillouion, transverse electric, transverse magnetic)

INTRODUCTION

The study of electromagnetic wave propagation through a spatially non-homogeneous medium is interesting and complicated (Ong 1993). Different methods have been used in studying wave propagation in a non-homogeneous medium. These include Abele's 2 x 2 matrix method, a scheme used for computation of the optical effect of layered non-homogeneous media [Abele's 1950].

Geometrical optical approximation, the phase-integral method, generalized geometrical-optic approximation, and the method of perturbation theory have been also used to study propagation in non-homogeneous media [Ong 1083]. The great advantage in studying wave propagation in a non-homogeneous medium is that it permits an understanding of unified treatment of guided and radiation modes and thus permits the propagation of a given field in rather general refractive index structures. Apart from permitting an assessment of various properties of the propagated field, the method can again be used in calculating the propagation constant [Thyle'n et al., 1992; Feit et al., 1978]. However, the concept involved in wave propagation in a non-homogeneous media usually involves formulation which has certain shortcomings [Thyle'n et al., 1983] as approximation is always involved though the approximation can be improved by introducing certain corrections and applying them to the formalism [Feit et al., 1980].

The Wentzel-Kramers-Brillouin (W.K.B.) approximation is one of those approximation techniques that can be used in the problem of wave propagation through a non-uniform medium. It is also widely used in quantum mechanics especially in scattering problems [Davydov, 1991; Fitzpatrick 2002] when a wave is propagated normally incident on an interface where the refractive index suddenly changes, there is generally significant reflection of the wave. The WKB approximation is a solution that is obtained by assuming that the incident wave propagating on a medium with slowly varying refractive index causes change in the wavelength of the propagating wave. Invariably, this change brings about a change in the amplitude of the wave or change in the phase of the wave. The amplitude of the electric field component varies inversely to $n^{½}$ ($Ø_E \propto 1/n^{½}$) whereas, the amplitude of the magnetic field component varies directly to $n^{½}$ ($Ø_{BX} \propto n^{½}$).

$$\mathbf{Ø_{Ey} \approx A\, n^{-½}}$$

We embarked on using this approximation in the study of wave propagation on thin film as it is possible here to introduce the Jeffries connection formula which enables one to compute reflection co-efficient, a concept that favours a proper understanding of the reflection properties of the thin film [Ugwu et al., 2005].

In this work we considered a slowly varying refractive index thin film medium, as a function of position (i.e., the thickness of the thin film). Obviously, though we did not explicitly treat the problem of the relative amplitude of the propagating wave, it induces a very strong reshaping of a propagated wave through such film and thus might eventually offer the opportunity of manipulating and observing their shape.

COMPUTATIONAL METHODS

The fields associated with electro magnetic wave propagating in a uniform dielectric medium E on constant refractive index satisfies:

$$\left\{\frac{\varepsilon}{c^2}\frac{\partial^2}{\partial t^2} = \nabla^2\right\} \Phi = O \quad \textbf{(1)}$$

Let us consider a non-homogeneous dielectric film as a plane dielectric layer with thickness Z' and dielectric susceptibility ε(Z),z ≤ z'. A linearly polarized E.M. wave, that we assume to propagate in the z- direction (normal incidence conditions) is described by Maxwell equations, linking the $Ø_{Ey}$ and $Ø_{Bx}$ components of the wave; the plane wave solution to the equation are respectively know for both magnetic and electric field components which are given in Equation (2) and (3) below:

$$\Phi\, E_y = \Phi E_o \exp\, i\, [k.\, r - \omega t] \quad \textbf{(2)}$$

$$\Phi\, B_x = \Phi Eo \exp\, i\, [k.r - \omega t] \quad \textbf{(3)}$$

with
$$\frac{\omega^2}{k^2} = \frac{c^2}{\varepsilon} \quad \textbf{(4)}$$

and the phase velocity of the wave given as:

$$v = \frac{\omega}{k} = \frac{c}{n} \quad \textbf{(5)}$$

where **n =**
$$\sqrt{\varepsilon} \quad \textbf{(6)}$$

$$\nabla \wedge \Phi\, E_y = íkc\, \Phi_{Bx} \quad \textbf{(7)}$$

and
$$\nabla \wedge c\, \Phi_{Bx} = íkn^2\, \Phi_{Ey} \quad \textbf{(8)}$$

n is the non-uniform refractive index of the film medium related to the dielectric medium of the thin film as in Equation (6). As the wave is assumed to be initially aligned along the 7-axis and n=n, we expect all field components to be functions of z only so that:

$$\frac{\delta}{\delta x} \equiv \frac{\delta}{\delta x} = O \quad \textbf{(9)}$$

In this case; equations 7 and 8 reduce to:

$$\Phi_{EZ} = C\Phi_{BZ} = O \quad \textbf{(10)}$$

with
$$\frac{\partial \varphi_{Ey}}{\partial z} = ikc\,\varphi_{Bx} \quad \textbf{(11)}$$

$$\frac{\partial \varphi_{Ey}}{\partial z} = ikc\,\varphi_{Ex} \quad \textbf{(12)}$$

$$\frac{\partial \varphi_{Ex}}{\partial z} = -ikn^2\,\varphi_{Ex} \quad \textbf{(13)}$$

Equations 11 to 13 are completely independent of one another and are assumed to be linearly polarized with its electric field vector parallel to the y-axis. Combining equations 11 and 12 we obtain:

$$\frac{\partial^2 \varphi_{Ey}}{\partial z^2} + k^2 n^2 \varphi_{Ey} = o \quad \textbf{(14)}$$

If the medium is a uniform one with a constant refractive index n,

$$\varphi_{Ey} = A \exp i\zeta z \quad \textbf{(15)}$$

where $\zeta = \pm$ iknz

And since n = n(z) is a slowly varying function then Equation 15 can be substituted in 14 to obtain :

$$\left(\frac{d\zeta}{dz}\right)^2 = n^2 k^2 + i\frac{d^2\zeta}{dz^2} \quad \textbf{(16)}$$

which is non.- linear differential equation and is not easy to solve without approximation. Already we have assumed that n (z) is slowly varying function, e.g.:

$$\frac{d^2 \zeta}{dz^2} = o \tag{17}$$

$$\frac{d\zeta}{dz} \approx \pm kn \tag{18}$$

provided that:

$$\frac{d^2 \zeta}{dz^2} \approx \pm K \frac{dn}{dz} \tag{19}$$

which gives the condition that the relative amplitude of the wave is less than or equal to a unity.

Another approximation to the solution is obtained when Equation 18 is substituted into Equation 16, which gives:

$$\left(\frac{dn}{dz}\right) / kn^2 << \mathit{1} \tag{20}$$

$$\frac{d\zeta}{dz} \approx Kn$$

$$\left(1 + \frac{1}{kn^2}\frac{dn}{dz}\right)^{1/2} \tag{21}$$

We take the first two terms expansion of right hand side of Equation 21 and solve it to obtain:

$$\zeta \approx \pm K \int^z ndz + i\log\left(n^{1/2}\right) \tag{22}$$

and if substituted into Equation 15 yields:

$$\varphi_{Ey} \approx A\, n^{-1/2} \exp (\pm ik \int^z \boldsymbol{n\, dz} \tag{23}$$

with a corresponding,

$$\varphi_{Bx} \approx A\, n^{½} \exp ((\pm ik \int^z \boldsymbol{n\, dz} \tag{24}$$

[Fittpatrick, 2002] for Equation 23 and 24 to be a good solution to Equation 16,

$$\frac{1}{k} / \frac{3}{4}\left(\frac{1}{n^2}\frac{dn}{dz}\right)^2 - \frac{1}{2n^3}\frac{d^2n}{dz^2} / << \mathit{1} \tag{25}$$

REFLECTION CO-EFFICIENT

The thin film of our interest is the one with a varying function of z (thickness) of the film surface bounded by air to the substrate. If the incident wave of amplitude $Ø_{E0}$ is generated at a source (z_o), then the complex amplitude of the incident wave in the region 0 < z < z' is given by W.K.B. solution as:

$$\varphi_{Ey} = \varphi_{Eo}\, n^{-1/2} \exp (ik \int^z ndz \tag{26}$$

and

$$\varphi_{Bx} = -\varphi_{Eo}\, \boldsymbol{n}^{\mathit{-1/2}} \exp (ik \int_o^z ndz \tag{27}$$

This incident wave is either absorbed or reflected in the film as it propagates through the film thickness. Since the complex amplitude of the reflected wave in the region 0 < z < z' is give in Equations 29 and 30, the W.K.B. solution, is then:

$$\varphi_{Ey} = \varphi_{Eo}\, \boldsymbol{Rn}^{\mathit{-1/2}} \exp (-ik \int_o^z ndz \tag{28}$$

and

$$\varphi_{Bx} = \varphi_{Eo}\, \boldsymbol{Rn}^{\mathit{+1/2}} \exp (-ik \int_o^z ndz \tag{29}$$

where, R is the co-efficient of reflection and is given by:

$$\boldsymbol{R} = -\exp\, z\, ik \int_o^z ndz \tag{30}$$

which is called Jeffries connection formula.

RESULT/DISCUSSION

In Figure 1, the graph of the relative amplitude R_a for both real and complex transverse electric mode (T.E) in relation to the thickness of film along which the wave is propagated is shown.

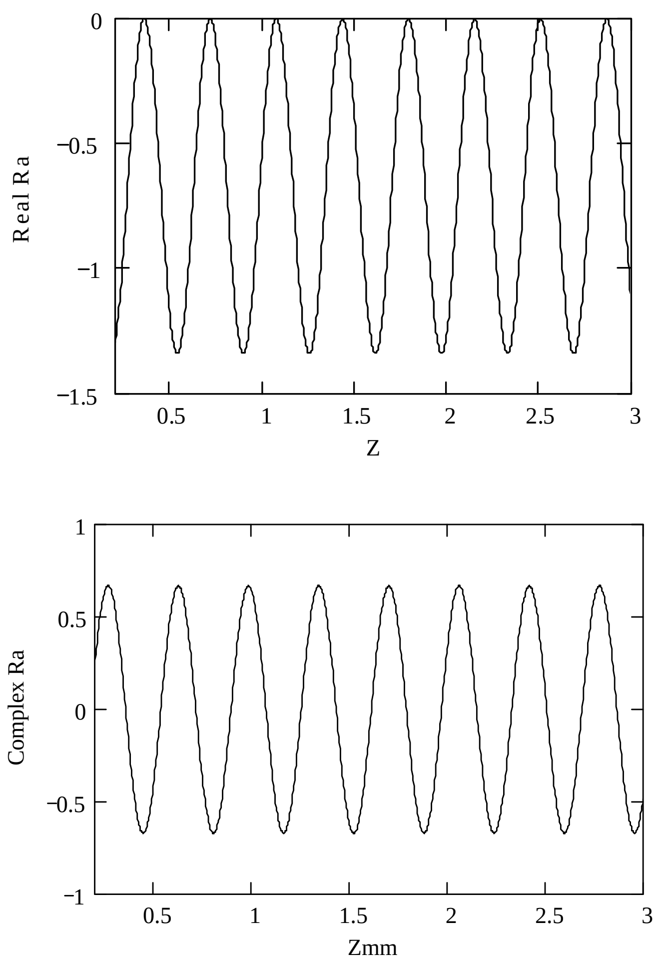

Figure 1: Relative Amplitude Ra for both Real and Complex Transverse Electric (T.E) Mode relating to Thickness of the Film at Wavelength λ = 0.8 mm, n = 2.2325.

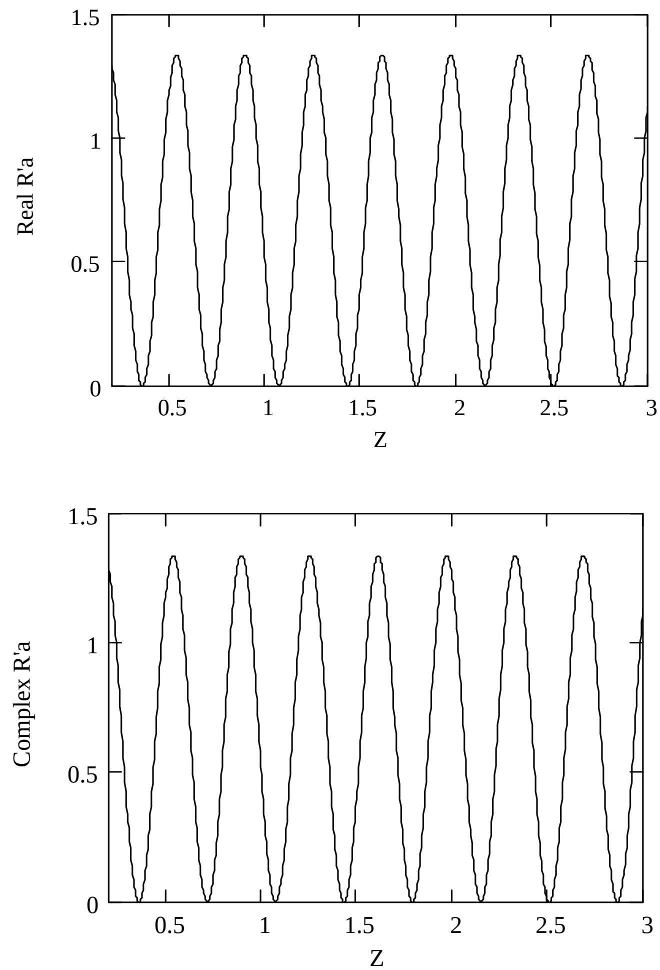

Figure 2: Relative Amplitude Ra for both Real and Complex Transverse Magnetic (T.M) Mode relating to Thickness of the Film at Wavelength λ = 0.8 mm, n = 2.2325.

Similarly, that of transverse magnetic mode (T.M) was presented in Figure 2. These figures clarified the condition outlined by Equation 20 that the amplitude is less or equal to a unity and again the relative amplitude for the real component of T.E. mode is observed to oscillate between zero and 1.4 and the complex relative amplitude oscillate between 0.5 and –0.5. However, in the case of TM mode, the relative amplitude oscillates between zero and +1.4 while the complex part oscillates in phase with the real part.

Figures 3 and 4 graph of the variation of reflection co-efficient with propagation distance z at a wavelength 0.8μm. The real component of the graph Real (R_{el}) showed that the reflection co-efficient increased from zero to a maximum of unity as the thickness of the film increased.

It is observed that the film has a low intensity of reflection co-efficient over a very small thickness of the film indicating that when the film is very thin ($z \leq$ 1.5 micron) the propagating wave is transmitted with small fraction of the radiant energy being absorbed at each reflection event and the other fractions transferred to the particles (electrons) in the film.

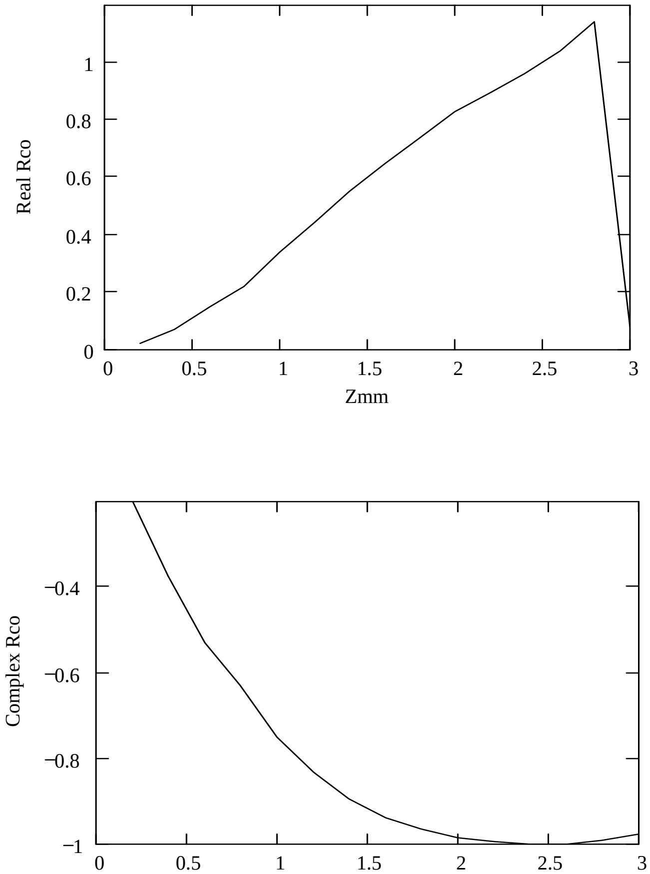

Figure 3: Variation of Reflection Coefficient with Film Thickness.

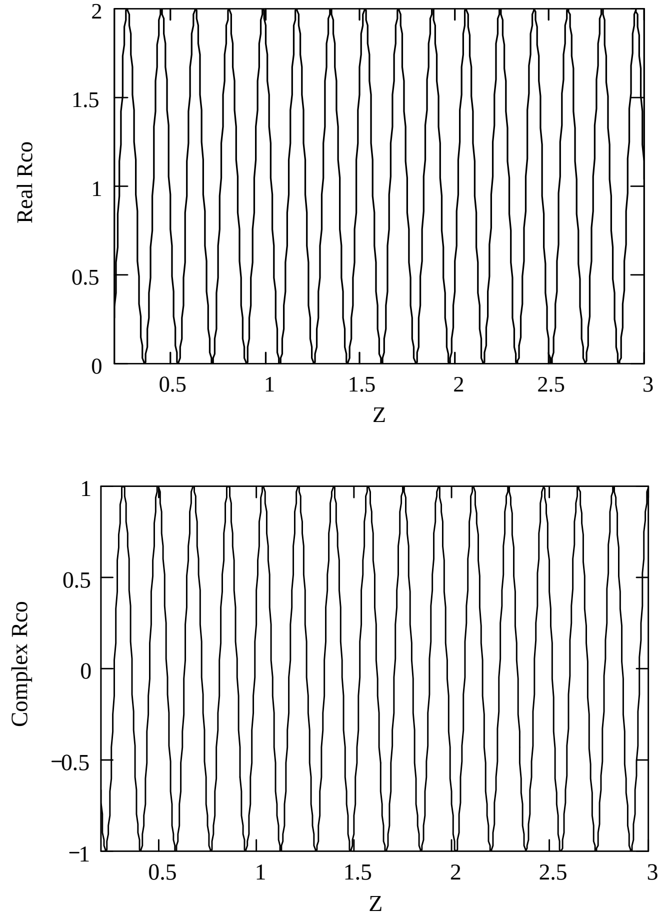

Figure 4: Reflection Coefficient for both Real and Complex Components at Wavelength λ = 0.8 mm, n = 2.2325.

The same graph shows that the film has a high reflection co-efficient at the thicker range (z > 1.5 micron) and hence will like exhibit antireflection properties in the far infrared (6.0μm ≤ 15.5μm) where the reflection coefficient is low.

CONCLUSION

In this work, we studied the reflection co-efficient and other behaviour of the thin film. This study was made possible by solving electromagnetic wave equation propagating through a thin film media presenting slow variation in refractive index. The solution of the plane wave equation for both magnetic and electric field components were obtained and are considered to be aligned along the z-axis in the film medium with non-uniform refractive index.

To realize the computation, it was assumed that the relative amplitude of the wave is less than a unity and n is slowly varying function of the propagation distance z, these equations were written in terms of (T.E) and (T.M) components along z and the W.K.B. solution enabled us to introduce the Jeffries connection formula, which led to the computation of the reflection co-efficient of the wave

This work also indicated that our result did not agree with W.K.B. approximation which explained that when a wave is propagated incident on a medium of which the refractive index changes slowly along the direction of the propagation of the wave, then the wave is not reflected at all or a small amount of reflection is experienced [Fitzpatrick 2002].

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