

# Neural Network Based Analysis of Induction Motors with Saturable Inductances

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## ABSTRACT

This paper describes the development of a Neural Network based model to simulate an induction motor including the effects of saturation on leakage and magnetizing inductances. The machine dynamics are presented by a set of nonlinear, time-varying differential equations. The saturable inductances of the machine are modeled by a Neural Network. An 1100 W induction motor is tested and the machine parameters are estimated. The identified model is simulated using the variable step ODE method of MATLAB<sup>®</sup> for unsaturable and saturable inductances modeled by both curve fitting method and Neural Network. The model is validated by comparing a simulated sample result with the dynamic performance measured in the laboratory.

(Keywords: induction machine, saturation modeling, neural networks, electrical engineering, induction motor)

## INTRODUCTION

The major loads of power systems are induction motors. The transient behavior of these motors plays an important role in the analysis of power systems. To investigate the transient operation of the induction motor, a model is required to predict current, torque, and speed characteristics of the motor. Also it is desired to develop models that are compatible with Vector Control structure because of its performance and abilities.

In the simple models for analysis of the motor, motor inductances are assumed to be constant and consequently, magnetic saturation is negligible. In some applications, such unsaturated models can be used successfully [1-3]. However, in a number of applications, the success of the modeling approach depends to a great extent, on the accuracy with which the equivalent-circuit inductances can be determined. The inductances can vary widely depending on the state of the flux in different parts of the machines. In this respect, magnetic

saturation remains one of the major factors influencing the winding inductances which are extremely difficult to model analytically. In addition, during transient operation the saturation levels within the machine are changing with time; the fluxes in turn are determined by the machine currents [4]. For this reason several authors [5-7] have been developing dynamic machine models based on numerical methods such as the finite-element procedure. This is a flexible and powerful technique, but is relatively expensive in terms of computer processing time.

Other scientists have modeled saturable motor directly. For instance, a new  $\pi$  model has been developed to achieve some control performances [8]; however, this model isn't general and loses its accuracy for 2 pole motors [8]. Another model has been proposed by using Curve Fitting [9].

Saturation of the magnetic circuit of the motor influences the magnetic permeability of the iron parts (reduction), the magnitude of the starting current, the duration of the starting period, the thermal capacity of the motor, the winding insulation, the pulsating torque, and the starting current on the power system. However, a mathematical model for a saturated motor is justifiable on the basis of the required accuracy of the above-mentioned factors. Therefore, to study the transient behavior of a motor, a set of nonlinear time-varying differential equations is required. The nonlinear time-varying parameters will characterize the saturated and unsaturated modes of the motor operation.

Different analytical methods for a saturated model may be considered [10, 11]. The object of this paper is to present a modeling method for the saturated motor in which the magnetization curves of the motor is represented by a set of nonlinear functions. Consequently, the leakage and magnetizing inductances of the motor are converted into nonlinear functions against

excitation currents. These functions are implemented by a MLP Neural Network. A MATLAB<sup>®</sup> program is developed based on the model and the model equations are solved directly. Finally a sample experimental result is compared with the simulated result in order to validate the proposed model.

### MACHINE EQUATIONS FOR THE UNSATURATED CASE

The transient behavior of an induction motor is normally represented in the d,q reference frame. The motor performance may be expressed by Equations (1)- (6), where  $\omega$  is the angular speed of the reference axes as described in [1,2].

$$V_{ds} = r_s i_{ds} + \frac{d\lambda_{ds}}{dt} - \omega \lambda_{qs} \quad (1)$$

$$V_{qs} = r_s i_{qs} + \frac{d\lambda_{qs}}{dt} + \omega \lambda_{ds} \quad (2)$$

$$V_{dr} = r_r i_{dr} + \frac{d\lambda_{dr}}{dt} - (\omega - \omega_r) \lambda_{qr} \quad (3)$$

$$V_{qr} = r_r i_{qr} + \frac{d\lambda_{qr}}{dt} + (\omega - \omega_r) \lambda_{dr} \quad (4)$$

$$V_{0s} = r_s i_{0s} + \frac{d\lambda_{0s}}{dt} \quad (5)$$

$$V_{0r} = r_r i_{0r} + \frac{d\lambda_{0r}}{dt} \quad (6)$$

where  $\lambda$  is the total flux linkage of a particular winding. For a three-phase balanced system, Equations (5) and (6) are eliminated. The voltages  $V_{ds}$ ,  $V_{qs}$ ,  $V_{dr}$  and  $V_{qr}$  are the applied stator and rotor voltages in the d,q model. The voltages  $V_{dr}$  and  $V_{qr}$  are equal to zero.  $V_{ds}$  and  $V_{qs}$  depend upon the transformation and speed of the reference frame.

Transformation is made from a three-phase system to the d,q system in which the power of the system is invariant. The reference axes are assumed fixed in the stator. For  $V_a = \sqrt{2}V_1 \cos(\omega_1 t)$ , the voltages  $V_{ds}$  and  $V_{qs}$  are:

$$V_{ds} = -\sqrt{3}V_1 \sin(\omega_1 t) \quad (7)$$

$$V_{qs} = \sqrt{3}V_1 \cos(\omega_1 t) \quad (8)$$

where  $V_1$  is the r.m.s. voltage of the line and  $\omega_1$  is the frequency of the supply. The torque equation is:

$$T_e = P(\lambda_{ds} i_{qs} - \lambda_{qs} i_{ds}) \quad (9)$$

where p is the number of pole pairs.

The performance of the motor connected to a three-phase balanced system can be simulated using Equations (1)-(4), where  $\omega = \omega_1$ . The mechanical equation of the motor, by considering  $\omega_r$  as electrical speed of rotor, is:

$$\frac{J}{P} \frac{d\omega_r}{dt} + \frac{B}{P} \omega_r + T_L = T_e \quad (10)$$

Substituting  $T_e$  from Eq. (9) into Eq. (10) yields

$$\frac{d\omega_r}{dt} = \frac{P}{J} \left[ P(\lambda_{ds} i_{qs} - \lambda_{qs} i_{ds}) - \frac{B}{P} \omega_r - T_L \right] \quad (11)$$

The motor flux linkage equations are:

$$\lambda_{ds} = L_s i_{ds} + L_m i_{dr} \quad (12)$$

$$\lambda_{qs} = L_s i_{qs} + L_m i_{qr} \quad (13)$$

$$\lambda_{dr} = L_r i_{dr} + L_m i_{ds} \quad (14)$$

$$\lambda_{qr} = L_r i_{qr} + L_m i_{qs} \quad (15)$$

The motor's total leakage factor  $\sigma$ , the stator's leakage factor  $\sigma_1$  and the rotor's leakage factor  $\sigma_2$  are defined as follows:

$$\sigma = 1 - \frac{L_m^2}{L_s L_r} \quad (16)$$

$$\sigma_1 = \frac{L_s}{L_m} - 1 \quad (17)$$

$$\sigma_2 = \frac{L_r}{L_m} - 1 \quad (18)$$

Using Equations (12)-(15) and the leakage factors, relationships between the flux linkages and currents can be expressed as follows:

$$\begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{dr} \\ i_{qr} \end{bmatrix} = \left( \frac{1-\sigma}{\sigma L_m} \right) \begin{bmatrix} 1+\sigma_2 & 0 & -1 & 0 \\ 0 & 1+\sigma_2 & 0 & -1 \\ -1 & 0 & 1+\sigma_1 & 0 \\ 0 & -1 & 0 & 1+\sigma_1 \end{bmatrix} \begin{bmatrix} \lambda_{ds} \\ \lambda_{qs} \\ \lambda_{dr} \\ \lambda_{qr} \end{bmatrix} \quad (19)$$

Using Equation (19), Equations (1)-(4) and (11) can be solved by the MATLAB<sup>®</sup> ODE function. A computer program based on the variable step ODE function has been developed and the transient current, speed, and torque have been evaluated.

### INDUCTION MACHINE MODEL INCLUDING SATURATION EFFECTS

Inductances  $L_s$  and  $L_r$  are the sum of the leakage and mutual inductances [12]:

$$L_s = L_{ls} + L_m = (L_{lsa} + L_{lsi}) + L_m \quad (20)$$

$$L_r = L_{lr} + L_m = (L_{lra} + L_{lri}) + L_m \quad (21)$$

where  $L_{lsa}$  and  $L_{lra}$  are the air-dependent end-winding leakage inductances [13]. The terms  $L_{lsi}$  and  $L_{lri}$  correspond to the sum of the saturated iron-dependent leakage inductances. By substituting the inductances  $L_s$  and  $L_r$  from Equations (20) and (21) into Equations (12)-(15), a new set of equations may be derived.

For a saturation case, the inductances  $L_m$ ,  $L_{lsa}$  and  $L_{lra}$  are nonlinear and vary with the excitation current. The procedure described in Ref. [14] leads to the following equations:

$$V_{ds} = r_s i_{ds} + (L_{lds} + \overline{L_{lds}}) \frac{di_{ds}}{dt} + \overline{L_{md}} \frac{di_{md}}{dt} - v_1 \quad (22)$$

$$V_{qs} = r_s i_{qs} + (L_{lqs} + \overline{L_{lqs}}) \frac{di_{qs}}{dt} + \overline{L_{mq}} \frac{di_{mq}}{dt} + v_2 \quad (23)$$

$$0 = r_r i_{dr} + (L_{ldr} + \overline{L_{ldr}}) \frac{di_{dr}}{dt} + \overline{L_{md}} \frac{di_{md}}{dt} - v_3 \quad (24)$$

$$0 = r_r i_{qr} + (L_{lqr} + \overline{L_{lqr}}) \frac{di_{qr}}{dt} + \overline{L_{mq}} \frac{di_{mq}}{dt} + v_4 \quad (25)$$

where  $v_1$ ,  $v_2$ ,  $v_3$ , and  $v_4$  are:

$$v_1 = (\lambda_{lqsa} + \lambda_{lqsi}^* + \lambda_{mq}^*) \omega_1 \quad (26)$$

$$v_2 = (\lambda_{lds} + \lambda_{ldsi}^* + \lambda_{md}^*) \omega_1 \quad (27)$$

$$v_3 = (\lambda_{lqra} + \lambda_{lqri}^* + \lambda_{mq}^*) (\omega_1 - \omega_r) \quad (28)$$

$$v_4 = (\lambda_{ldra} + \lambda_{ldri}^* + \lambda_{md}^*) (\omega_1 - \omega_r) \quad (29)$$

and  $(\cdot)^*$  represents the flux leakage nonlinear effects which may be expressed as:

$$\lambda_{md}^* = L_{md}^* i_{md} \quad (30)$$

$$\lambda_{ldsi}^* = L_{ldsi}^* i_{ds} \quad (31)$$

$$\lambda_{ds}^* = \lambda_{ldsi}^* + \lambda_{md}^* \quad (32)$$

for the d-axis (similar equations can be written for the q-axis). They are functions of their respective exciting currents. Also note the following terms used in Equations (26)-(29):

$$\overline{L_{lds}} = \frac{d\lambda_{lds}^*}{di_{ds}} \quad (33)$$

$$\overline{L_{md}} = \frac{d\lambda_{md}^*}{di_{md}} \quad (34)$$

which are the incremental inductances for the d-axis. These equations cannot be solved directly using the ODE function and they must be transformed into a suitable form. Equations (22)-(25) can be rewritten as follows:

$$\frac{di_{ds}}{dt} = \frac{1}{L_{lds} + \overline{L_{md}}} (V_{ds} - r_s i_{ds} - \overline{L_{md}} \frac{di_{dr}}{dt} + v_1) \quad (35)$$

$$\frac{di_{qs}}{dt} = \frac{1}{L_{lqs} + \overline{L_{mq}}} (V_{qs} - r_s i_{qs} - \overline{L_{mq}} \frac{di_{qr}}{dt} - v_2) \quad (36)$$

$$\frac{di_{dr}}{dt} = \frac{1}{L_{ldr} + L_{md}} (-r_r i_{dr} - \overline{L_{md}} \frac{di_{ds}}{dt} + v_3) \quad (37)$$

$$\frac{di_{qs}}{dt} = \frac{1}{L_{lqr} + L_{mq}} (-r_r i_{qr} - \overline{L_{mq}} \frac{di_{qs}}{dt} - v_4) \quad (38)$$

Knowing  $v_{dr} = v_{qr} = 0$ , Equations (35)- (38) can be solved and the following values obtained:

$$\frac{di_{qs}}{dt} = \frac{1}{1 - \left( \frac{\overline{L_{mq}^2}}{L_2 L_4} \right)} \left[ \frac{(V_{qs} - r_s i_{qs} - v_2) \overline{L_{mq}}}{L_2} - \frac{\overline{L_{mq}}}{L_2 L_4} (-r_r i_{qr} - v_4) \right] \quad (39)$$

$$\frac{di_{ds}}{dt} = \frac{1}{1 - \left( \frac{\overline{L_{md}^2}}{L_1 L_3} \right)} \left[ \frac{(V_{ds} - r_s i_{ds} + v_1) \overline{L_{md}}}{L_1} - \frac{\overline{L_{md}}}{L_1 L_3} (-r_r i_{dr} + v_3) \right] \quad (40)$$

$$\frac{di_{qr}}{dt} = \frac{1}{1 - \left( \frac{\overline{L_{mq}^2}}{L_2 L_4} \right)} \left[ \frac{(-r_r i_{qr} - v_4) \overline{L_{mq}}}{L_4} - \frac{\overline{L_{mq}}}{L_2 L_4} (V_{qs} - r_s i_{qs} - v_2) \right] \quad (41)$$

$$\frac{di_{dr}}{dt} = \frac{1}{1 - \left( \frac{\overline{L_{md}^2}}{L_1 L_3} \right)} \left[ \frac{(-r_r i_{dr} + v_3) \overline{L_{md}}}{L_3} - \frac{\overline{L_{md}}}{L_1 L_3} (V_{ds} - r_s i_{ds} + v_1) \right] \quad (42)$$

where

$$L_1 = L_{lds} + \overline{L_{dsi}} + \overline{L_{md}} \quad L_2 = L_{lqs} + \overline{L_{qsi}} + \overline{L_{mq}}$$

$$L_3 = L_{ldr} + \overline{L_{ldr}} + \overline{L_{md}} \quad L_4 = L_{lqr} + \overline{L_{lqr}} + \overline{L_{mq}}$$

and finally, the mechanical equation is:

$$\frac{d\omega_r}{dt} = \frac{P}{J} \left[ P(\lambda_{ds}^* i_{qs} - \lambda_{qs}^* i_{ds}) - \frac{B}{P} \omega_r - T_L \right] \quad (43)$$

## SATURATION MODELING

In this section the above mentioned model has been divided into nonlinear and linear parts. the nonlinear block is static and calculates parameters

$\overline{L_{md}}$ ,  $\overline{L_{mq}}$ ,  $L_1$ ,  $L_2$ ,  $L_3$ ,  $L_4$ ,  $\lambda_{ds}$ ,  $\lambda_{qs}$  and inputs  $v_1$ ,  $v_2$ ,  $v_3$ ,  $v_4$ . However, the linear section includes Equations (39)-(43).

In this method, induction motors can be modeled easier, because the nonlinear part can be trained by a Neural Network and modeling the linear part is straightforward.

The reason for this separation is that Neural Networks are very efficient in mapping static functions, but in dynamic modeling, their performance is low. In other words, Neural Networks can implement static mappings with reasonable number of layers. On the other hand, to model nonlinear dynamics, either the number of necessary layers for modeling is very high or recursive networks should be used. In both situations, there is problem of network convergence. In addition to this, in recursive networks some other problems such as stability, sensitivity, and complexity exist. Finally, Neural Networks are unable to model all nonlinear dynamics. So in this paper, our model is composed of a dynamic linear part, which is straightforward, and a MIMO static function that can be modeled with a simple network.

Also, this structure improves the procedure of some control methods such as input-output feedback linearization.

The separated model is as follows (Figure 1):

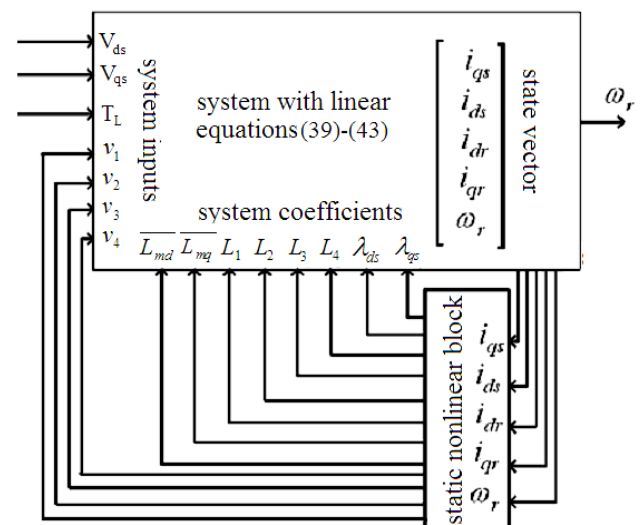


Figure 1: Proposed Model for Induction Motor.

## MODELING IMPLEMENTATION

To implement the nonlinear block shown in Figure 1, a feed-forward network with a back-propagation learning algorithm has been used. Then the network is trained and the proposed model is verified by example of [9] with following Name Plate (Table 1):

**Table 1:** Specification of the Sample Induction Motor

Motor type	TYPZK 90L-6 IEC 34-1	
Output power $P_{out}$	1100W	
Voltage	220/380 V	
Current	5.7/3.3 A	
Frequency	50 Hz	
Speed	910 rpm	
Stator resistance $r_s$	5.85 $\Omega$	
Rotor resistance $r_r$	5.87 $\Omega$	
Stator and rotor unsaturated inductances $L_s$ and $L_r$	0.252 H	
Magnetizing inductance $L_m$	0.2346 H	
Moment of inertia $J$	0.005	
Damping coefficient $B$	0.0008	
Rated torque $T_r$	11.45 N.m	

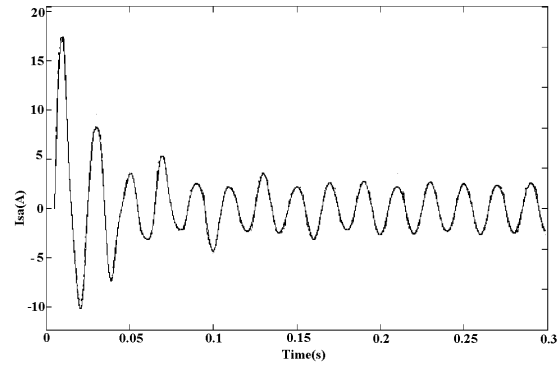
## SIMULATION AND EXPERIMENTAL RESULTS

To validate the accuracy of the simulation method a sample experimental result is compared with the saturated and unsaturated simulation responses. The experimental one is taken from [9].

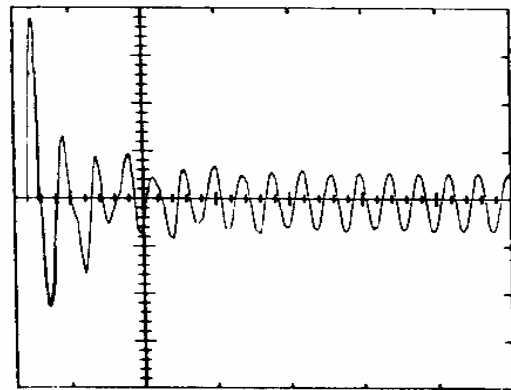
Figure 2(a) presents the starting current of the proposed model. The result is in very good agreement with the measured one, shown in Figure 2(b). Such excellent agreement confirms that the suggested model is very accurate.

Figure 3(a) compares starting current of unsaturated model and saturated one, modeled with both curve fitting and Neural Network methods. For better comparison a zoomed view is illustrated in Figure 3(b).

Figure 4 shows the electromagnetic torque when the machine is accelerated at no load and the saturated parameters and unsaturated parameters are used.



(a)



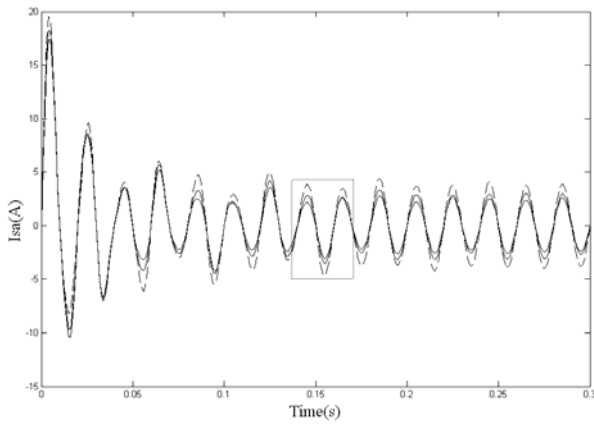
(b)

**Figure 2:** No-Load Starting Current-Time Characteristic: (a) Neural Network Based Model Response; (b) Experimental Result.

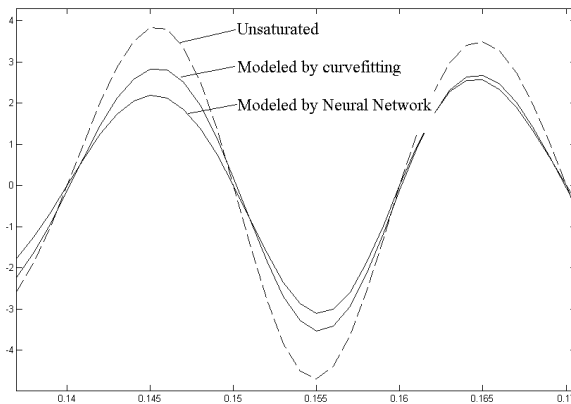
The results indicate that the first peak torque of the unsaturated case is higher than that of the saturated case. On the other hand, the oscillation frequency of the unsaturated model is larger than that of the saturated case and the latter is damped slightly sooner. Figure 5 also shows a similar trend for the variation of the motor speed.

## CONCLUSIONS

This paper has developed a Neural Network based model for dynamic simulation of an induction motor. In this method effects of exciting currents on machine Inductances and cross coupling effects of field components on motor performance can be considered.

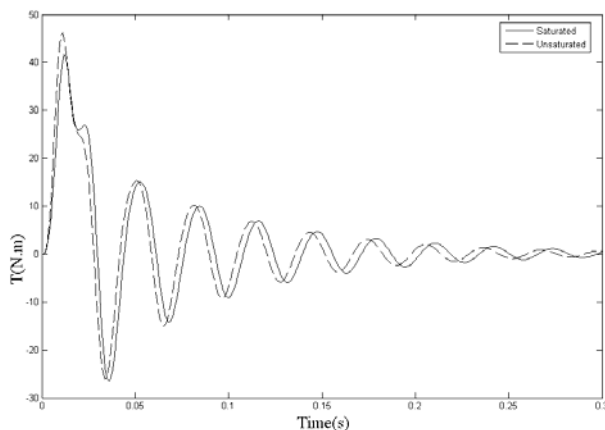


(a)

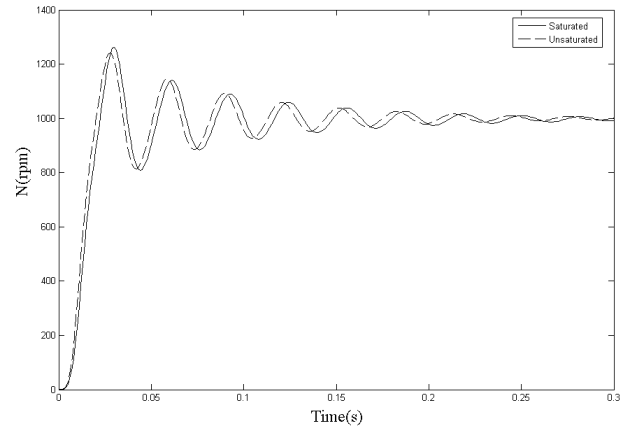


(b)

**Figure 3:** Comparison of Starting Current for Saturated and Unsaturated Models. (a) Full Scale View, (b) Zoomed View.



**Figure 4:** Starting Electromagnetic Torque at No-Load Condition.



**Figure 5:** No-Load Speed of Motor.

However, the most important benefit of this method is possibility of online training of Neural Network for adaptive control methods. Then the proposed model has been verified with an experimental result and finally simulation of the model has been compared with unsaturated and curve fitting method models.

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