Existence of Periodic Solutions for a Certain Boundary Value Problem of a Nonlinear Forth Order Differential Equation

Hilary Mbadiwe Ogbu, M.Sc.

Department of Industrial Mathematics and Applied Statistics
Ebonyi State University, Abakaliki, Nigeria

E-mail: ohilary2006@yahoo.com

ABSTRACT

Existence of periodic solutions for the equation, $x^{(4)} + a_1\dddot{x} + a_2\ddot{x} + a_3\dot{x} + h(x) = p(t)$ with boundary conditions, $D^{(r)}x(0) = D^{(r)}x(2\pi), r = 0,1,2,3, D = \frac{d}{dt}$ have been obtained via an auxiliary equation approach. The procedure consists essentially of first determining a set of appropriate conditions in respect of a general constant - coefficient ordinary differential equation of order four and then generalizing these conditions to a suitable corresponding non-linear ordinary differential equation of order four.

(Keywords: boundary value problems, a priori bounds, compact and equicontinuity, integrated and so called energy equation)

INTRODUCTION

Consider the nonlinear differential equation:

$$x^{(4)} + a_1\dddot{x} + a_2\ddot{x} + a_3\dot{x} + h(x) = p(t) \quad \text{..................} \quad \textbf{(1)}$$

with boundary conditions

$$D^{(r)}x(0) = D^{(r)}x(2\pi), r = 0,1,2,3, D = \frac{d}{dt} \quad \text{..................} \quad \textbf{(2)}$$

where a_1, a_2, a_3 are constants and h, p are continuous functions with $p - 2\pi$ periodic in t.

In the special case,

$$x^{(4)} + a_1\dddot{x} + a_2\ddot{x} + a_3\dot{x} + a_4 x = p(t) \quad \text{..................} \quad \textbf{(3)}$$

subject to the condition (2), Reissig, Sansone, and Conti (1974), proved the existence of exactly one periodic solution for the equations (3) under conditions (2) or the equivalent system:

$$\dot{x} = y, \dot{y} = z, \dot{z} = u, \dot{u} = p(t) - a_4 x - a_3 y - a_2 z - a_1 u \text{......} \textbf{(4)}$$

where,

$$a_1 > 0, a_1 a_2 - a_3 > 0, a_1 a_2 a_3 - a_3{}^2 - a_1^2 a_4 = K > 0, a_4 > 0$$

The forcing term is assumed to be 2π periodic in t and bounded.

In the literature, existence of periodic solutions for the equations (1) under conditions (2) had its origin following the investigations of Ezeilo (1963) performed on a differential equation of the third order. Harrow (1967) extended the ideas to the behavior of the system of a certain fourth order differential equation:

$$\dot{x} = y, \dot{y} = z, \dot{z} = u, \dot{u} = -au - bz - cy - f(x) + P(t) \text{......} \textbf{(5)}$$

Further studies have been done by Ezeilo (1977), Tiryaki (1990 & 1991), Ezeilo (2000), and Ezeilo and Tejumola (2001).

The auxiliary equation to (3) which is:

$$r^4 + a_1 r^3 + a_2 r^2 + a_3 r + a_4 = 0$$

has a root of the form $r = i\beta$ if the two equations $\beta^4 - a_2\beta^2 + a_4 = 0$ and $\beta(a_3 - a_1\beta^2) = 0$ are simultaneously satisfied.

The Equation (3) subject to the boundary conditions (2) has no non trivial solution if either

$$\chi(\beta) = \beta^4 - a_2\beta^2 + a_4 \neq 0, \ \beta = 0,1,2,...$$

or

$$a_3 - a_1\beta^2 \neq 0 \qquad \beta = 0, 1, 2, \ldots$$

but,

$$\beta^4 - a_2\beta^2 + a_4 = \left(\beta^2 - \frac{1}{2}a_2\right)^2 + a_4 - \frac{1}{4}a_2^2$$

Thus, $\chi(\beta) \neq 0$ implies that

$$a_4 > \frac{1}{4}a_2^2 \quad \text{(6)}$$

On generalization of Equation (3) to its nonlinear terms subject to the condition (6), Ezeilo (2000) commented, "On the unsatisfactory state of affairs concerning the generalizations [(3) – (2)] subject to the condition

$$\chi(\beta) = \beta^4 - a_2\beta^2 + a_4 \neq 0, \quad \beta = 0, 1, 2, \cdots \quad \text{(7)}$$

for arbitrary a_1 and a_3".

Ezeilo (2000) commented further to say, "the problem is still open whether we can work out a more acceptable method for involving $\chi(\beta)$ directly or in some other way so that results obtained can be valid not only where a_2 and a_4 are such that $\chi(\beta) > 0$ for all integers $\beta \geq 0$ but also for a_2 and a_4 when $\chi(\beta)$ is negative for some $\beta's$ and positive for other $\beta's$".

The above ideas can be illustrated by considering the linear differential equation,

$$x^{(4)} + \dddot{x} + 6\ddot{x} + \dot{x} + 4x = \text{cost} \quad \text{(8)}$$

subject to condition (2), which has 2π periodic solutions but Equation (8) is neither dissipative nor does it satisfy condition (6). The Equation (8) satisfies the condition,

$$a_4 < \frac{1}{4}a_2^2 \quad \text{(9)}$$

On full details of Equation (3) subject to the condition (9), see Ogbu (2006).
The objective of this paper is to propose a theorem that would guarantee the existence of 2π periodic solutions for Equation (1) – (2) subject to the condition:

$$\frac{h(x)}{x} < \frac{1}{4}a_2^2 \quad \text{(10)}$$

As a consequence, we prove the following:

Theorem 1:

Suppose further to the assumptions stated above that,

(i) $a_4 > 0, \ \frac{h(0)}{x} = 0, \ \frac{h(x)}{x} > 0, \quad x \neq 0$ **(11)**

(ii) $h(x)\operatorname{sgn} x \to \infty$ as $|x| \to \infty$ **(12)**

(iii) $a, > 0, a_2 > 0, a_1a_2 - a_3 > 0 (a_1a_2 - a_3)a_3 - a_1^2\frac{h(x)}{x} > 0$ **(13)**

(iv) $\frac{h(x)}{x} < \frac{1}{4}a_2^2, \quad x \neq 0$ **(14)**

Then equations (1) – (2) are at least are 2π periodic for arbitrary $a_1 = a_3 \neq 0$.

Remark: The above result is a generalization of (9) to its non-linear terms and provides an answer to Ezeilo's comments.

NOTATIONS

Throughout the proof which follows denote capitals $D_0, D_1, D_2 \cdots$ which depend at most on a_1, a_2, a_3, h and p. The $D_{i's}$ retain a fixed identity throughout the proof of Theorem 2.

The symbols $|\square|_\infty$ $|\square|_1$ and $|\square|_2$ with respect to the mapping: $[0, 2\pi] \to \square$ will have their usual meaning. That is for any given function $\theta : [0, 2\pi] \to \square$ say,

$$|\theta|_\infty := \max_{0\le + \le 2\pi} |\theta(t)| \quad |\theta|_1 := \int_0^{2\pi} |\theta(s)| ds$$

$$|\theta|_2 := \left(\int_0^{2\pi} \theta^2(s)\, ds\right)^{\frac{1}{2}}$$

THE PROOF OF THEOREM 2

The proof of Theorem 2 is by the Leray-Schauder fixed point technique and the starting point is the parameter λ dependant equation.

$$x^{(4)} + a_1\dddot{x} + a_2\ddot{x} + a_3\dot{x} + (1-\lambda)a_4 x + \lambda h(x) = \lambda p \quad \text{(15)}$$

or

$$x^{(4)} + a_1\dddot{x} + a_2\ddot{x} + a_3\dot{x} + h_\lambda(x) = \lambda p \quad \text{(16)}$$

where,

$$h_\lambda(x) = (1-\lambda)a_4 x + \lambda h(x)$$

by setting

$$\left.\begin{array}{l} \dot{x} = y, \dot{y} = z, \dot{z} = u, \\ \dot{u} = -a_1 u - a_2 z - a_3 y - (1-\lambda)a_4 x - \lambda h(x) + \lambda p \end{array}\right\} \quad \text{(17)}$$

Equation (15) can be written compactly in matrix form

$$\dot{X} = AX + \lambda F(t \;\; X) \quad \text{(18)}$$

where,

$$X = \begin{bmatrix} x \\ y \\ z \\ u \end{bmatrix} \quad A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -a_4 & -a_3 & -a_2 & -a_1 \end{bmatrix} \quad F = \begin{bmatrix} 0 \\ 0 \\ 0 \\ Q \end{bmatrix} \quad \text{(19)}$$

$$Q = a_4 x - h(x) + p(t)$$

We remark that Equation (15) reduces to a linear equation,

$$x^{(4)} + a_1\dddot{x} + a_2\ddot{x} + a_3\dot{x} + a_4 x = 0 \quad \text{(20)}$$

when $\lambda = 0$ and to (2.1.1) when $\lambda = 1$

The eigenvalues of A can be verified to be the roots of the auxiliary Equation (20) namely,

$$r^4 + a_1 r^3 + a_2 r^2 + a_3 r + a_4 = 0 \quad \text{(21)}$$

has no roots of the form $r = i\beta$, (β real) and so the matrix $\left(e^{-2\pi A} - I\right)$ (I being the identity 4×4 matrix) is invertible.

Therefore $X = X(t)$ is a 2π periodic solution of (18) if and only if X satisfies the equation,

$$X = \lambda TX \quad \text{(22)}$$

where the transformation T is defined by:

$$(TX)(t) = \int_t^{t+2\pi} \left(e^{-2\pi A} I\right)^{-1} e^{(t-s)A} F(s, X(s))\, ds \quad \text{(23)}$$

Let S be the space of all real continuous and 4–vector function $\bar{X}(t) = (x(t), y(t), z(t), u(t))$ which are of period 2π and with norm $\|\bar{X}\|_s$ of $\bar{X}(t) = (x(t), y(t), z(t), u(t))$ defined by

$$\|\bar{X}\|_s = \sup_{0\le t\le 2\pi} \{|x(t)| + |y(t)| + |z(t)| + |u(t)|\} \quad \text{(24)}$$

If the mapping of T is completely continuous mapping of S into itself, it suffices for the proof of Theorem 1 merely to establish a priori bounds. Next is to show that the conditions of Scheafer's Lemma [1955] are satisfied under the hypotheses of Theorem 1. This requires the proof the following Lemma 1: Let T be a compact transformation of a normed linear space into itself. Let $\lambda \in [0,1]$. Then either there is an $x \in S$ such that $x = \lambda_1 T(x)$ or the set $\{x \in S : x = \lambda_1 T(x), \quad 0 < \lambda, < 1\}$ is not bounded. For further details on the above Lemma see Tejumola (1966).

Finally, the proof of Theorem 1 will suffice to concentrate on Equation (15) and to prove simply that there exists a constant $D_0 > 0$ independent of λ such that,

$$|x|_\infty \le D_0, |\dot{x}|_\infty \le D_0, |\ddot{x}|_\infty \le D_0, |\dddot{x}|_\infty D_0 \quad \text{(25)}$$

VERIFICATION OF THE A PRIORI BOUNDS

Let $x(t)$ be a possible 2π periodic solution of (15). The main tool to be used here in this verification is the integrated and so called energy equation W defined by,

$$W = xu - yz + a_1xz - \frac{1}{2}a_1y^2 + \int_0^x a_3 s ds \quad \text{(26)}$$

The time derivative $\dot{W}$ along the solution paths of (17) is:

$$\dot{W} = z^2 - a_2xz - \lambda h(x)x + \lambda xp \quad \text{(27)}$$

$$\dot{W} \le z^2 - a_2xz - \lambda h(x)x + px \quad \text{(28)}$$

Integrating

$$\int_0^{2\pi} \dot{W}dt \le -\int_0^{2\pi} z^2 dt - \int_0^{2\pi} a_2 xz dt - \int_0^{2\pi} h(x)x dt + \int_0^{2\pi} p(x) dt$$

$$\le -\int_0^{2\pi} \ddot{x}^2 dt + \int_0^{2\pi} a_2 \dot{x}^2 dt + \int_0^{2\pi} p(x) dt - \int_0^{2\pi} h(x)x dt$$

Let $\alpha = \min(-1 \pm a_2)$. Therefore,

$$\int_0^{2\pi} \dot{W}dt \le -\alpha \int_0^{2\pi} \ddot{x}^2 dt - \alpha \int_0^{2\pi} \dot{x}^2 dt + \int_0^{2\pi} px dt - \int_0^{2\pi} h(x)x dt$$

$$\alpha \int_0^{2\pi} \ddot{x}^2 dt + \alpha \int_0^{2\pi} \dot{x}^2 dt \le \int_0^{2\pi} px dt - \int_0^{2\pi} h(x)x dt$$

In particular,

$$\alpha \int_0^{2\pi} \ddot{x}^2 dt \le \int_0^{2\pi} |p||x| dt + \int_0^{2\pi} |h(x)||x| dt \le (D_1 + D_2) \int_0^{2\pi} |x| dt$$

$$\alpha \int_0^{2\pi} \ddot{x}^2 dt \le D_3 \int_0^{2\pi} |x| dt$$

That

$\int_0^{2\pi} \ddot{x}^2 dt \le D_4 \int_0^{2\pi} |x| dt$ by Notation Section

$$\le D_4 |x|_1 \quad \text{(29)}$$

Now, since $x(0) = x(2\pi)$ implies that there exists $\tau \in [0, 2\pi]$ such that $\dot{x}(\tau) = 0$. Then the identity $\dot{x}(t) = \dot{x}(\tau) + \int_\tau^t \ddot{x}(s) dt$ holds.

Therefore,

$$\dot{x}(t) = \int_\tau^t \ddot{x}(s) ds$$

Thus,

$$\max_{0 \le t \le 2\pi} |\dot{x}(t)| \le \int_0^{2\pi} |\ddot{x}(t)| ds$$

$$\le (2\pi)^{\frac{1}{2}} \left(\int_0^{2\pi} \ddot{x}^2(t) dt \right)^{\frac{1}{2}}$$

by Schwartz's inequality. From (29)

$$\max_{0 \le t \le 2\pi} |\dot{x}(t)| \le (2\pi)^{\frac{1}{2}} (D_5)^{\frac{1}{2}} \equiv D_6$$

Therefore,

$$|\dot{x}|_\infty \le D_6 \quad \text{(30)}$$

Integrating, (15) with respect to t from $t = 0$ to t from $t = 0$ to $t = 2\pi$ and using the 2π periodicity condition we obtain:

$$\int_0^{2\pi} (1-\lambda) a_4 x dt + \int_0^{2\pi} \lambda h(x) dt = \int_0^{2\pi} \lambda p dt$$

Now let,

$$|(1-\lambda) a_4 x + \lambda h(x)| \le D_7 \quad \text{(31)}$$

case (i) x be positive

let x be sufficiently large.

Then, $\left|(1-\lambda)a_4x+\lambda h(x)\right|=(1-\lambda)a_4x+\lambda h(x)$

h being positive by virtue of hypothesis on h.

Take $x \ge D_7$ $\left(D_7\right.$ sufficiently large)

$(1-\lambda)a_4x+\lambda h_{(4)} \ge (1-\lambda)a_4D_7+\lambda D_8$

(D_8 arbitrarily large)

$(1-\lambda)a_4D_7+D_8=a_4D_7+\lambda\left(D_8-a_4D_7\right)$

Thus,
$(1-\lambda)a_4D_7+\lambda D_8 \ge a_4D_7$.

Case (ii) $x<0$, Let x be negative and sufficiently large

$x<-D_9$ $\left(D_9\right.$ Constant, positive and sufficiently large):

$$\left|(1-\lambda)a_4x+\lambda h(x)\right|=-\left[(1-\lambda)a_4x+\lambda h(x)\right]$$

$$=(1-\lambda)a_4|x|+\lambda h\operatorname{sgn}x$$

$$\ge a_4D_9+\lambda\left(D_{10}-D_9\right)$$

Thus

$$|x(\tau)| \le D_{11} \quad \textbf{(32)}$$

for some τ. Therefore, the identity,

$x(t)=x(\tau)+\int_\tau^t \dot{x}(s)ds$ holds in view of Equation (30).

Thus, $\max\limits_{0\le t\le 2\pi}|x(t)| \le |x(\tau)|+\int_0^{2\pi}\dot{x}(t)dt$

$$\le |x(\tau)|+(2\pi)^{\frac{1}{2}}\left(\int_0^{2\pi}\dot{x}^2(x)dt\right)^{\frac{1}{2}}$$

by Schwartz's inequality. From (30),

$$\max_{0\le t\le 2\pi}|x(t)| \le D_{11}+(2\pi)^{\frac{1}{2}}D_{12} \equiv D_{13}$$

Therefore

$$|x|_\infty \le D_{13} \quad \textbf{(33)}$$

It remains only to establish the third and forth inequalities in Equation (25). For this purpose let Equation (15) be written in the form:

$$x^{(4)}+a_1\dddot{x}+a_2\ddot{x}=k \quad \textbf{(34)}$$

Note that k is bounded. That is,

$$|k| \le |\lambda p|+a_3D_6+(1-\lambda)a_4D_{13}=D_{14} \quad \textbf{(35)}$$

then multiply both sides of (34) by $\dddot{x}$ and integrate with respect to t from $t=0$ to $t=2\pi$

$$\int_0^{2\pi}\dddot{x}x^{(4)}dt+\int_0^{2\pi}a_1\dddot{x}^2dt+\int_0^{2\pi}a_2\ddot{x}\,\dddot{x}dt=\int_0^{2\pi}k\dddot{x}dt$$

using the periodicity condition on $x(t)$, we have

$$a_1\int_0^{2\pi}\dddot{x}^2dt=\int_0^{2\pi}k\,\dddot{x}\,dt$$

$$a_1\int_0^{2\pi}\dddot{x}^2dt \le |k|\int_0^{2\pi}|\dddot{x}|dt$$

$$\int_0^{2\pi}\dddot{x}^2dt \le D_{14}\int_0^{2\pi}|\dddot{x}|\,dt$$

$$\le D_{14}(2\pi)^{\frac{1}{2}}\left(\int_0^{2\pi}\dddot{x}^2dt\right)^{\frac{1}{2}}$$

by Schwartz's inequality.

Therefore,

$$\left(\int_0^{2\pi}\dddot{x}^2dt\right)^{\frac{1}{2}} \le D_{14}(2\pi)^{\frac{1}{2}} \equiv D_{15} \quad \textbf{(36)}$$

since $\dot{x}(0)=\dot{x}(2\pi)$, there exist $\tau\in[0,2\pi]$ such that $\ddot{x}(\tau)=0$. Thus the identity

$\ddot{x}(t)=\ddot{x}(\tau)+\int_\tau^t \dddot{x}(s)\,ds$ holds.

Therefore,

$$\max_{0\le t\le 2\pi}|\ddot{x}(t)| \le \int_0^{2\pi}|\dddot{x}|\,dt$$

$$\leq (2\pi)^{\frac{1}{2}} \left(\int_0^{2\pi} \ddot{x}^2 dt \right)^{\frac{1}{2}}$$

by Schwartz's inequality. By Equation (36)

$$\max_{0 \leq t \leq 2\pi} |\ddot{x}(t)| \leq (2\pi)^{\frac{1}{2}} D_{15} \equiv D_{16}$$

Thus,

$$|\ddot{x}|_\infty \leq D_{16} \qquad \textbf{(37)}$$

We are now left only with the fourth inequality for (25) to be fully established. Consider again Equation (15) in the form,

$$x^{(4)} + a_1 \dddot{x} = \eta \qquad \textbf{(38)}$$

where

$$|\eta| \leq |\lambda p| + a_2 D_{16} + a_3 D_6 + (1-\lambda) a_4 D_{13} \equiv D_{17} \qquad \textbf{(39)}$$

multiply both sides of (38) by $x^{(4)}$ and integrate with respect to t from $t = 0$ to $t = 2\pi$; that is,

$$\int_0^{2\pi} x^{(4)^2} dt + \int_0^{2\pi} a_1 \dddot{x}\, x^{(4)} dt = \int_0^{2\pi} \eta x^{(4)} dt$$

Using the 2π periodicity condition on $x(t)$, we have

$$\int_0^{2\pi} x^{(4)^2} dt \leq \eta \int_0^{2\pi} \left| x^{(4)} \right| dt$$

$$\leq D_{17} (2\pi)^{\frac{1}{2}} \left(\int_0^{2\pi} x^{(4)^2} dt \right)^{\frac{1}{2}}$$

Thus

$$\left(\int_0^{2\pi} x^{(4)^2} dt \right)^{\frac{1}{2}} \leq D_{17} (2\pi)^{\frac{1}{2}} \equiv D_{18} \qquad \textbf{(40)}$$

Since $\ddot{x}(0) = \ddot{x}(2\pi)$ there exist $\tau \in [0, 2\pi]$ Such that $\dddot{x}(\tau) = 0$. Thus the identity $\dddot{x}(t) = \dddot{x}(\tau) = 0 + \int_\tau^t x^{(4)} dt$ holds.

Therefore,

$$\max_{0 \leq t \leq 2\pi} |\dddot{x}(t)| \leq \int_0^{2\pi} \left| x^{(4)} \right| dt$$

$$\leq (2\pi)^{\frac{1}{2}} \left(\int_0^{2\pi} x^{(4)^2} dt \right)^{\frac{1}{2}}$$

by Schwartz's inequality. From equation (40)

$$\max_{0 \leq t \leq 2\pi} |\dddot{x}(t)| \leq (2\pi)^{\frac{1}{2}} D_{18} \equiv D_{19}$$

$$|\dddot{x}|_\infty \leq D_{19} \qquad \textbf{(41)}$$

The estimates (30), (33), (37), and (41) verify (25) and Theorem 2 is established.

DISCUSSIONS

From (27) we have,

$\dot{W} = -z^2 - a_2 xz - \lambda h(x)$ for $p \equiv 0$

Note that

$$-z^2 - a_2 xz - \lambda h(x) x$$

$$= -\left(z + \frac{1}{2} a_2 x \right)^2 + \frac{1}{4} a_2^2 x^2 - \lambda h(x) x$$

$$= -\left(z + \frac{1}{2} a_2 x \right)^2 - x^2 \left(\frac{\lambda h(x)}{x} - \frac{1}{4} a_2^2 \right)$$

by condition (13),

if $\frac{h(x)}{x} < \frac{1}{4} a_2^2$ then $\dot{W}$ which comprises of the first term

$-\left(z + \frac{1}{2} a_2 x \right)^2$ which is negative and the second term

$-x^2 \left(\frac{\lambda h(x)}{x} - \frac{1}{4} a_2^2 \right)$ which is positive

when $\frac{h(x)}{x}$ for $\lambda = 1$ is less than $\frac{1}{4} a_2^2$.

The composition of $\dot{W}$ could be positive or negative depending on whether

$$-\left(z+\frac{1}{2}a_2x\right)^2 > -x^2\left(\frac{\lambda h(x)}{x}-\frac{1}{4}a_2^2\right) \text{ or}$$

$$-\left(z+\frac{1}{2}a_2x\right)^2 < -x^2\left(\frac{\lambda h(x)}{x}-\frac{1}{4}a_2^2\right)$$

respectively. The illustration above clearly shows that $\chi(\beta)$ is positive for some $\beta's$ and negative for other $\beta's$.

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ABOUT THE AUTHOR

Hillary Mbadiwe Ogbu, M.Sc. is a lecturer in the Department of Industrial and Applied Statistics at Ebonyi State UNIversity in Abakaliki, Nigeria. His area of research interests are in the area of ordinary differential equation, systems, and applications.

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