

Analytical Method of Computation of Rectangular-Rotor Bar Impedance for Skin – Effect Using MATLAB.

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ABSTRACT

This paper presents an analytical method for the computation of rectangular-rotor bar impedance for skin-effect. A MATLAB m-file is developed and used to compute the bar impedance and reactance at varying rotor frequencies. It was found that the skin-depth varies exponentially with the bar frequency and inductance but proportionately with the rotor bar resistance. From the MATLAB program, the bar complex impedance per unit length of the conductor was determined and hence, also the effective resistance and the inductance.

(Key words: Skin-depth, Skin-effect, Rectangular-rotor bar, Complex impedance, Squirrel-cage induction machine)

INTRODUCTION

The problem of calculating the impedance of a rectangular rotor bar has long been established. Several authors have devoted attention to the calculation of the effective resistance and reactance of a variety of shapes of deep bars, ranging from rectangular, trapezoidal, tapered, T-shaped, composite, and circular cross-sections. Babb and Williams [1,2] developed a method of computing the impedance of a rectangular bar, T-shaped bar, and a rectangular bar plus an idle bar using the equivalent transmission line approach. Unfortunately, the computed reactance from this method [1,2] is always lower than the measured one.

An analytical approach based on Bessel functions for the computation of the impedance of rectangular and trapezoidal bar conductors has been reported by Swann and Salmon [3]. This approach has been noted to be complex and generally requires rigorous computational

efforts [4]. The method presented in this work not only simplifies the analysis method but also presents an elegant and modern way of computing the rotor bar impedance through the use of MATLAB [5].

ANALYTICAL METHOD DEVELOPMENT

Skin-effect in rotor bar conductors can be analyzed by using Maxwell's field equations, circuit theory, a transmission line approach, or by the use of analytical expressions in terms of Bessel functions [1,2,3,4,6]. In this paper, the circuit theory approach is adopted since the rotor winding is usually assumed to be a circuit element. Figure 1 shows a rectangular rotor bar positioned in the slot. The bar width b_L must be slightly less than the rotor-slot width b_{Nut} , such that the bar can be driven into the slot without significant upsetting of the rotor lamination [6].

$$\text{Let's define } k = b_L/b_{Nut} \dots\dots\dots (1)$$

$$b_L = kb_{Nut} \dots\dots\dots (2)$$

The D.C. resistance of unit length of the bar, with no skin effect is,

$$R_{dc} = \left(\frac{\rho}{b_L h_L} \right) \text{ Ohm/meter of axial length } (3)$$

At any point in the slot,

$$B = \mu_o H = \left(\frac{4\pi}{10^7} \right) H \text{ wb/m}^2 \dots\dots\dots (4)$$

The flux density B is, in general, equal to:

$$B = \left(\frac{4\pi}{10^7} b_{Nut} \right) \int kb_{Nut} Jdy \text{ wb/m}^2 \dots\dots (5)$$

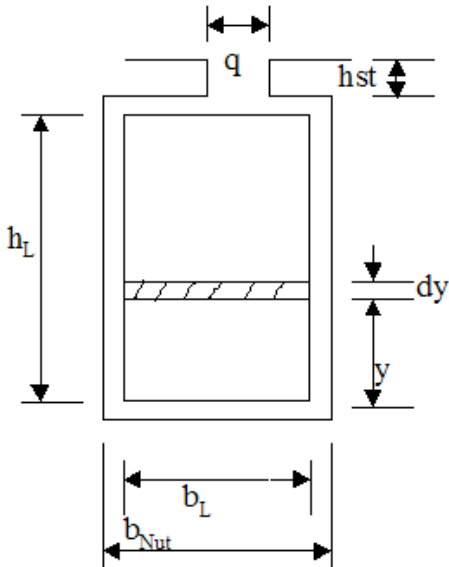


Figure 1: Cross-sectional view of a rotor bar.

$$B = \left(4\pi k / 10^7\right) \int J dy \quad \text{-----} \quad (6)$$

At some point y, the flux density is equal to:

$$B = \left(4\pi k / 10^7\right) \int_0^y J dy \quad \text{-----} \quad (7)$$

The rms flux linking the conductor filament at height y is:

$$\phi = \int_y^{h_L} B dy \quad \text{-----} \quad (8)$$

The rms voltage induced by this flux is given by:

$$E = -j2\pi f \phi = -j2\pi f \int_y^{h_L} B dy \text{ Volts} \quad \text{-----} \quad (9)$$

The current density at height y is given by:

$$J = (V-E)/\rho \quad \text{-----} \quad (10)$$

Therefore,

$$\frac{dJ}{dy} = -\frac{dE}{\rho dy} \quad \text{-----} \quad (11)$$

From equation (9), equation (11) becomes,

$$\frac{dJ}{dy} = j2\pi f B / \rho \text{ A/m}^3 \quad \text{-----} \quad (12)$$

Substituting equation (7) into equation (12) yields:

$$\frac{dJ}{dy} = \left(j8\pi^2 kf / \rho 10^7 \right) \int J dy \quad \text{-----} \quad (13)$$

Differentiating equation (13) with respect to y gives,

$$\frac{d^2 J}{dy^2} = \left(j8\pi^2 kf / \rho 10^7 \right) J \text{ A/m}^4 \quad \text{-----} \quad (14)$$

$$\text{Let } \delta = \sqrt{4\pi^2 kf / \rho 10^7} \text{ m}^{-1} \quad \text{-----} \quad (15)$$

Substituting equation (15) into equation (14) yields the following:

$$\frac{d^2 J}{dy^2} = j2\delta^2 J \quad \text{-----} \quad (16)$$

The fundamental relationship between B and y is also needed. To get that, we have to differentiate equation (6) with respect to y,

$$\frac{dB}{dy} = 4\pi k J / 10^7 \quad \text{-----} \quad (17)$$

But from equation (12) we derive,

$$J = \frac{j2\pi f}{\rho} \int B dy \quad \text{-----} \quad (18)$$

Combining equation (18) with equation (17) yields the following:

$$\frac{dB}{dy} = \frac{j8\pi^2 kf}{\rho 10^7} \int B dy \quad \text{-----} \quad (19)$$

Differentiating equation (19) with respect to y gives,

$$\frac{d^2 B}{dy^2} = \frac{j8\pi^2 kf}{\rho 10^7} B \quad \dots\dots\dots (20)$$

$$\frac{d^2 B}{dy^2} = j2\delta^2 B \quad \dots\dots\dots (21)$$

It can be seen from [7] that equation (21) represents a second order differential equation with a general solution given as:

$$B = A_1 \text{Cosh}(1+j)\delta y + A_2 \text{Sinh}(1+j)\delta y \quad (22)$$

Equation (22) is solved by making use of the physical initial conditions that exist in the machine. At the bottom of the rotor bar, where y equals zero, the cross-slot flux, B is equal to zero. On evaluation, A_1 is found to be zero.

Similarly, at the top of the slot, $y = h_L$ and,

$$B = \frac{4\pi I}{10^7 b_{Nut}} \quad \dots\dots\dots (23)$$

On simplification, we have the following:

$$A_2 = \frac{4\pi I 10^{-7}}{b_{Nut} \text{Sinh}(1+j)\delta h_L} \quad \dots\dots\dots (24)$$

and

$$B = \frac{4\pi I}{10^7 b_{Nut}} \left[\frac{\text{Sinh}(1+j)\delta y}{\text{Sinh}(1+j)\delta h_L} \right] \quad \dots\dots\dots (25)$$

The general solution of equation (16) is:

$$J = B_1 \text{Cosh}(1+j)\delta y + B_2 \text{Sinh}(1+j)\delta y \quad (26)$$

From equations (12) and (25),

$$\frac{dJ}{dy} = 0, \text{ when } y = 0, \text{ so that } B_2 \text{ must be equal to zero.}$$

$$B_1(1+j)\delta = \frac{4\pi I 10^{-7}}{b_{Nut} \text{Sinh}(1+j)\delta h_L} \left(\frac{j2\pi f}{\rho} \right) \quad (27)$$

So that,

$$J = \frac{(1+j)\delta I}{kb_{Nut}} \left[\frac{\text{Cosh}(1+j)\delta y}{\text{Sinh}(1+j)\delta h_L} \right] \quad \dots\dots\dots (28)$$

Equation (28) gives the eddy-current density that exists in the rotor bar as a function of the radial position in the bar. Again, equation (25) gives the cross-slot leakage flux density as a function of radial position in the rotor bar.

The effects of the eddy currents on I^2R loss and reactance can be obtained by calculating the total voltage drop along the bar, as the sum of the voltages due to IR drop and the linkages produced by the cross slot flux. The real component of this voltage drop will give the effective resistance of the bar in alternating current and the imaginary component will give the effective reactance.

For a unit length of the bar, the voltage drop along the bar at any height y is:

$$V_r = J \rho \quad \dots\dots\dots (29)$$

Placing equation (28) into equation (29) yields:

$$V_r = \frac{(1+j)\delta I \rho}{kb_{Nut}} \frac{\text{Cosh}[(1+j)\delta y]}{\text{Sinh}[(1+j)\delta h_L]} \quad \dots\dots\dots (30)$$

For the same unit length of the bar, the corresponding reactive component of the voltage drop is:

$$V_{\text{reactive}} = j2\pi f \phi \quad \dots\dots\dots (31)$$

But,

$$\phi = \int_y^{h_L} B dy \text{ Webers} \quad \dots\dots\dots (32)$$

Placing equation (25) into equation (32) yields,

$$\phi = \frac{4\pi I}{10^7 b_{Nut}} \int_y^{h_L} \frac{\text{Sinh}[(1+j)\delta y]}{\text{Sinh}[(1+j)\delta h_L]} dy \quad \dots\dots\dots (33)$$

$$\phi = \frac{4\pi I}{10^7 b_{Nut}} \frac{\text{Cosh}[(1+j)\delta h_L] - \text{Cosh}[(1+j)\delta y]}{(1+j)\delta \text{Sinh}[(1+j)\delta h_L]} \quad (34)$$

Therefore, the total voltage drop for unit length along the axial dimension of the bar at some point y in the bar cross-section is given by:

$$V_T = V_r + V_{\text{reactive}} = V_r + j2\pi f \phi \dots\dots\dots (35)$$

Substituting equations (30) and (34) into equation (35) yields the following:

$$V_T = \frac{(1+j)\delta I \rho}{k b_{\text{Nut}}} \frac{\text{Cosh}[(1+j)\delta y]}{\text{Sinh}[(1+j)\delta h_L]} + (j2\pi f) \frac{4\pi I}{10^7 \rho} \frac{\text{Cosh}[(1+j)\delta h_L] - \text{Cosh}[(1+j)\delta y]}{(1+j)\delta \text{Sinh}[(1+j)\delta h_L]} \dots\dots\dots (36)$$

Multiplying the numerator and denominator of the second term of equation (36) by ρk and rearranging the expression yields the following:

$$V_T = (1+j)\delta \frac{I \rho}{b_{\text{Nut}} k} \frac{\text{Cosh}[(1+j)\delta y]}{\text{Sinh}[(1+j)\delta h_L]} + 2j \frac{4\pi^2 k f}{10^7 \rho} \frac{I \rho}{b_{\text{Nut}} k} \frac{\text{Cosh}[(1+j)\delta h_L] - \text{Cosh}[(1+j)\delta y]}{(1+j)\delta \text{Sinh}[(1+j)\delta h_L]} \dots\dots\dots (37)$$

Recalling that $\sqrt{2j} = 1+j$ and the expression of δ from equation (15), then equation (37) simplifies to:

$$V_T = (1+j)\delta \frac{I \rho}{b_{\text{Nut}} k} \frac{\text{Cosh}[(1+j)\delta y]}{\text{Sinh}[(1+j)\delta h_L]} + (1+j)^2 \delta^2 \frac{I \rho}{b_{\text{Nut}} k} \frac{\text{Cosh}[(1+j)\delta h_L] - \text{Cosh}[(1+j)\delta y]}{(1+j)\delta \text{Sinh}[(1+j)\delta h_L]} \dots\dots\dots (38)$$

Multiplying both the numerator and denominator of equation (38) by h_L yields the following:

$$V_T = (1+j)\delta \frac{I \rho}{b_{\text{Nut}} k h_L} \frac{h_L \text{Cosh}[(1+j)\delta y]}{\text{Sinh}[(1+j)\delta h_L]} + (1+j)^2 \delta^2 \frac{I \rho}{b_{\text{Nut}} k h_L} \frac{h_L \{ \text{Cosh}[(1+j)\delta h_L] - \text{Cosh}[(1+j)\delta y] \}}{(1+j)\delta \text{Sinh}[(1+j)\delta h_L]} \dots\dots\dots (39)$$

At low frequency, the ohmic resistance per unit length of the bar is given by:

$$R_{dc} = \frac{\rho}{b_{\text{Nut}} k h_L} \dots\dots\dots (40)$$

Placing equation (40) into equation (39) and simplifying the expression yields:

$$V_T = IR_{dc} \frac{(1+j)\delta h_L \text{Cosh}[(1+j)\delta h_L]}{\text{Sinh}[(1+j)\delta h_L]} \dots\dots\dots (41)$$

By using trigonometric identities and separating the expression into real and imaginary parts, equation (42) results [8].

$$V_T = IR_{dc} \delta h_L \frac{\text{Cosh} \delta h_L \text{Sinh} \delta h_L + \text{Cos} \delta h_L \text{Sin} \delta h_L}{\text{Sinh}^2 \delta h_L \text{Cos}^2 \delta h_L + \text{Cosh}^2 \delta h_L \text{Sin}^2 \delta h_L} + jIR_{dc} \delta h_L \frac{\text{Cosh} \delta h_L \text{Sinh} \delta h_L - \text{Cos} \delta h_L \text{Sin} \delta h_L}{\text{Sinh}^2 \delta h_L \text{Cos}^2 \delta h_L + \text{Cosh}^2 \delta h_L \text{Sin}^2 \delta h_L} \dots\dots\dots (42)$$

On further simplification of equation (42) using trigonometric identities, we have:

$$V_T = IR_{dc} \delta h_L \frac{\text{Sinh} 2\delta h_L + \text{Sin} 2\delta h_L}{\text{Cosh} 2\delta h_L - \text{Cos} 2\delta h_L} + jIR_{dc} \delta h_L \frac{\text{Sinh} 2\delta h_L - \text{Sin} 2\delta h_L}{\text{Cosh} 2\delta h_L - \text{Cos} 2\delta h_L} \dots\dots\dots (43)$$

Divide both sides of equation (43) by I to get the total impedance of unit length of the bar, Z_T .

$$Z_T = R_{dc} \delta h_L \frac{\text{Sinh} 2\delta h_L + \text{Sin} 2\delta h_L}{\text{Cosh} 2\delta h_L - \text{Cos} 2\delta h_L} + jR_{dc} \delta h_L \frac{\text{Sinh} 2\delta h_L - \text{Sin} 2\delta h_L}{\text{Cosh} 2\delta h_L - \text{Cos} 2\delta h_L} \dots\dots\dots (44)$$

Therefore, the total resistance per unit length of the bar is,

$$R_{ac} = R_{dc} \delta h_L \frac{\text{Sinh} 2\delta h_L + \text{Sin} 2\delta h_L}{\text{Cosh} 2\delta h_L - \text{Cos} 2\delta h_L} \dots\dots\dots (45)$$

The total reactance of unit length of the bar is,

$$X_{ac} = R_{dc} \delta h_L \frac{\text{Sinh} 2\delta h_L - \text{Sin} 2\delta h_L}{\text{Cosh} 2\delta h_L - \text{Cos} 2\delta h_L} \dots\dots\dots (46)$$

Equations (45) and (46) give the Alger's Analytical Solutions for the total resistance and reactance per unit length of the rectangular bar shown in Figure 1 [9].

METHODS OF SOLUTION AND RESULTS

In order to validate the accuracy of the analysis, this paper uses typical machine data from a KATT VDE 0530, class F insulation, surface cooled squirrel-cage induction machine with a rectangular shaped rotor as shown in Figures 2 and 3.



Figure 2: The machine Rotor with a rectangular bar.

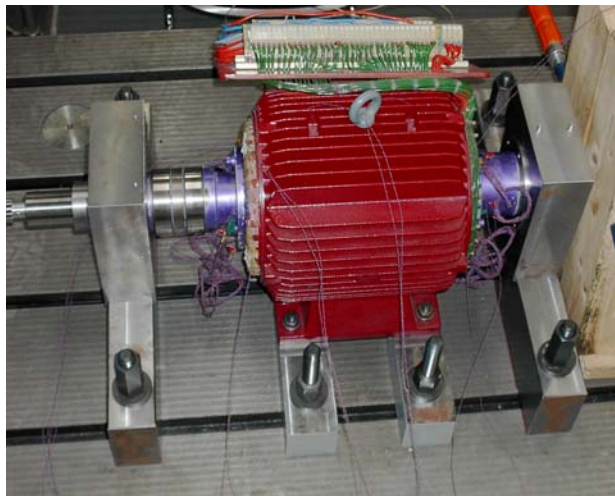


Figure 3: The 7.5KW test machine.

The rotor part of the 7.5KW squirrel-cage machine was decoupled (See Figure 2) to enable the rotor and stator parameters to be taken with the help of Vernier Caliper and Micrometer Screw Gauge. Table 1 shows the measured machine data obtained through this process.

Description	Rectangular Bar
Type of rotor cage	Steel (cast copper)
Conductivity of rotor bars	56Sm/mm ²
Number of pole pairs	2
Number of rotor slots	28
Number of stator slots	36
Relative permeability of copper	1000
Width of end ring	4.4mm
Height of end ring	13.2mm
Insulation thickness	0.3mm
Bar length	0.239m
Stator outside diameter	200mm
Iron core length	170mm
Inner diameter of rotor	30mm
Permeability of free space	$4\pi \times 10^{-7}$ H/m
Height of rotor bar	13.2mm
Width of rotor bar	4.4mm
Air gap	0.3mm
Slot insulation thickness	0.3mm

Table 1: Measured machine data.

The equations derived above for the skin-depth and rotor bar impedance are solved in 'skinEffect.m'. 'SkinEffect.m' is part of the MATLAB m-file developed for the purpose of the analysis of skin-effect in squirrel-cage induction machines [10]. The program is a general-purpose code that computes and plots the rotor bar impedance, resistance, reactance, inductance, etc. The program is highly interactive. When the program is loaded and run in the MATLAB environment, the user is requested to supply the machine parameters as shown in Table 1. These parameters form the program's input. The program runs and plots the graphs automatically as soon as the input values are complete. The program code is listed in Appendix A. Figure 4 shows the variation of rotor bar resistance, reactance, impedance and inductance with frequency.

From the figure, it is seen that the rotor bar resistance, impedance and reactance increase as the frequency increases. Conversely, the rotor bar inductance decreases as the frequency increases. Figure 5 shows that frequency varies exponentially with skin-depth.

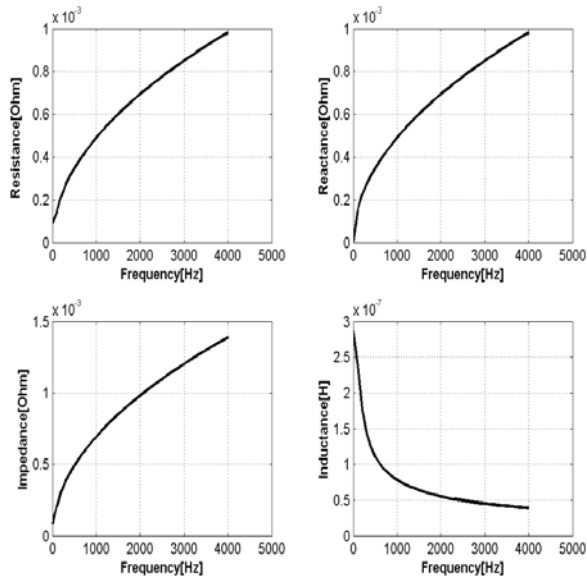


Figure 4: Rotor bar impedance plots.

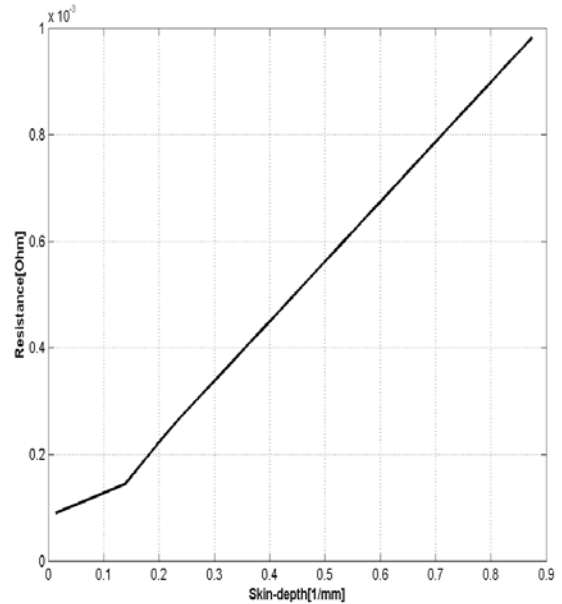


Figure 6: Variation of resistance with Skin-depth.

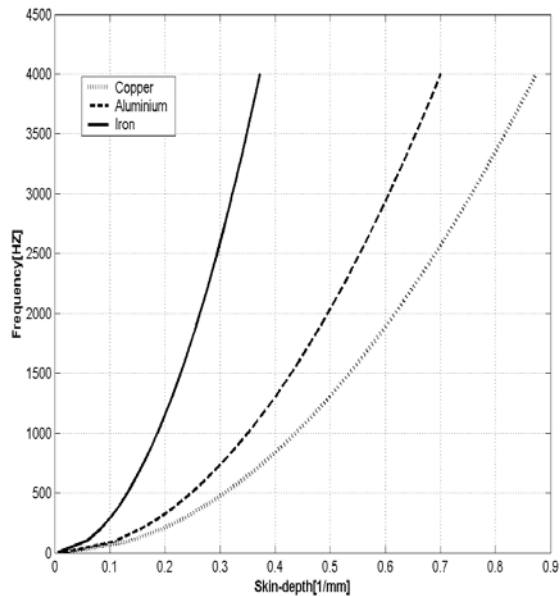


Figure 5: Variation of Skin-depth with frequency for different conductors.

Figure 5 also indicates that skin-depth is dependent on the material from which the bar is made. At 4KHZ, a rotor bar made of iron is expected to have about 2.6 mm skin-depth whereas in copper and aluminum of the same frequency, the skin-depth is respectively 1.1mm and 1.4mm. Figure 6 shows the variation of rotor resistance with skin-depth.

Figure 6 also shows an initial sluggish rise but thereafter the graph rises proportionately above 6.67mm. The rotor bar inductance as a function of skin-depth is depicted in Figure 7.

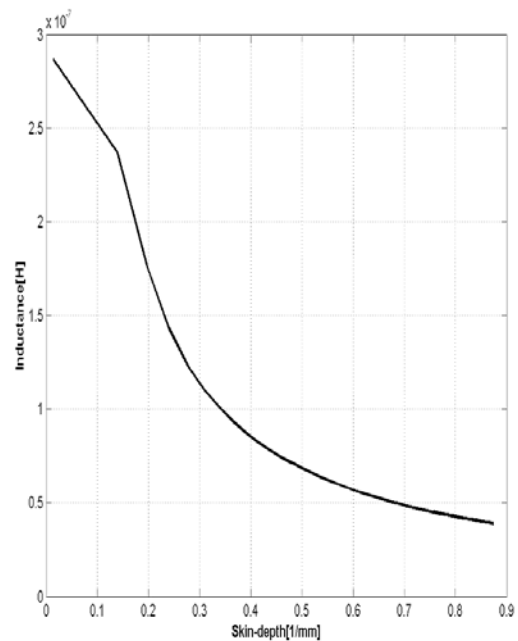


Figure 7: Variation of inductance with Skin-depth.

Figure 7 shows that the rotor bar inductance decreases as the skin-depth increases. Figure 8 shows the trend of skin-depth against frequency for different k values (ratio of bar width to slot width).

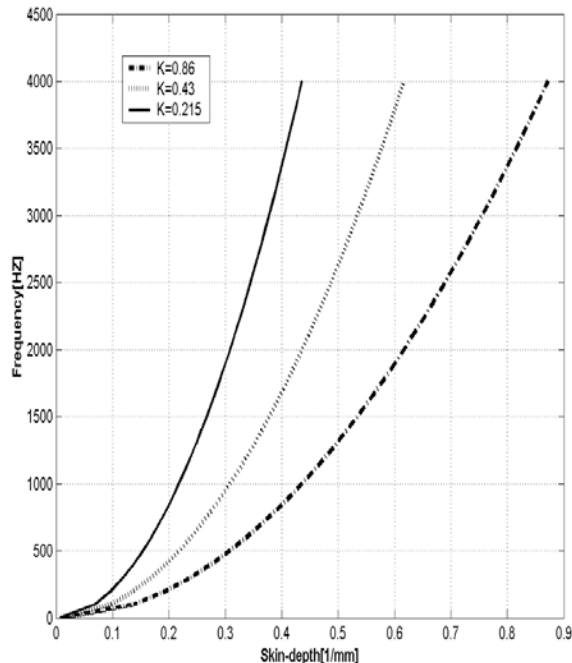


Figure 8: Graph of Frequency against Skin-depth for different k values.

It can be seen from the Figure 8 that as the value of k increases, the greater the curve tilts towards the skin-depth axis.

CONCLUSION

The analysis presented in this paper permits the skin-effect in a rectangular rotor bar to be simulated by the use of MATLAB. The MATLAB program developed is simple, effective, highly interactive and can be modified for any rotor form other than rectangular shaped rotors. The simulation results presented in this paper are therefore useful for a wide variety of conditions encountered with the variable-frequency and transient operation of induction machines and have the merit of accounting for non-uniform current distribution in the rotor bar.

NOMENCLATURE

h_L = depth of bar in m

b_L = width of slot in m

f = frequency in Hz

ρ = resistivity of bar conductor in $\Omega - m$

k = ratio of bar width to slot width

y = distance up from the bottom of the bar in m

B = rms flux density across the slot at height y , in wb/m^2

J = rms current density at height y , in A/m^2

H = magnetic field strength, in A/m

I = bar current

δ = penetration depth or skin-depth.

ZT = total rotor bar impedance

X_{ac} = total rotor bar reactance

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APPENDIX A:

```

%*****
%The name of the Program is
%SkinEffect.m. This Program computes
%and plots the bar impedance, resistance,
%reactance and inductance for the
%rectangular rotor bar of a 7.5KW
%Squirrel-cage induction machine. The
%Program requests the user to supply the
%input values as shown in
%Table 1 of
%this paper. The user also has the option
%to choose which variable he wishes to
%compute. The Program is written using
%the latest version of MATLAB-
%MATLABR12.*****
%*****
%SUPPLY THE INPUT PARAMETERS AS
INDICATED IN TABLE 1
%*****

format long e
clear;
clc;
a=input('Enter the slot insulation thickness =')
hNut=input('Enter the height of rotor bar in mm
=')
bNut=input('Enter the width of rotor bar in mm =')
mu_zero=input('Enter the Permeability of free
space =')
xcu=input('Enter the conductivity of copper =')
Ls=input('Enter Bar length in m =')
hL=hNut-2*a;
bL=bNut-2*a;

%***COMPUTATION BASED ON THE
FOLLOWING SELECTION
MODES*****

disp('****PLEASE CHOOSE AMONG THE
FOUR OPTIONS LISTED
BELOW*****')

```

```

test=0;
while test==0
disp(' ')
disp(['1. SKIN-DEPTH PLOTS AT VARYING
FREQUENCY']);
disp(['2. PLOTS OF SKIN-DEPTH FOR
VARIOUS MATERIALS']);
disp(['3. ROTOR BAR IMPEDANCE PLOTS']);
disp(['4. END OF PROGRAM']);
disp(' ');
flag=input('PLEASE INDICATE THE MODE OF
OPERATION YOU WANT BY
TYPING[1,2,etc]:');
if flag<1
disp(' ');
disp('SORRY THE SELECTION IS BETWEEN 1
AND 4'); *
elseif flag>4
disp(' ');
disp('SORRY THE SELECTION IS BETWEEN 1
AND 4');
elseif flag==4
disp(' ');
disp('THANK YOU. YOU HAVE COME TO THE
END OF THE PROGRAM. ');
return;
else
test=1;
end
end

%MODE 1: SKIN-DEPTH PLOTS AT VARYING
FREQUENCY
if flag==1
K1=0.86;
K2=0.43;
K3=0.215;
f=1:100:4001;
Alpha1=sqrt(4.*pi.*pi.*K1.*f.*xcu.*1e-7);
delta1=(1.0)/(Alpha1); % Skin-depth[mm]
Alpha2=sqrt(4.*pi.*pi.*K2.*f.*xcu.*1e-7);
delta2=(1.0)/(Alpha2); % Skin-depth[mm]
Alpha3=sqrt(4.*pi.*pi.*K3.*f.*xcu.*1e-7);
delta3=(1.0)/(Alpha3); % Skin-depth[mm]
end

%MODE 2: PLOTS OF SKIN-DEPTH FOR
VARIOUS MATERIALS(Cu,Al,iron)
if flag==2
K=bL/bNut;
xcu=56;
xAl=36;
xfe=10.2;
f=1:100:4001;
Alpha1=sqrt(4.*pi.*pi.*K.*f.*xcu.*1e-7);
delta1=((1.0)/(Alpha1)); % Skin-depth[mm]for
Copper

```



```

Alpha2=sqrt(4.*pi.*pi.*K.*f.*xAl.*1e-7);
delta2=((1.0)/(Alpha2)); % Skin-depth[mm]for
Aluminium
Alpha3=sqrt(4.*pi.*pi.*K.*f.*xfe.*1e-7);
delta3=((1.0)/(Alpha3)); % Skin-depth[mm] for
Iron
end
%MODE 3: ROTOR BAR IMPEDANCE PLOTS
if flag==3
K=bL/bNut;
xcu=56;
f=1:100:4001;
Alpha=sqrt(4.*pi.*pi.*K.*f.*xcu.*1e-7);
delta=((1.0)/(Alpha)); % Skin-depth[mm]for
Copper
Ad=hL.*Alpha;
Ro=(Ls)/(xcu*hL*bL); %D.C. RESISTANCE AT
ZERO-FREQUENCY
Lo=(mu_zero.*Ls.*hL)/(bNut);%D.C.
INDUCTANCE AT ZERO-FREQUENCY
Ro1=Ad.*Ro;
A1=(sinh(2.*Ad)+sin(2.*Ad));
A2=(cosh(2.*Ad)-cos(2.*Ad));
Rac=Ro1.*(A1./A2); %A.C. RESISTANCE
EQUATION
Lo1=(Ro.*Ad)/(2*pi.*f);
B1=sinh(2.*Ad)-sin(2.*Ad);
Lac=Lo1.*(B1./A2); % A.C. INDUCTANCE
ZAL=Rac+j*(2*pi.*f.*Lac); %Rotor Bar
Impedance
ZTa=abs(ZAL); %Magnitude of the rotor bar
impedance
Zima=imag(ZAL);%Rotor Bar Reactance
Zr=real(ZAL); %Rotor Bar Resistance
end

%*****MODE 1*****
if flag==1
figure(1);
plot(f,delta1,'r') % For K =0.86
hold on
grid on
plot(f,delta2,'b') % For K =0.43
plot(f,delta3,'g') % For K =0.215
ylabel('Skin-depth[mm]')
xlabel('Frequency[HZ]')
figure(9);
plot(Alpha1,f,'r') % For K =0.86
hold on
grid on
plot(Alpha2,f,'b') % For K =0.43
plot(Alpha3,f,'g') % For K =0.215

```

```

xlabel('Skin-depth[1/mm]')
ylabel('Frequency[HZ]')
legend('K=0.86','K=0.43','K=0.215')
end

%*****MODE 2*****
if flag==2
figure(2);
plot(Alpha1,f,'r') % For Copper
hold on
grid on
plot(Alpha2,f,'b') % For Aluminium
plot(Alpha3,f,'g') % For Iron
xlabel('Skin-depth[1/mm]')
ylabel('Frequency[HZ]')
legend('Copper','Aluminium','Iron')
end

%*****MODE 3*****
if flag==3
figure(3);
subplot(2,2,1);
plot(f,Rac,'r') %Rotor bar resistance Plot
grid on
ylabel('Resistance[Ohm]')
xlabel('Frequency[Hz]')
subplot(2,2,2)
plot(f,Zima,'b') %Rotor bar Reactance plot
grid on
ylabel('Reactance[Ohm]')
xlabel('Frequency[Hz]')
subplot(2,2,3)
plot(f,ZTa,'.-') %Rotor bar Impedance plot
grid on
ylabel('Impedance[Ohm]')
xlabel('Frequency[Hz]')
subplot(2,2,4)
plot(f,Lac,':') %Rotor bar inductance plot
grid on
ylabel('Inductance[H]')
xlabel('Frequency[Hz]')

figure(4);
plot(Alpha,f,'r')
grid on
xlabel('Skin-depth[1/mm]')
ylabel('Frequency[HZ]')

figure(5);
plot(Alpha,Rac,'b')
grid on
ylabel('Resistance[Ohm]')
xlabel('Skin-depth[1/mm]')

figure(6);
plot(Alpha,Lac,':')
grid on

```

```
ylabel('Inductance[H]')
xlabel('Skin-depth[1/mm]')
end
```

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