Stochastic Approach for Evaluating the Effectiveness of Antiretroviral Therapy (ART) in HIV/AIDS Patients in Nasarawa State

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ABSTRACT

The Human Immunodeficiency Virus and Acquired Immune Deficiency Syndrome (HIV/AIDS) are becoming increasingly drug-resistant in some patients, leading to treatment failure and the spread of drug-resistant strains. This undermines the effectiveness of limited treatment options such as antiretroviral therapy (ART). Therefore, it is important to assess the future progress and predict the efficacy of ART treatment. To achieve this, a Markov chain model was formulated using the CD4 counts of a sample of 13,710 patients from the HIV Counselling and Testing (HCT) Federal Medical Center Keffi, Nasarawa State who receive treatment every six months.

Nasarawa State has a high prevalence rate of HIV in the North Central region of Nigeria. The methodology used in this study is suitable for evaluating and predicting treatment performance in a group of HIV patients or a cohort study. The progression of patient response to therapy was assessed by examining the transition probability matrix from one CD4 count state to another.

The efficacy of the therapy, which is the maximum patient response to treatment, was evaluated by considering the long-term chances of patients being in each CD4 count state and the average time it takes for patients to return to each CD4 count state. The CD4 count states considered in the study are as follows: less than 200 cells/μL (Poor health), between 200 - 499μL (Moderate health), and greater than or equal to 500μL (Good health), denoted as I, II, and III, respectively. The model predicts that, over the long term, patients will spend 45% of their time in good health, 40% in a moderate health state, and 15% in a poor health state. The corresponding mean recurrence times for these states are 1.24 years, 1.13 years, and 3.21 years, respectively. The study concludes that the differences in patient health state probabilities may be attributed to factors such as antiretroviral drug resistance. The authors recommend identifying and considering these factors when administering antiretroviral therapy to ensure very high probabilities of good and moderate health states.

(Keywords: stochastic, model, efficacy, ART, HIV/AIDS, CD4, Markov chain)

INTRODUCTION

The Human Immunodeficiency Virus (HIV) causes acquired immunodeficiency syndrome (AIDS) and has been spreading widely across the global population, making it one of the most serious public health crises in the world. It poses a huge challenge to modern medicine and humanity. The medical community has recognized HIV/AIDS as a distinct clinical condition since 1981. In 2007, the global HIV population was around 33 million, with two million deaths related to HIV/AIDS and three million new infections occurring annually (UNAIDS, 2010).

According to UNAIDS (2014), Sub-Saharan Africa is home to around 67% of the world's HIV/AIDS (PLWHA) population. Nigeria has the third-largest infected population, with 2 to 3.2 million individuals, following South Africa and India. HIV is naturally present in blood, sperm, vaginal and anal fluids, as well as breast milk, and can be transmitted through intercourse,

sharing of sharp tools, blood transfusions, and from mother to child through breastfeeding. The increased spread of the disease has led to a response from global health initiatives such as the United States (US) President's Emergency Program for AIDS Relief (PEPFAR) and the Global Fund to Fight AIDS, Tuberculosis, and Malaria (GFTAM). These initiatives have increased the provision of antiretroviral therapy (ART) to a growing number of affected patients (reference to the plate). According to the author, this has led to the expansion of treatment and preventive initiatives, improving access to ART for communities that were previously underserved. In 2017, it was estimated that 59% of people living with HIV worldwide were receiving ART (UNAIDS, 2014).

Sub-Saharan Africa (SSA), which includes Nigeria, has been disproportionately affected by the pandemic, accounting for more than half of people living with HIV (PLHIV) (1 and 2). Nigeria is one of the target countries for International Health Initiatives Programs, with strong local political support. Over the last 15 years, significant progress has been achieved in combating the Human Immunodeficiency Virus (HIV) epidemic. However, a recent survey released by the Nigerian government suggests a nationwide HIV prevalence of 2.0% among individuals aged 14 to 49 (Nasarawa/SURGE, May 2019). The research results indicate that there is a significant HIV epidemic among adolescents aged 10-18 who undergo HIV testing at the Virology laboratory of General Hospital Keffi in Nasarawa State. The study was conducted from January 2013 to December 2028.

In a recent development, Dr. Ruth Bello, Executive Director of the Nasarawa State AIDS Control Agency (SACA), revealed in an interview with the News Agency of Nigeria on June 15, 2023, that the inhabitants of Nasarawa faced one of the greatest risks of developing HIV/AIDS in Nigeria. She stated that Nasarawa has only seen a 2% decline in HIV/AIDS prevalence, whereas Keffi, a satellite town in the area, has a prevalence of over 25%. This indicates that the inhabitants of Nasarawa are at a higher risk of developing HIV/AIDS. Unfortunately, there is currently no known cure for HIV, but with early diagnosis and proper antiretroviral (ARV) therapy, individuals with HIV can lead a normal, healthy life. Therefore, it is crucial to ensure that proper therapy is consistently administered. The ARV medications currently used for HIV therapy work by disrupting the virus's ability to reproduce at different stages of its life cycle. It might be possible to use a time series technique to analyze how the cycles transition from one stage to the next. This can be seen as a stochastic process.

Markov chain modeling has been applied in various medical studies to predict and estimate random or uncertain events associated with specific probabilities of occurrence. There have been practical applications in the analysis of genetics. For instance, Kuo, *et al*. (1999) determined the efficacy of noninsulin-dependent diabetes in a patient population, Davies (1975) predicted outcomes of dialysis treatment/kidney transplants in patients, and Kay (1986) and Pérez-Ocón (2001) analyzed longitudinal disease progression for liver cancer, breast cancer, Jackson et al (2002) looked at Markov modeling for bronchiolitis obliterans syndrome, and Commenges, *et al*. (2004) did so for Alzheimer's disease, among others.

Additionally, Bature, *et al.* (2010) described a Markov chain model that was used to track the movement of the HIV/AIDS virus from one generation to another over a 20-year period, and Sweeting et al. (2010) applied multistate Markov modeling to explain the rate at which Hepatitis C progresses. Another important work by Nwosu (2015) used stationary and smoothed non-stationary Markov Chain Models to determine the life expectancy of HIV/AIDS patients in Anambra State. The study's findings revealed that the smoothed non-stationary Markov Chain Model is theoretically superior to the stationary Markov Chain in predicting patient life expectancies.

Furthermore, Agada, Eneh, and Ikughur (2018) created a Markov chain model to evaluate and forecast the efficacy of ART in Wukari, Taraba state. He built upon previous work by forecasting the life expectancy of farmers living with HIV/AIDS in Makurdi, Benue state. The focus was on agricultural productivity and post-harvest losses.

This research aims to assess the progress of HIV/AIDS patients' response to ART treatments in Nasarawa State. It will utilize the transition probability matrix of the Markov chain to evaluate the efficacy of ART on patients living with HIV/AIDS in the region. Additionally, the study will use the steady step or long-run probability of the Markov chain to forecast the life expectancy

of patients living with HIV/AIDS in Nasarawa State, by employing the Fundamental matrix of the Markov chain.

METHODOLOGY

The study population comprises the CD4 count of 13710 HIV/AIDS patients receiving treatment every six (6) months at the ART Centre of the HIV Counseling and Testing (HCT) unit in selected hospitals in Nasarawa State from January 2010 to December 2022. The data for this study mainly comes from secondary sources. The CD4 counts of the 13710 HIV/AIDS patients, along with their identification numbers and visit dates, were meticulously organized to reflect their health state transition. The health states were classified as follows: less than 200 cells/μL for I (Poor health), between 200 and 499 cells/μL for II (Moderate health), and greater than or equal to 500 cells/μL for III (Good health) representing the different health states of the patients.

Basic Mathematical Concepts of the Markov Chain Model

A Markov chain is a sequence or chain of discrete states in time or space that have a defined probability of transitioning from one state to the next in the chain. A stochastic process $\{X_{(n)}; n \geq 0\}$ is a sequence of transitions from one state to another. The probabilities associated with each transition rely solely on the process's immediate previous state and not on how it reached that state.

A Markov chain is a sequence of random variables $X_0,\ X_1\ \ldots$ with values in a countable set S. At each moment n, the future state (or values) $X_{n+1},\ X_{n+1},\ \ldots$ depends only on the history $X_0,\ldots\ X_n$ through the current state X_n. The key characteristic of a Markov chain is that only the most recent point on the trajectory influences what happens next. This is known as the Markov property, indicating that X_{t+1}depends on Xt, but it also depends on $X_{t+1},\ldots,X_1,X_0$. A stochastic process $X = \{X_{(n)}; n \geq 0\}$ on a countable set S is a Markov chain if for every$i,j \in S$ and each $n \geq 0$

$$P\{X_{n+1} = j | X_0 \ldots, X_n\} = P\{X_{n+1} = j | X_n\} \quad \textbf{(1)}$$

At every time step n, the next state X_{n+1} is independent of the previous states $X_0,\ldots,X_{n-1}$ given the current state X_n. This property is known as the Markovian property and applies to a Markov-dependent stochastic process. It means that the probability of any future state $(X_{n+1} = j)$ given any previous states:

$$(X_0 = i0,\ X_1 = i1,\ \ldots, X_{n-1} = in-1)$$

and the present state (X_n) is independent of the past states. In essence, this property indicates that the process is memoryless.(Udom, 2010).

At any time n, the next state X_{n+1} is conditionally independent of the past $X_0 \ldots, X_{n-1}$ given the present state X_0.

A stochastic process that is Markov dependent is said to possess the Markovian property. This property is equivalent to the statement that the conditional probability of any future state $(X_{n+1} = j)$, given any past states (X_0=i_0, X_1=i_1,…,X_{n-1}=i_{n-1})), and the present state(X_n), is independent of the past states. The Markov property asserts that the process is memory-less. (Udom, 2010).

Transition Probability Matrix

In Markov chains, the transition probability matrix (or transition matrix) is a fundamental concept that defines how the system moves from one state to another over discrete time steps. The transition probability matrix P is a square matrix where each element P_{ij} represents the probability of moving from state i to state j in one time step.

Thus:

$$P = \begin{pmatrix} P_{11} & P_{12} & \ldots\ldots & P_{1r} \\ P_{21} & P_{22} & \ldots\ldots & P_{2r} \\ \vdots & \vdots & \vdots & \vdots \\ P_{r1} & P_{r2} & \ldots\ldots & P_{rr} \end{pmatrix} \quad \textbf{(2)}$$

r x r matrix

Properties

1. **Dimensions**: If the Markov chain has nnn states, then the transition matrix P is an nXn matrix.

2. **Element Interpretation**: P_{ij} denotes the probability that the system will transition from state i to state j in the next time step.

3. **Row Sum Property**: The sum of probabilities in each row of P equals 1. This reflects the fact that from any given state, the system must move to one of the possible states in the next time step.

$$\sum_{j=1}^{n} P_{ij} = 1 \ for\ all\ i$$

4. **Stationary Distribution**: If the Markov chain is irreducible and positive recurrent, it converges to a unique stationary distribution π. The stationary distribution satisfies

$\pi P = \pi$, where π is a row vector such that

$\sum_i \pi_i = 1$ and $\pi_i \geq 0$.

Steady State Probability of a Markov Chain

The steady-state probability of a Markov chain refers to the long-term probability distribution of states that the chain tends to approach as time progresses, assuming it exists, and the chain satisfies certain conditions. That is The steady-state probability distribution $\pi = (\pi_1, \pi_2, \ldots, \pi_n)$ of a Markov chain with n states is a vector where π_i represents the long-run proportion of time the chain spends in state i, given that the Markov chain has been running for a sufficiently long time.

Finding the Steady-state Distribution

Once we determine that the Markov chain satisfies irreducibility and aperiodicity, the steady-state distribution $\pi = (\pi_1, \pi_2, \ldots, \pi_n)$ can be found as the solution to the equation:

$$\pi P = \pi \qquad \textbf{(4)}$$

Where P is the transition probability matrix of the Markov chain. This equation essentially states that π is a left eigenvector of P corresponding to the eigenvalue 1 (assuming P is a stochastic matrix, where each row sums to 1). The iterative process is used to predict the future state of the system, and repeated multiplication by the transition matrix does not alter the long-term probability vector. The equilibrium probability matrix is shown below.

$$\pi = \begin{pmatrix} \pi_1 & \pi_2 & \cdots & \pi_r \\ \pi_1 & \pi_2 & \cdots & \pi_r \\ \vdots & \vdots & \vdots & \vdots \\ \pi_1 & \pi_2 & \cdots & \pi_r \end{pmatrix} \qquad \textbf{(5)}$$

r x r matrix

Mean Recurrence Time

Mean recurrence time is specifically refers to the expected time between consecutive visits to a particular state or set of states. If a state i is visited multiple times, the mean recurrence time μ_i is the average time between consecutive visits to state i. And mathematically, it is defined as $\mu_i = \frac{1}{\pi_i}$, where π_i is the steady-state probability of being in state i (assuming the chain is irreducible and positive recurrent).

Absorbing State of a Markov Chain

Absorbing state of a Markov chain is refers to a state from which once entered, the process cannot leave. Formally, a state i in a Markov chain is absorbing if the chain can transition from

any other state to state i and once the chain enters state i, it remains there indefinitely.

Canonical Form of an Absorbing Markov Chain

We have an $n\ x\ n$ transition matrix, denoted as P, which is created by an arbitrary absorbing Markov chain. This chain consists of K absorbing states and $(n-K)$ non-absorbing states. The transition matrix P has a simple form, as shown below, with alternating rows and columns, placing the absorbing states first (Robert, 1976; Kemeny et al., 1976). The transition matrix will have the following canonical form:

$$P = \begin{pmatrix} Q & R \\ 0 & I \end{pmatrix} \quad \textbf{(6)}$$

2 x 2 matrix

Where, Q is submatrix representing transitions between transient states. It is of size $(k\ X\ k)$\times where k is the number of transient states, R is submatrix representing transitions from transient states to absorbing states. It is of size $(k\ X\ r)$, where r is the number of absorbing states. 0 is $k\ X\ r$ zero matrix, representing transitions from absorbing states to transient states (which are zero because once absorbed, the chain stays in the absorbing state) and I is $r\ X\ r$ identity matrix, representing the absorbing states themselves (since once absorbed, the chain stays in the absorbing state).

The Fundamental Matrix (N)

The matrix $(1-Q)^{-1}$ is a foundational matrix, with I representing the identity matrix. In this study, it was used to calculate the expected time that a patient would spend in the transitional state j after starting in the transient state. This model was applied to determine the life expectancy of patients in state j who initiated ART treatment in state i. The underlying assumption is that life expectancy is a stochastic variable that influences the dynamics of HIV/AIDS (Nwosu, 2015). The basic matrix is represented as:

$$(I-Q)^{-1} = \begin{pmatrix} F_{11} & \cdots\cdots & F_{1N} \\ \vdots & \ddots & \vdots \\ F_{N1} & \cdots\cdots & F_{NN} \end{pmatrix} \quad \textbf{(7)}$$

N x N matrix

Where I is an identity matrix and Q is the initial probability matrix.

Use of Statistical Software

I used Microsoft Excel (MS Excel 2013) to convert the CD4 counts of HIV/AIDS patients into states (I, II, and III). I also used Turbo Pascal version 1.5 and SPSS 23 to calculate the transition matrix powers and the Steady-state probability Matrix.

RESULTS

In Table 1, the CD4 cell counts of 13,710 HIV/AIDS patients were tracked over time to reflect the transition among different health states (I, II, and III) over a period of six months. These states represent good, moderate, and poor health, respectively.

Table 1: CD4 Transition Counts of the HIV/AIDS Patients.

	I	II	III	Total
I	1562	1644	522	3728
II	3206	2196	854	6256
III	1456	1576	694	3726
Total	6224	5416	2070	13710

The initial transition probability matrix (P1) was calculated by dividing the elements of each row by their respective row totals, as shown in Table 4.1. This matrix shows the initial probability of a patient transitioning from one health status to another within the first six months of starting ART. The diagonal elements of the matrix indicate that a patient has a 42%, 35%, and 19% chance of remaining in Good, Moderate, and Poor health statuses, respectively. Meanwhile, the off-diagonal elements show the probability of a patient moving between the three health stages.

Initial Transition Probability Matrix in the First Six (6) Months

$$P^{I} = \begin{matrix} \\ I \\ II \\ III \end{matrix} \begin{pmatrix} I & II & III \\ 0.419 & 0.441 & 0.140 \\ 0.512 & 0.351 & 0.137 \\ 0.391 & 0.423 & 0.186 \end{pmatrix}$$

3 x 3 matrix

For the remaining six-month interval visits, we used the N-Step transition probability matrix, which was determined by calculating the powers of the original transition matrix (P^1). The resulting matrices for P^2, P^3, P^4,..., $P^{\&}$ show the probability of patients transitioning between the three health statuses at the second, third, and seventh six-month interval appointments. Therefore, P^7 represents the steady-state probability matrix of the patient's response to ART. The diagonal components of each matrix show the likelihood of a patient staying in the Good, Moderate, or Poor health states, while the off-diagonal elements represent the likelihood of a patient transitioning between the three health categories

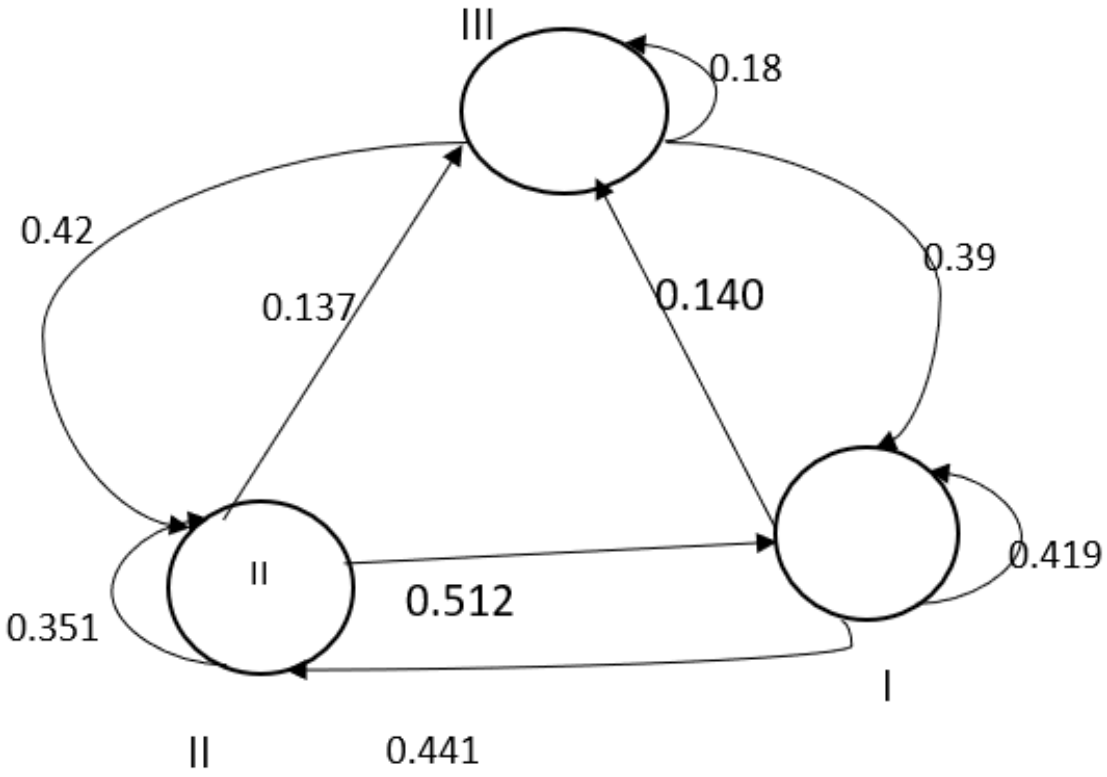

Figure 1: Transition Diagram of Patients between Health States in the First Six (6) Months.

Table 2: N-Step Transition Probabilities.

N^{th} Six Month interval appointments	Previous State	Transition Probability Actual State		
		I	II	III
2nd	I	0.456093	0.398790	0.145117
	II	0.447807	0.406944	0.145249
	III	0.453131	0.399582	0.147287
3rd	I	0.452024	0.402497	0.145479
	II	0.452779	0.401761	0.145461
	III	0.452037	0.402386	0.145576
4th	I	0.452359	0.402157	0.145485
	II	0.452291	0.402223	0.145486
	III	0.452346	0.402165	0.145489
5th	I	0.452327	0.402187	0.45486
	II	0.452333	0.402181	0.145486
	III	0.452328	0.402186	0.145486
6th	I	0.452330	0.402184	0.145486
	II	0.452329	0.402185	0.145486
	III	0.452330	0.402184	0.145486
7th	I	0.452330	0.402185	0.145486
	II	0.452330	0.402185	0.145486
	III	0.452330	0.402185	0.145486

Figure 2 shows the transition probabilities from states I, II, and III to state I after the first six (6) months of the appointment. The probability that a patient will remain in state I is 0.456093 (46%), while the chances of transitioning from states II and III to state I are 0.447807 (44%) and 0.453131 (45%) respectively in the second six (6) months of the appointment. This indicates a slight decline in the chances of the patient's health improving from moderate and poor health states to the good health state in the second six-month period under review. However, these chances decrease continually over the rest of the appointments and become constant at the 7th appointment, reaching a steady-state value of 0.452, explaining a 45% chance that a patient will be in the good health state in the long run. The graphical visualization illustrates the progression of patients' response to the ART based on the results obtained in Table 2.

The probabilities of patients transitioning from states I, II, and III to state II are illustrated in Figure 3 below. At the second six-month interval appointment, there is a 41% probability that a patient will remain in the Moderate health state. The probability of transitioning from the Good health state (I) to the Moderate health state (II) is 39.8%, while the probability of transitioning from the Poor health state (III) to the Moderate health state (II) is 39.9%. These probabilities fluctuated over subsequent appointments and stabilized at 40% during the 7th appointment, indicating a long-term 40% chance of a patient being in a moderate health state.

In Figure 4, you can see the transition probabilities of patients from states I, II, and III to III. After the second six-month appointment, the probability that a patient will remain in a poor health state is 14.7%. The chances of transitioning from the Good and Moderate health states to the poor health state are 14.5% and 14.5% respectively. These probabilities continue to vary over the subsequent appointments and become steady at the 7th appointment, with a value of 14.5%. This shows that there is a 14.5% chance that a patient will remain in a poor health state in the long run.

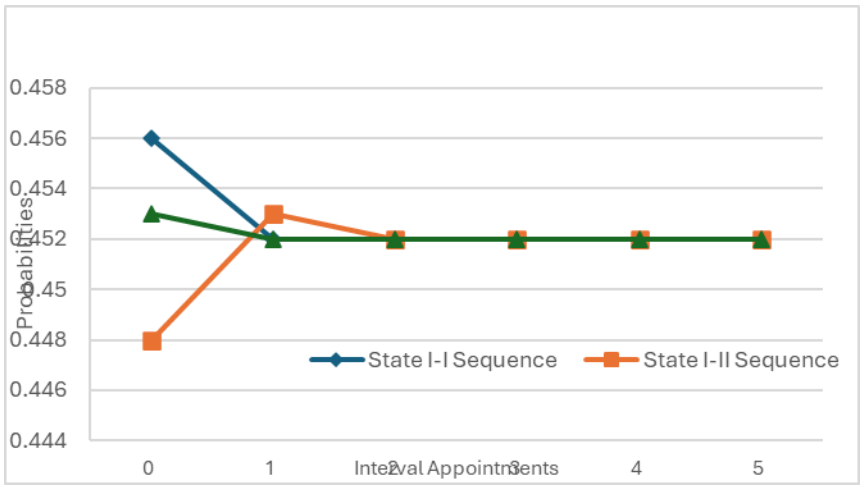

Figure 2: Transition Probabilities of Patients from States I, II, III to State I.

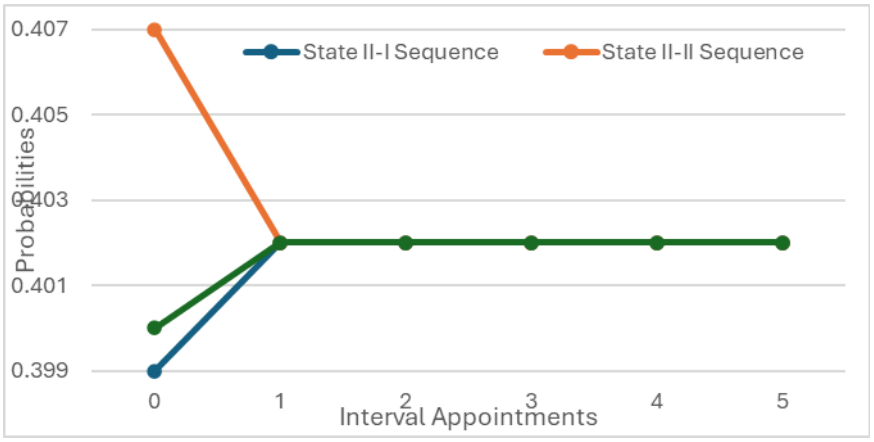

Figure 3: Transition Probabilities of Patients from States I, II, III to II.

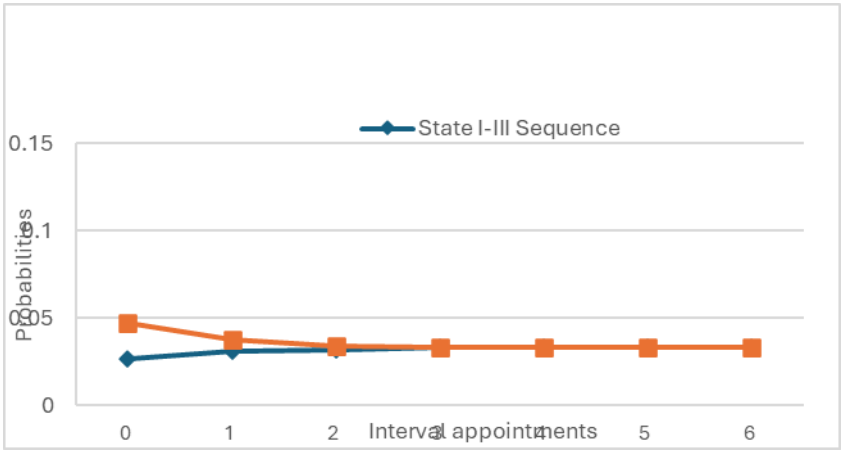

Figure 4: Transition Probabilities of Patients from States I, II, III to III.

Assessing the Efficacy of ART on Patients

The efficacy of a treatment or therapy is a measure of the maximum response of patients to treatments (Agada et al., 2018). In this study, efficacy refers to the long-term potential of the ART to produce optimal effects on the patients. Therefore, in forecasting the efficacy of the ART, the long-term probabilities or steady-state probabilities of the patient's health status are used, as shown in the matrix below.

Steady States Probability Matrix of Patients in the 7th Six (6) Months

$$P^7 = \begin{matrix} \\ I \\ II \\ III \end{matrix} \begin{pmatrix} I & II & III \\ 0.452 & 0.402 & 0.145 \\ 0.452 & 0.402 & 0.145 \\ 0.452 & 0.402 & 0.145 \end{pmatrix}$$

Hence, this matrix allows us to forecast the long-term (steady state) probabilities of patients being in a specific health state. It's important to note that the overall effectiveness of the ART results in a patient being in a good health state 45% of the time, in a moderate health state 40% of the time, and in a poor health state 15% of the time.

Forecasting the Life Expectancy of the HIV/AIDS Patients

In order to account for death, we need to adjust the transition matrix for the three health states in the initial probability matrix and calculate a new transition matrix $(\hat{P})$ for an absorbing Markov Chain. To achieve this, we need to consider that patients in state III (CD4 count < 200) might die and not transition out of the system.

The computation $(I - \hat{P})^{-1}$ is called the fundamental matrix, where I is an identity matrix as defined above. Before the elements of the fundamental matrix are computed, it is necessary to first determine the canonical form of the original transition probability matrix (P), as this will aid in the computation. The mathematical steps are shown below.

Given the Canonical form of matrix P be denoted as P_{can}. Then,

$$P^7 = \begin{matrix} \\ I \\ II \\ III \end{matrix} \left(\begin{array}{ccc} I & II & III \\ 0.419 & 0.441 & 0.140 \\ 0.512 & 0.351 & 0.137 \\ \hline 0.000 & 0.000 & 1.000 \end{array}\right)$$

Therefore,

$$\hat{P} = \begin{matrix} \\ I \\ II \end{matrix} \begin{pmatrix} I & II \\ 0.419 & 0.441 \\ 0.512 & 0.351 \end{pmatrix}$$

The required fundamental matrix now becomes:

$$(I - \hat{P})^{-1} = \left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 0.419 & 0.441 \\ 0.512 & 0.351 \end{pmatrix}\right)^{-1}$$

$$\begin{pmatrix} 7.263 & 5.513 \\ 6.400 & 8.113 \end{pmatrix}$$

Adding the row elements, we have:

$$\begin{pmatrix} 12.78 \\ 14.51 \end{pmatrix}$$

The fundamental matrix indicates the life expectancies of patients in each health state before their passing, with the total life expectancies for patients in good and moderate health states being 12.78 and 14.51 years, respectively. It's important to note that these life expectancies are not 100%, as evident from the fundamental matrix. The differences in life expectancies between patients in states I and II may be a significant factor in the patients' progression to state III and the limitations of the therapy.

CONCLUSION

The initial probabilities of a patient being in Good, Moderate, and Poor health states in the first six (6) months of ART initiation were 0.419, 0.351, and 0.186 respectively. The overall effectiveness of the ART treatment results in a patient being in

Good health 45% of the time, Moderate health 40% of the time, and Poor health 15% of the time. Patient health was regularly evaluated during appointments to determine transitions between health states. The life expectancy for patients in the Good and Moderate health states was 12.78 and 14.51 years, respectively.

REFERENCES

1. Afolabi, M.O., K.T. Ijadunola, A.O. Fatusi, and O.A. Olasode. 2009. "Determinants of Adherence to Antiretroviral Drugs Among People Living with HIV/AIDS in the Ife-Ijesa of Osun State, Nigeria". *African Journal of Primary Health Care & Family Medicine*. 1(1): 1-6 DOI:10.4102/phcfm.v1i1.6.

2. Agada, P.O., J. Eneh, and J.A. Ikughur. 2018. "The Process for Assessing the Progress and Predicting the efficacy of Antiretroviral Therapy (ART)". *FUW Trends in Science and Technology Journal*. 3(2): 12-26.

3. Agada, P.O., B.B. Apeagee, and I.O. Ogwuche. 2019. "A Stochastic Model for Assessing the Efficacy of Antiretroviral Therapy (ART) on Farmers Living with HIV/AIDS". *Nigerian Annals of Pure and Applied Science*. 2: 248 - 258.

4. Antiretroviral Therapy. 2006. "Antiretroviral Therapy Coverage Rate in Low- and Middle-Income Countries". http://www.aidsportal.org/Article_Details.aspx?ID=3121

5. Bature, R.S., A.A. Obiniyi, E.E. Absalom, and O.O. Sule. 2010. "Markov Chain Simulation of HIV/AIDS Movement Pattern". *Int J Comput Sci.* 8:156-67.

6. Commenges, D., P. Joly, L. Letenneur, and J.F. Dartiques. 2004. "Incidence and mortality of Alzheimer's Disease or Dementia using an Illness-Death Model". *Stat Med.* 23:199-210.

7. Davies, R., D. Johnson, and S. Farrow. 1975. "Planning Patient Care with a Markov Model". *Opl Res*. Q. 26:599-607.

8. Jackson, C.H. and L.D. Sharples. 2002. "Hidden Markov Models for the Onset and Progression of *Bronchiolitis obliterans* Syndrome in Lung Transplant Recipients". *Stat Med*. 21:113-28.

9. Kay, R.A. 1986. "Markov Model for Analyzing Cancer Markers and Disease States in Survival Studies". *Biometrics*. 42: 855-65.

10. Kuo, H.S., H.J. Chang, P. Chou, L. Teng, and T.H. Chen. 1999. "A Markov Chain Model to Assess the Efficacy of Screening for Non-Insulin Dependent Diabetes Mellitus (NIDDM)". *Int. J. Epidemiol.* 28: 233-40.

11. Rajesh, T.G., R. Bedimo, J.F. Hoy, *et al*. 2022. "Antiretroviral Drugs for Treatment and Prevention of HIV Infection in Adults". *JAMA*. 329(1): 63-84. doi:10.1001/jama.2022.22246.

12. Saag, M.S., R.T Gandhi, J.F. Hoy, *et al.* 2020. "Antiretroviral Drugs for Treatment and Prevention of HIV Infection in Adults". *JAMA*. 324(16): 1651-1669. doi:10.1001/jama.2020.17025.

13. Nasarawa State AIDS Control Agency (SACA). 2023. June 16, 2023.

14. National Center for AIDS and STI Control. 2009. "Cumulative HIV/AIDS Situation of Nepal, Facts and Figure". Teku, Kathmandu, Nepal.

15. National Center for AIDS and STI Control. 2006. "National Guidelines on Anti-Retroviral (ARV) Therapy". Revised Edition.

16. National HIV/AIDS Strategy (2006-2011). Department of Health Services. Federal Ministry of Health. "National Sentinel Surveillance Survey Technical Report (2005)". Federal Ministry of Health, Abuja, Nigeria.

17. Nwosu, C.A. 2015. "Determining the Life Expectancy of HIV/AIDS Patients in Anambra using Stationary and Smoothed Non-Stationary Markov Chain Models". *ABACUS*. 42(2): 93-107.

18. Pérez-Ocón R., J.E. Ruiz-Castro, and M.L. Gámiz-Pérez. 2001. "Non-Homogeneous Markov Models in the Analysis of Survival after Breast Cancer". *J R Stat Soc Ser C Appl Stat*.;50: 111-24.

19. Sweeting, M.J., V.T. Farewell, and D. De Angelis. 2010. "Multi-State Markov Models for Disease
20. Progression in the Presence of Informative Examination Times: An Application to Hepatitis C". *Stat Med*. 29: 1161-74.

21. Udom, A.U. 2010. *Elements of Applied Mathematical Statistics*. ICIR: Akwa-Ibom, Nigeria.

22. UNAIDS. 2014. "Global AIDS Response Progress Reporting: UNAIDS: New York, NY.

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