Investigation of the Residual Stress and Ultimate Tensile Stress of Mild Steel Plate having Multiple Holes

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ABSTRACT

The durability of mild steel plate is usually influenced by the presence of drilled holes. These holes are required in engineering design for weight reduction, quick dissipation of heat, etc. There are usually inbuilt locked-in stresses in materials as a result of drilling operations. In this study, residual stress and ultimate tensile strength were studied to ascertain the mechanical behavior of a 6mm mild steel plate having multiple holes.

Selected input variables namely drilling angle, operating current and cutting speed were subjected to statistical design of experiment using the CCD method. The modelling and optimization results showed that the drilled multiple holed mild steel plates samples had an optimum solution of 30o for angle, 53.74amps for operating current and 13.23mm/min for cutting speed. The yielded best overall performance response values were residual stress of 111.784MPa and ultimate tensile stress of 320.054MPa.

(Keywords: residual stress, ultimate tensile stress, mild steel plate)

INTRODUCTION

The importance of monitoring residual stresses on different metals used in engineering applications cannot be overstated in modern development, as this will determine whether a particular metal will fulfill design and service requirements when used in different manufacturing or production processes. The mechanical and physical characteristics of a metal determine its ability to satisfy these specifications.

According to Huber and Beedle (1954), Fukumoto and Itoh (1981), and Chenenko and Kennedy (1991), residual stresses are formed mainly during the section production process and are linked to non-uniform plastic deformation and differential cooling. Relative to structural members, residual stresses generally result in early yielding, which reduces stiffness and load-carrying capability. According to a thorough benchmark assessment of surface and subsurface integrity, the most frequently assessed surface integrity metrics are residual stresses, surface hardness, and microstructural alterations (Jawahir, *et al.*, 2011). The use of cutting models to forecast residual stresses has generally received a lot of attention since it can overcome many of the drawbacks of experimental residual stress measurement (Yang and Liu, 2001).

Temperature gradients, surface treatments, material forming, shaping techniques, non-uniform plastic deformation during processing, and phase transformations during heat treatment are some of the mechanisms that can lead to misfits or non-homogenous stresses among the various parts of a material or structure, which can result in residual stresses in composite materials, welds, quenched components, semiconductor fabrication, and thin films. They may be compressive or tensile. The applied stresses are added up algebraically. Advantageous compressive residual stresses are intentionally added in some cases, such as pre-stressed concrete, brittle materials that can be toughened, shot peening, quenching, and tempered glass. In general, residual tensions are undesirable. (Gdoutos, 2022).

Residual stresses can be identified by the length scale over which they stabilize or equilibrate, according to Withers and Bhadeshia (2001). Long-range macro-stresses and short-range micro-stresses are two categories into which they can be divided. It's common to refer to macro-stressors as Type I stresses.

In 2020, Etin-Osa et al. investigated the proof of principle for determining the weld residual stresses in a material. A magnetic residual stress instrument was used to measure the samples in order to do this. Then, the weld samples were placed through a mechanical tensile test in the hopes that the materials with the highest residual stresses would break first. Finally, a correlation was found by comparing the results of the two processes. Two specimens, A11 and B11, each measuring 200 × 40 × 3 mm, were produced when four (4) mild steel coupons measuring 100 × 40 × 3 mm were welded together. Tensile testing was performed on the welded specimens, and the results were analyzed following measurement of the specimens using the built-in magnetic device, yielding 37 signals per specimen. This simple experiment indicates that the Magnetic RS device was able to reproduce the behavior of residual stress and predicted that A11 would fail first. This study aims to model and optimize the residual stress and ultimate tensile stress of multiple holed mild steel plate with laboratory generated and simulated data using RSM and FEM.

MATERIALS AND METHODS

Material Selection and Design Consideration

Some of the materials used for this study include mild steel plate of 6mm thickness, portable drilling machine, punch, angle grinder and X-ray diffraction machine.

To prepare the sample, selected input variables including drilling angle, operating current and cutting speed were subjected to statistical design of experiment using the CCD method. Using the experimental design, twenty pieces of mild steel plates with multiple holes were produced and tested using X-ray diffraction technique to determine the residual stress, the tensometer for ultimate tensile strength.

The various methods used in this study included Design of Experiment (DoE), sample preparation, laboratory tests to generate data, data analysis and simulation, and modelling, optimization and validation of data.

Design of Experiment

The durability of mild steel plate is usually influenced by the presence of drilled holes. Often, these holes are required in engineering design for weight reduction, quick dissipation of excess heat, frictional increase between plates or other design requirement needed to improve efficiency. To optimize the process involved in the preparation of the mild steel in order to achieve the desired residual stress and ultimate tensile strength, Response Surface Methodology (RSM) was employed. The specific objective of the optimization model was to:

i. Minimize the residual stress;

ii. Maximize the ultimate tensile strength;

The optimization process's ultimate goal was to identify the ideal values for the input variables—the drilling angle (degree), operating current (amp), and cutting speed (mm/min)—that would optimize the ultimate tensile strength and minimize residual stress. The experimental data for the optimization process was created using the following procedures.

i. A statistical design of experiment (DOE) was conducted by employing the central composite design approach (CCD).

ii. Using the DOE as a guide, an experimental design matrix was created with twenty (20) experimental runs, six (6) center points, six (6) axial points, and eight (8) factorial points.

Table 1 lists the experimental variables' range and level that were employed in the statistical design of the experiment.

Table 1: Range and Levels of Independent Variables.

Independent Variables	Range and Levels of Input Variables	
	Lower Range (-1)	Upper Range (+1)
Drilling Angle (degree) X_1	30	90
Operating Current (amp) X_2	30	80
Cutting Speed (mm/min) X_1	5	20

Selection of Appropriate Experiment Design Method

A central composite design (CCD) statistical Design of Experiment (DoE) was conducted utilizing the range and values of the independent variables listed in Table 1. With the help of design expert version 7.01, experimental design was completed. Montegomery (2005) defines the total number of experimental runs that can be produced with the CCD as follows:

$$N= 2^n + n_o + 2n \quad \textbf{(1)}$$

where,
N = the number of experimental runs based on CCD design
2^n = the number of factorial points
n_o = the number of center points
2n = the number of axial points,
n = the number of variables.

Determination of the Number of Experimental Runs

Using Equation (1), twenty (20) experimental runs were generated based on the CCD method and presented in Table 2.

Table 2: Design of experiment (DOE).

Run	Type	Factor 1 Drilling Angle (degree)	Factor 2 Operating Current (amp)	Factor 3 Drilling Speed (mm/min)	Response 1 Residual Stress (MPa)	Response 2 Ultimate Tensile Stress (MPa)
1	Center	60	55	12.5		
2	Center	60	55	12.5		
3	Center	60	55	12.5		
4	Center	60	55	12.5		
5	Center	60	55	12.5		
6	Center	60	55	12.5		
7	Axial	60	55	10.5		
8	Axial	45	55	12.5		
9	Axial	60	55	10		
10	Axial	60	40	12.5		
11	Axial	60	50	12.5		
12	Axial	45	55	12.5		
13	Fact	30	30	5		
14	Fact	90	30	5		
15	Fact	90	30	20		
16	Fact	30	30	20		
17	Fact	90	80	20		
18	Fact	30	80	5		
19	Fact	90	80	5		
20	Fact	30	80	20		

Preparation of Sample

In preparing and testing the residual stress and Ultimate Tensile Stress of the specimens, some of the experimental procedures were carried out in the Production Engineering Workshop of the University of Benin, while the others were done in the Metallurgical Department of Ahmadu Bello University (ABU), Zaria.

Drilling along the targeted angles lines of 30^o, 45^o, 60^o and 90^o were performed using a portable drilling machine and cleaned with angle grinder. The punch was used to dent the exact spots for the drilling path of the drill bit. The corresponding drilling speed and operating current were set for the various drilling angles during the drilling operations with respect to the DoE. Figure 1 shows a sample of the drilled mild steel plate.

Figure 1: The Drilled 6mm Mild Steel Plate.

Residual Stress Test (RST)

In the measurement of residual stresses on the samples, the non-destructive measurement method was done using the Rigaku X-ray diffraction machine.

Ultimate Tensile Test (UTS)

The test specimens were cut out of the plate with dimensions as shown in Figure 2. l_2 = 140mm, b_l = 30mm, l_0 = 80mm, b = 20mm l_l = 100mm and t = 6mm.

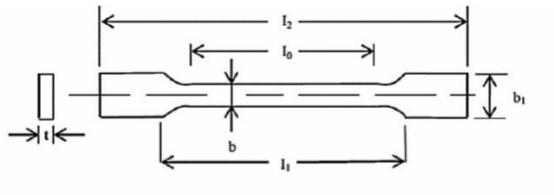

Figure 2: Tensile Test Piece Specimen.

Using Equation (2), the ultimate tensile stresses were calculated for the five samples.

$$\sigma = F/A \qquad \textbf{(2)}$$

where,
σ = Stress,
F = Applied force,
A = Cross-sectional Area of the test specimen.

A total of sixty (60) pieces of holed mild steel coupons having a dumbbell shape, were prepared for the experiment (Figure 3).

Figure 3: Dumbbell Samples with Drilled Holes.

The control of experiment samples were also produced that has no drilled holes on the mild steel plate (Figure 4).

Figure 4: Control of Experiment Sample.

Analyses of Data

The results obtained from the laboratory experiments were imported into the experimental design done by using the CCD, so as to analyze the generated data.

RESULTS AND DISCUSSION

The result obtained from the laboratory analysis of the sample is presented in Table 3. The study investigated the impact of changing input variables—drilling angle, operating current, and cutting speed—on the response variables, which included residual stress and ultimate tensile strength of a mild steel plate with multiple holes. Variations in the drilling angle had a noticeable effect on the response variables. For example, an axial drilling angle of 60 degrees generally resulted in higher residual stress and ultimate tensile strength compared to other angles. Lower operating currents were associated with lower residual stresses and strains, while higher currents tended to increase these response variables. When the cutting speed was reduced, an increase in residual stress and ultimate tensile strength was observed. Specifically, the mild steel plates experienced significant changes in residual stress, ranging from 110MPa to 185MPa, and ultimate tensile strength, ranging from 270.8MPa to 380.42MPa, based on the input conditions. These results demonstrated the intricate relationship between the input variables and the mechanical properties of the mild steel plates being investigated, providing valuable insights for optimizing manufacturing processes.

The results of the study had several significant implications for the mechanical properties of mild steel plates with multiple holes. Changes in the input variables; drilling angle, operating current, and cutting speed affected the residual stress of the plates. Increased residual stress, which was observed at higher operating currents and lower cutting speeds, could lead to decreased fatigue life and an increased likelihood of crack formation. This implied that optimizing these parameters was crucial to minimizing residual stress and enhancing the durability of the plates. The ultimate tensile strength of the mild steel plates also varied with the input conditions. Higher ultimate tensile strengths were generally observed with specific combinations of drilling angles and cutting speeds. This indicated that the mechanical integrity and load-bearing capacity of the plates could be optimized by carefully selecting these variables. Ensuring higher tensile strength was important for applications where the steel plates were subjected to significant mechanical loads, as it would prevent premature failure and ensure structural integrity.

Table 3: Experimental Test Results.

Run	Type	Factor 1 Drilling Angle (degree)	Factor 2 Operating Current (amp)	Factor 3 Cutting Speed (mm/min)	Response 1 Residual Stress (MPa)	Response 2 Ultimate Tensile Stress (MPa)
1	Center	60	55	12.5	110	284.92
2	Center	60	55	12.5	111	275.33
3	Center	60	55	12.5	115	277.61
4	Center	60	55	12.5	110	270.8
5	Center	60	55	12.5	115	273.44
6	Center	60	55	12.5	115	273.22
7	Axial	60	55	10.5	121	298.5
8	Axial	45	55	12.5	165	298.65
9	Axial	60	55	10	145	307.58
10	Axial	60	40	12.5	135	300.33
11	Axial	60	50	12.5	121	280
12	Axial	45	55	12.5	120	380.42
13	Fact	30	30	5	140	350.54
14	Fact	90	30	5	172	308.71
15	Fact	90	30	20	120	298.34
16	Fact	30	30	20	131	299.87
17	Fact	90	80	20	160	298.5
18	Fact	30	80	5	125	370.65
19	Fact	90	80	5	185	290.22
20	Fact	30	80	20	145	345.91

Table 4: Descriptive Statistics of Experimental Data.

Index	Drilling Angle (Degree)	Operating Current (amp)	Cutting Speed (mm/min)	Residual Stress (MPa)	Ultimate Tensile Strength (MPa)
Mean	58.500	54.000	12.275	133.050	304.177
Standard Error	4.474	3.710	1.099	5.007	7.259
Median	60.000	55.000	12.500	123.000	298.500
Mode	60.000	55.000	12.500	115.000	298.500
Standard Deviation	20.007	16.591	4.916	22.392	32.463
Sample Variance	400.263	275.263	24.170	501.418	1053.822
Kurtosis	-0.544	-0.494	-0.388	0.030	0.737
Skewness	0.201	0.156	0.141	1.010	1.275
Range	60.000	50.000	15.000	75.000	109.620
Minimum	30.000	30.000	5.000	110.000	270.800
Maximum	90.000	80.000	20.000	185.000	380.420
Sum	1170.000	1080.000	245.500	2661.000	6083.540
Count	20.000	20.000	20.000	20.000	20.000

The descriptive statistics of the experimental data are presented in Table 4.

The descriptive statistics for the drilling angle revealed that the mean value was 58.5^{o}, indicating that, on average, the drilling angle used in the experiments was close to this value. The standard error was 4.474, suggesting that the sample mean was fairly precise. The median and mode of the drilling angle were both 60^{o}, showing that this value was the most frequently occurring and central value in the data set. The standard deviation was 20.007^{o}, indicating significant variability in the drilling angles used. The range was 60^{o}, with a minimum of 30^{o} and a maximum of 90^{o}, highlighting the broad spectrum of drilling angles tested.

The operating current had a mean of 54amps, with a standard error of 3.710, suggesting reasonable precision in the sample mean. The median and mode were both 55amps, showing that this was the most common and central value. The standard deviation of 16.591amps indicated considerable variation in the operating currents applied. The range of operating currents was 50amps, with values spanning from a minimum of 30amps to a maximum of 80amps. The kurtosis and skewness values, both close to zero, suggested a relatively normal distribution of the operating current data.

The cutting speed's mean was 12.275mm/min, with a standard error of 1.099, indicating a relatively precise sample mean. The median and mode values were 12.5mm/min, reflecting the central tendency and most frequent value. The standard deviation was 4.916mm/min, showing moderate variability in the cutting speeds used. The range was 15mm/min, with a minimum of 5 mm/min and a maximum of 20mm/min. The negative kurtosis and skewness values suggested a slightly symmetrical distribution.

For residual stress, the mean value was 133.05MPa, with a standard error of 5.007, indicating a precise estimate of the sample mean. The median residual stress was 123MPa, and the mode was 115MPa, showing the central and most frequent values. The standard deviation was 22.392MPa, indicating considerable variability in residual stress measurements. The range of residual stress values was 75MPa, with a minimum of 110MPa and a maximum of 185MPa. The positive skewness and near-zero kurtosis indicated a slight skew towards higher values and a near-normal distribution.

The ultimate tensile strength had a mean of 304.177MPa and a standard error of 7.259, suggesting a precise sample mean. The median value was 298.5MPa, and the mode was also 298.5MPa, reflecting the central and most frequent values. The standard deviation was 32.463MPa, showing considerable variability in tensile strength measurements. The range was 109.62MPa, with a minimum of 270.8MPa and a maximum of 380.42MPa. The positive skewness and kurtosis values indicated a skew towards

higher tensile strength values and a leptokurtic distribution.

The descriptive statistics of the data provide several important implications for understanding and optimizing the mechanical properties of mild steel plates with multiple holes. Firstly, the wide ranges observed in drilling angle, operating current, and cutting speed highlight the necessity for precise control and optimization of these parameters. Variability in these input variables directly influences mechanical properties such as residual stress and tensile strength. Engineers and manufacturers can use this information to fine-tune production processes to achieve desired mechanical outcomes. Moreover, the range and distribution of values for residual stress and ultimate tensile strength underscore how variations in manufacturing parameters can impact the structural integrity and performance of mild steel plates. Higher variability may indicate potential challenges in maintaining consistent mechanical properties across production batches, necessitating rigorous quality control measures.

The descriptive statistics provide a baseline for evaluating and improving manufacturing processes. Identifying outliers or trends in the data can signal areas for improvement in production techniques, material selection, or equipment calibration, ultimately leading to enhanced product quality and customer satisfaction.

Optimization Results using RSM

The randomized design comprising of three input variables, namely drilling angle (degree), operating current (amp) and the cutting speed (mm/min) coupled with two response variables, namely, residual stress (MPa) and ultimate tensile strength (MPa) in coded values and real values are presented in Tables 3 and 5 respectively.

Table 5: Design Matrix showing the Coded Values and the Experimental Values.

Run	Type	Factor 1 Drilling Angle (degree)	Factor 2 Operating Current (amp)	Factor 3 Cutting Speed (mm/min)	Response 1 Residual Stress (MPa)	Response 2 Ultimate Tensile Stress (MPa)
1	Center	0	0	0	110	284.92
2	Center	0	0	0	111	275.33
3	Center	0	0	0	115	277.61
4	Center	0	0	0	110	270.8
5	Center	0	0	0	115	273.44
6	Center	0	0	0	115	273.22
7	Axial	0	0	-0.27	121	298.5
8	Axial	1.68	0	0	165	298.65
9	Axial	0	0	1.68	145	307.58
10	Axial	0	1.68	0	135	300.33
11	Axial	0	-1.68	0	121	280
12	Axial	-1.68	0	0	120	380.42
13	Fact	-1	-1	-1	140	350.54
14	Fact	1	-1	-1	172	308.71
15	Fact	1	-1	1	120	298.34
16	Fact	-1	-1	1	131	299.87
17	Fact	1	1	1	160	298.5
18	Fact	-1	1	-1	125	370.65
19	Fact	1	1	-1	185	290.22
20	Fact	-1	1	1	145	345.91

Table 6: RSM Design Summary.

Study Type	Response Surface	Runs	20							
Initial Design	Central Composite	Blocks	No Blocks							
Design Model	Quadratic									

Factor	Name	Units	Type	Low Actual	High Actual	Low Coded	High Coded	Mean	Std. Dev.		
A	Drilling Angle	Degree	Numeric	30	90	-1	1	60	24.79029		
B	Operating Current	Amp	Numeric	30	80	-1	1	55	20.65857		
C	Cutting Speed	mm/min	Numeric	5	20	-1	1	13.03	5.511201		

Response	Name	Units	Obs	Analysis	Minimum	Maximum	Mean	Std. Dev.	Ratio	Trans	Model
Y1	Residual Stress	MPa	20	Polynomial	110	185	133.05	21.825	1.681818	None	Quadratic
Y2	Tensile Strength	Mpa	20	Polynomial	270.8	380.42	304.18	31.640	1.404801	None	Quadratic

The model summary which shows the factors and their lowest and highest values including the mean and standard deviation is presented in Table 6.

Result of Table 6 showed that the model is of the quadratic type. The minimum drilling angle was observed to be 30^{o} with a maximum value of 90^{o} and mean ± standard deviation of 60.00±24.79029. For operating current, the minimum value was 30amp with a maximum value of 80amp and mean ± standard deviation of 55.00±20.65857. For cutting speed, the minimum value was 5mm/min with a maximum value of 20mm/min and mean ± standard deviation of 13.03±5.511201. Each response variable's sequential model sum of squares was computed to confirm that the quadratic model was appropriate for interpreting the experimental data. The results are shown in Tables 7 and 8, respectively.

As terms are added, the sequential model sum of squares table displays the cumulative improvement in the model fit. The highest order polynomial with significant additional terms and an unaliased model was chosen as the best fit based on the computed sequential model sum of squares.

It was determined from Tables 7–8 that the cubic polynomial was aliased and could not be used to fit the final model. The choice of the quadratic polynomial in this study was also justified because the quadratic and 2FI model was proposed as the best fit.

The lack of fit test was calculated for the response variable in order to assess how well the quadratic model can account for the underlying variation linked to the experimental data. A model that does not fit well enough cannot be used to make predictions. Hence, it is required in model analysis that the lack of fit remains non-significant. Results of the computed lack of fit is presented in Tables 9 and 10, respectively.

The results of Tables 9–10 showed that the cubic polynomial is aliased to model analysis, whereas the quadratic polynomial had a non-significant lack of fit and was recommended for model analysis. Tables 11–12 display the model summary statistics that were calculated for the response variable using the various model sources.

For the chosen model, the model summary statistics of models fit display the coefficient of determination (R2), adjusted R-squared, predicted R-squared, the standard deviation (Root MSE), and the predicted error sum of squares (PRESS) statistic. The best model for final optimization should be chosen based on low standard deviation, R-Squared close to 1, and relatively low PRESS.

Table 7: Sequential model sum of square for Residual Stress

Response	1	Residual Stress	Transform:	None		
*** WARNING: The Cubic Model is Aliased! ***						
Sequential Model Sum of Squares [Type I]						
	Sum of		Mean	F	p-value	
Source	Squares	df	Square	Value	Prob > F	
Mean vs Total	354046.1	1	354046.1			
Linear vs Mean	2744.84	3	914.9465	2.158494	0.1329	
2FI vs Linear	1724.5	3	574.8333	1.477542	0.2666	
Quadratic vs 2FI	4966.887	3	1655.629	182.4924	< 0.0001	Suggested
Cubic vs Quadratic	57.38986	5	11.47797	1.721696	0.2828	Aliased
Residual	33.33333	5	6.666667			
Total	363573	20	18178.65			
"Sequential Model Sum of Squares [Type I]"0+: Select the highest order polynomial where the additional terms are significant and the model is not aliased						

Table 8: Sequential model sum of square for UltimateTensile Strength

Response	1	Ultimate Tensile Strength	Transform:	None		
*** WARNING: The Cubic Model is Aliased! ***						
Sequential Model Sum of Squares [Type I]						
	Sum of		Mean	F	p-value	
Source	Squares	df	Square	Value	Prob > F	
Mean vs Total	1850473	1	1850473			
Linear vs Mean	7928.407	3	2642.802	3.496288	0.0402	
2FI vs Linear	1812.509	3	604.1696	0.763901	0.5343	
Quadratic vs 2FI	9820.133	3	3273.378	70.919	< 0.0001	Suggested
Cubic vs Quadratic	337.7133	5	67.54267	2.726742	0.1475	Aliased
Residual	123.8523	5	24.77047			
Total	1870496	20	93524.78			
"Sequential Model Sum of Squares [Type I]"0+: Select the highest order polynomial where the additional terms are significant and the model is not aliased						

Table 9: Lack of Fit Test for Residual Stress.

Lack of Fit Tests	Response Variable: Residual Stress					
	Sum of		Mean	F	p-value	
Source	Squares	df	Square	Value	Prob > F	
Linear	6748.777	11	613.5252	92.02878	< 0.0001	
2FI	5024.277	8	628.0346	94.20519	< 0.0001	
Quadratic	57.38986	5	11.47797	1.721696	0.2828	Suggested
Cubic	0	0				Aliased
Pure Error	33.33333	5	6.666667			
"Lack of Fit Tests: Want the selected model to have insignificant lack-of-fit.						

Table 10: Lack of Fit Test for Ultimate Tensile Strength.

Lack of Fit Tests	Response Variable: Ultimate Tensile Strength					
	Sum of		Mean	F	p-value	
Source	Squares	df	Square	Value	Prob > F	
Linear	11970.36	11	1088.214	43.93192	0.0003	
2FI	10157.85	8	1269.731	51.25987	0.0002	
Quadratic	337.7133	5	67.54267	2.726742	0.1475	Suggested
Cubic	0	0				Aliased
Pure Error	123.8523	5	24.77047			
"Lack of Fit Tests: Want the selected model to have insignificant lack-of-fit.						

Table 11: Model Dummary Statistics for Residual Stress.

Model Summary Statistics	Response Variable: Residual Stress					
	Std.		Adjusted	Predicted		
Source	Dev.	R-Squared	R-Squared	R-Squared	PRESS	
Linear	20.58839	0.288113	0.154634	-0.15945	11046.05	
2FI	19.72427	0.469126	0.224107	-1.1095	20097.05	
Quadratic	3.012029	0.990477	0.981907	0.953068	447.1206	Suggested
Cubic	2.581989	0.996501	0.986704		+	Aliased
"Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared" and the "Predicted R-Squared".						

Table 12: Model summary statistics for Ultimate Tensile Strenght

Model Summary Statistics	Response Variable: Ultimate Tensile Strength					
	Std.		Adjusted	Predicted		
Source	Dev.	R-Squared	R-Squared	R-Squared	PRESS	
Linear	27.49342	0.395973	0.282717	0.049165	19038.19	
2FI	28.12294	0.486496	0.249494	-0.51596	30353.5	
Quadratic	6.793862	0.976948	0.956201	0.95043	992.5274	Suggested
Cubic	4.976994	0.973814	0.976495		+	Aliased
"Model Summary Statistics": Focus on the model maximizing the "Adjusted R-Squared" and the "Predicted R-Squared".						

Because the quadratic polynomial model had the lowest predicted error sum of square value, the lowest standard deviation, the highest adjusted R-square value, and the predicted R-square value with a relatively high coefficient of determination, it was chosen over the cubic polynomial model, which was aliased based on the results of Tables 11–12.

To evaluate the quadratic model's ability to optimize residual stress (MPa), ultimate tensile strength (MPa), strain (mm), wear rate (mm2/kg), and natural frequency (Hz), a one-way analysis of variance (ANOVA) was conducted. The results are shown in Tables 13–14.

Analysis of variance (ANOVA) was required to determine the significance of the model and to assess the substantial contributions of each individual variable, the combined effect, and the quadratic effect in reducing the residual stress. The model's F-value of 115.57 indicates that it is significant based on Table 13's results.

A "Model F-Value" this enormous could only be the result of noise in 0.01% of cases. When "Prob > F" is less than 0.0500, it means that the model terms are important. In this instance, important model terms are A, B, C, AB, AC, BC, A2, B2, and C2. Model terms that are not significant are indicated by values higher than 0.1000.

Table 13: ANOVA Table for Validating Model Significance towards Minimizing Residual Stress.

Response					1	Residual Stress
ANOVA for Response Surface Quadratic Model						
Analysis of variance table [Partial sum of squares - Type III]						
	Sum of		Mean	F	p-value	
Source	Squares	df	Square	Value	Prob > F	
Model	9436.227	9	1048.47	115.568	< 0.0001	significant
A-Drilling Angle	2158.202	1	2158.202	237.8887	< 0.0001	
B-Operating Current	417.8899	1	417.8899	46.06209	< 0.0001	
C-Cutting Speed	698.4163	1	698.4163	76.98322	< 0.0001	
AB	364.5	1	364.5	40.17716	< 0.0001	
AC	968	1	968	106.6982	< 0.0001	
BC	392	1	392	43.20836	< 0.0001	
A^2	1626.902	1	1626.902	179.326	< 0.0001	
B^2	437.8767	1	437.8767	48.26513	< 0.0001	
C^2	2489.043	1	2489.043	274.3557	< 0.0001	
Residual	90.72319	10	9.072319			
Lack of Fit	57.38986	5	11.47797	1.721696	0.2828	not significant
Pure Error	33.33333	5	6.666667			
Cor Total	9526.95	19				

Table 14: ANOVA Table for Validating Model Significance towards Maximizing Ultimate Tensile Strength.

Response					2	Ultimate Tensile Strength
ANOVA for Response Surface Quadratic Model						
Analysis of variance table [Partial sum of squares - Type III]						
	Sum of		Mean	F	p-value	
Source	Squares	df	Square	Value	Prob > F	
Model	19561.05	9	2173.45	47.08864	< 0.0001	significant
A-Drilling Angle	6978.779	1	6978.779	151.198	< 0.0001	
B-Operating Current	492.4838	1	492.4838	10.66985	0.0085	
C-Cutting Speed	945.4626	1	945.4626	20.48381	0.0011	
AB	892.1088	1	892.1088	19.32788	0.0013	
AC	671.9778	1	671.9778	14.55866	0.0034	
BC	248.4221	1	248.4221	5.38216	0.0428	
A^2	6634.226	1	6634.226	143.7331	< 0.0001	
B^2	236.0888	1	236.0888	5.114955	0.0472	
C^2	2276.495	1	2276.495	49.32116	< 0.0001	
Residual	461.5657	10	46.15657			
Lack of Fit	337.7133	5	67.54267	2.726742	0.1475	not significant
Pure Error	123.8523	5	24.77047			
Cor Total	20022.61	19				

In comparison to the pure error, the "Lack of Fit F-value" of 1.72 indicates that the Lack of Fit is not significant. A significant "Lack of Fit F-value" has a 28.28% probability of being caused by noise. A non-significant lack of fit is a symptom of a substantial model, which is excellent.

Based on Table 14's results, the model's F-value of 47.09 suggests that it is significant. The likelihood that a "Model F-Value" this large might be caused by noise is 0.01%. "Prob > F" values below 0.0500 signify the significance of the model terms. A, B, C, AB, AC, BC, A2, B2, and C2 are important model terms in this instance. The model terms are not significant if the values are higher than 0.1000. The "Lack of Fit F-value" of 2.73 indicates that, in comparison to the pure error, the Lack of Fit is not significant. Noise has a 14.75% chance of producing a "Lack of Fit F-value" this high. A substantial model is indicated by a non-significant lack of fit, which is excellent.

To validate the adequacy of the quadratic model based on its ability to minimize the residual stress and maximize the ultimate tensile strength, the goodness of fit statistics presented in Tables 15 – 16 was employed.

Table 15: GOF for Validating Model Significance towards Minimizing Residual Stress.

Std. Dev.	3.012029		**R-Squared**	0.990477
Mean	133.05		**Adj R-Squared**	0.981907
C.V. %	2.263832		**Pred R-Squared**	0.953068
PRESS	0.1206		**Adeq Precision**	33.31775
The "Pred R-Squared" of 0.953068 is in reasonable agreement with the "Adj R-Squared" of 0.981907				

From the result of Table 15, it was observed that the "Predicted R-Squared" value of 0.953068 is in reasonable agreement with the "Adj R-Squared" value of 0.981907. Adequate precision measures the signal to noise ratio. A ratio greater than 4 is desirable. The computaed ratio of 33.31775 as observed in Table 15 indicates an adequate signal. This model can be used to navigate the design space and adequately minimize the residual stress.

From the result of Table 16, it was observed that the "Predicted R-Squared" value of 0.95043 is in reasonable agreement with the "Adj R-Squared" value of 0.956201. Adequate precision measures the signal to noise ratio. A ratio greater than 4 is desirable. The computaed ratio of 20.5801 as observed in Table 16 indicates an adequate signal. This model can be used to navigate the design space and adequately maximize the tensile strength.

Table 16: GOF for Validating Model Significance towards Maximizing Ultimate Tensile Strength.

Std. Dev.	6.793862		**R-Squared**	0.976948
Mean	304.177		**Adj R-Squared**	0.956201
C.V. %	2.233523		**Pred R-Squared**	0.95043
PRESS	0.5274		**Adeq Precision**	20.5801
The "Pred R-Squared" of 0.95043 is in reasonable agreement with the "Adj R-Squared" of 0.956201				

To obtain the optimal solution, we first consider the coefficient statistics and the corresponding standard errors. The computed standard error measures the difference between the experimental terms and the corresponding predicted terms. Coefficient statistics for the response variables is presented in Tables 17 – 18.

Variance inflation factor (VIF) value of 1.00 for the individual and combine terms with 1.023733022 for the quadratic terms as observed in Table 17 indicate a significant model in which the variables are adequately correlated with the response variable (residual stress).

Table 17: Coefficient Estimates Statistics for Minimizing Residual Stress.

	Coefficient		Standard	95% CI	95% CI	
Factor	**Estimate**	**Df**	**Error**	**Low**	**High**	**VIF**
Intercept	113.288	1	1.14183	110.7438	115.8321	
A-Drilling Angle	12.57103	1	0.815049	10.75498	14.38707	1
B-Operating Current	5.531662	1	0.815049	3.715619	7.347705	1
C-Cutting Speed	-8.79725	1	1.002649	-11.0313	-6.56321	1
AB	6.75	1	1.064913	4.377226	9.122774	1
AC	-11	1	1.064913	-13.3728	-8.62723	1
BC	7	1	1.064913	4.627226	9.372774	1
A^2	10.6535	1	0.795556	8.880889	12.42611	1.023733022
B^2	5.526973	1	0.795556	3.754365	7.299582	1.023733022
C^2	17.13061	1	1.034227	14.82621	19.43501	1.023733022

Table 18: Coefficient Estimates Statistics for Maximizing Ultimate Tensile Strength.

Factor	Coefficient Estimate	Df	Standard Error	95% CI Low	95% CI High	VIF
Intercept	278.5115	1	2.575484	272.773	284.2501	
A-Drilling Angle	-22.6055	1	1.838406	-26.7017	-18.5093	1
B-Operating Current	6.005105	1	1.838406	1.908882	10.10133	1
C-Cutting Speed	-10.2356	1	2.261552	-15.2746	-5.19652	1
AB	-10.56	1	2.401993	-15.912	-5.20803	1
AC	9.165	1	2.401993	3.813026	14.51697	1
BC	5.5725	1	2.401993	0.220526	10.92447	1
A^2	21.51328	1	1.794437	17.51502	25.51153	1.023733
B^2	4.058346	1	1.794437	0.060092	8.056601	1.023733
C^2	16.38287	1	2.332778	11.18512	21.58062	1.023733

Variance inflation factor (VIF) value of 1.00 for the individual and combine terms with 1.023733 for the quadratic terms as observed in Table 18 indicate a significant model in which the variables are adequately correlated with the response variable (ultimate tensile strength).

The model equations based on actual factors, which show the individual effects, and the combine interactions of the selected input variables, namely; drilling angle (degree), operating current (amp) and the cutting speed (mm/min) against the mesured response variable, namely; residual stress (MPa) and ultimate tensile strength (MPa) are shown in Equations (3) – (4).

$$y_1 = 226.2879 - 0.885x_1 - 1.758x_2 - 7.907x_3 + 0.009x_1x_2 - 0.049x_1x_3 + 0.037x_2x_3 + 0.012{x_1}^2 + 0.009{x_2}^2 + 0.305{x_3}^2 \quad \textbf{(3)}$$

$$y_2 = 483.293 - 3.357x_1 - 0.001x_2 - 12.725x_3 - 0.014x_1x_2 + 0.041x_1x_3 + 0.030x_2x_3 + 0.024{x_1}^2 + 0.006{x_2}^2 + 0.291{x_3}^2 \quad \textbf{(4)}$$

where,
y_1 = residual stress,
y_2 = ultimate tensile strenght,
y_3 = Strain,
y_4 = wear rate,
x_1 = the drilling angle (degree),
x_2 = the operating current (amp), and
x_3 = the cutting speed (mm/min)

The diagnostic case statistics which shows the observed values of the response variables; against the predicted values is presented in Tables 19 – 20.

Model Diagnostics

To accept any model, its satisfactoriness must first be checked by an appropriate statistical analysis output. To diagnose the statistical properties of the response surface model, the normal probability plot of residual presented in Figures 4 – 5 were employed.

The normal probability plot of studentized residuals was employed to assess the normality of the calculated residuals.

To determine the presence of a possible outlier in the data, the cook's distance plot was generated for the response variable. The cook's distance is a measure of how much the regression would change if the outlier is omitted from the analysis. A point that has a very high distance value relative to the other points may be an outlier and should be investigated. The generated cook's distance is presented in Figures 6 – 7.

Table 19: Diagnostics Case Statistics Report of Observed and Predicted Residual Stress.

Standard Order	Actual Value	Predicted Value	Residual	Leverage	Internally Studentized Residual	Externally Studentized Residual	Influence on Fitted Value DFFITS	Cook's Distance	Run Order
1	140	140.0436	-0.0436	0.763873	-0.02979	-0.02826	-0.05083	0.000287	13
2	172	173.6857	-1.68565	0.763873	-1.15169	-1.17316	* -2.11	0.429092	14
3	125	123.6069	1.393075	0.763873	0.951794	0.946857	1.703032	0.293064	18
4	185	184.249	0.751021	0.763873	0.513122	0.493328	0.887308	0.085176	19
5	131	130.4491	0.550902	0.676128	0.321387	0.306482	0.442825	0.021563	16
6	120	120.0912	-0.09115	0.676128	-0.05318	-0.05045	-0.0729	0.00059	15
7	145	142.0124	2.987578	0.676128	1.742904	1.981612	* 2.86	0.634165	20
8	160	158.6545	1.345525	0.676128	0.784957	0.768735	1.11072	0.128631	17
9	120	122.2787	-2.27874	0.632595	-1.24814	-1.28871	-1.69101	0.268228	12
10	165	164.5625	0.437539	0.632595	0.239654	0.228011	0.29919	0.009889	8
11	121	119.6175	1.38251	0.632595	0.757245	0.739913	0.970891	0.098731	11
12	135	138.2237	-3.22371	0.632595	-1.76573	-2.01921	* -2.65	0.536818	10
13	121	116.8521	4.147932	0.145155	1.489459	1.601836	0.66007	0.03767	7
14	145	146.9455	-1.94548	0.702207	-1.18362	-1.2109	-1.85944	0.330349	9
15	110	113.288	-3.28796	0.143709	-1.17966	-1.20619	-0.49414	0.023355	4
16	115	113.288	1.712042	0.143709	0.614249	0.594042	0.24336	0.006332	3
17	115	113.288	1.712042	0.143709	0.614249	0.594042	0.24336	0.006332	5
18	115	113.288	1.712042	0.143709	0.614249	0.594042	0.24336	0.006332	6
19	110	113.288	-3.28796	0.143709	-1.17966	-1.20619	-0.49414	0.023355	1
20	111	113.288	-2.28796	0.143709	-0.82088	-0.8064	-0.33035	0.011309	2

Table 20: Diagnostics Case Statistics Report of Observed and Predicted Utlimate Tensile Strength.

Standard Order	Actual Value	Predicted Value	Residual	Leverage	Internally Studentized Residual	Externally Studentized Residual	Influence on Fitted Value DFFITS	Cook's Distance	Run Order
1	350.54	351.4795	-0.93952	0.763873	-0.28459	-0.27108	-0.48758	0.026201	13
2	308.71	309.0585	-0.34849	0.763873	-0.10556	-0.1002	-0.18022	0.003605	14
3	370.65	373.4647	-2.81473	0.763873	-0.8526	-0.83996	-1.51076	0.235165	18
4	290.22	288.8037	1.416296	0.763873	0.429008	0.41079	0.738854	0.05954	19
5	299.87	301.5334	-1.66338	0.676128	-0.43022	-0.41197	-0.59524	0.03864	16
6	298.34	295.7724	2.567645	0.676128	0.664097	0.644388	0.931054	0.09207	15
7	345.91	345.8086	0.101408	0.676128	0.026228	0.024883	0.035953	0.000144	20
8	298.5	297.8076	0.692435	0.676128	0.179092	0.170174	0.245879	0.006696	17
9	380.42	377.3781	3.041925	0.632595	0.738684	0.720717	0.945702	0.09395	12
10	298.65	301.3425	-2.69249	0.632595	-0.65383	-0.63398	-0.83189	0.073606	8
11	280	279.8909	0.109058	0.632595	0.026483	0.025125	0.032968	0.000121	11
12	300.33	300.0896	0.240372	0.632595	0.058371	0.055385	0.072674	0.000587	10
13	298.5	282.406	16.09396	0.145155	2.562138	4.14698	1.708849	0.111468	7
14	307.58	307.6352	-0.0552	0.702207	-0.01489	-0.01412	-0.02169	5.23E-05	9
15	270.8	278.5115	-7.71155	0.143709	-1.22663	-1.26254	-0.51722	0.025252	4
16	277.61	278.5115	-0.90155	0.143709	-0.1434	-0.13619	-0.05579	0.000345	3
17	273.44	278.5115	-5.07155	0.143709	-0.8067	-0.79149	-0.32425	0.010922	5
18	273.22	278.5115	-5.29155	0.143709	-0.8417	-0.82839	-0.33936	0.01189	6
19	284.92	278.5115	6.408452	0.143709	1.019356	1.021576	0.418507	0.017439	1
20	275.33	278.5115	-3.18155	0.143709	-0.50607	-0.48637	-0.19925	0.004298	2

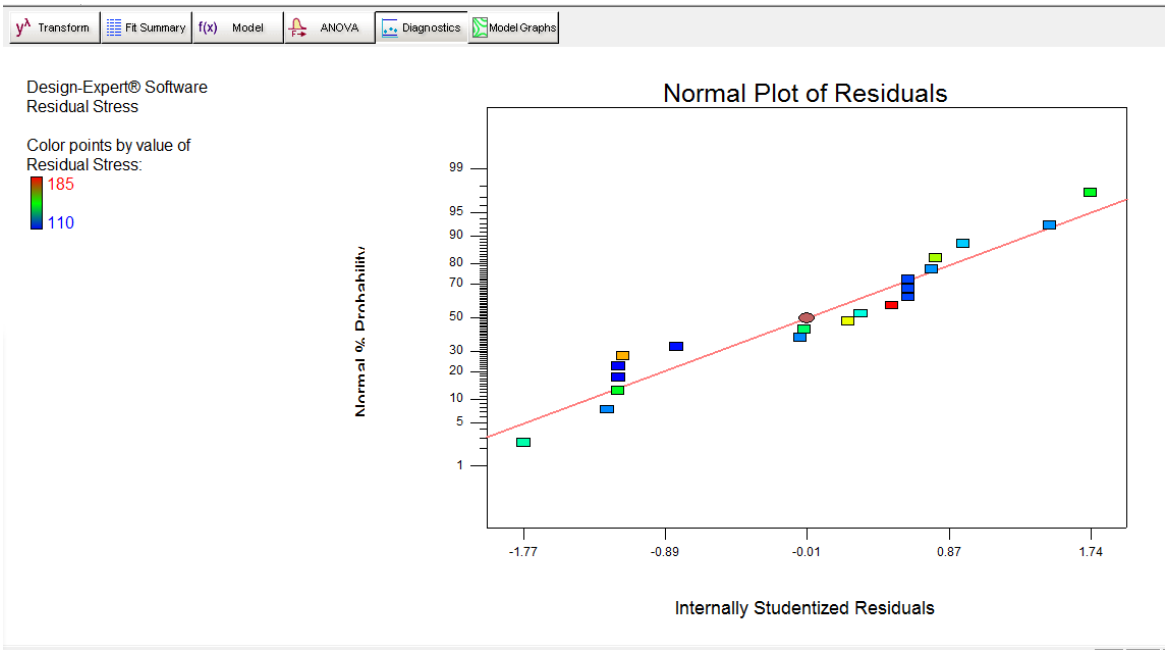

Figure 4: Normal Probability Plot of Studentized Residuals for Residual Stress.

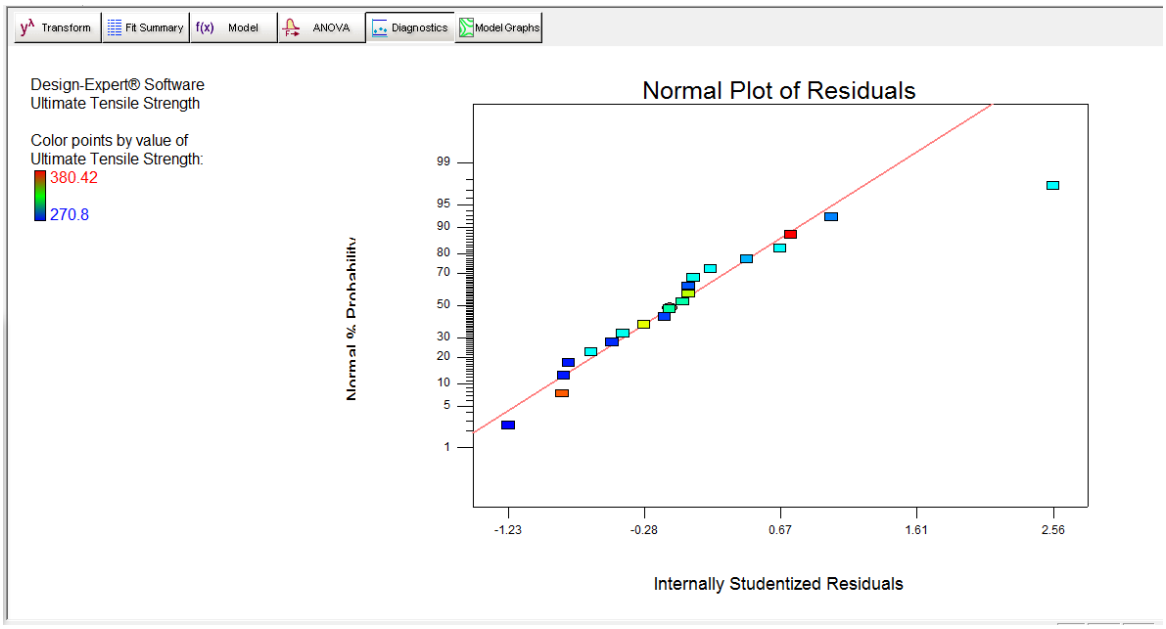

Figure 5: Normal Probability Plot of Studentized Residuals for Ultimate Tensile Strength.

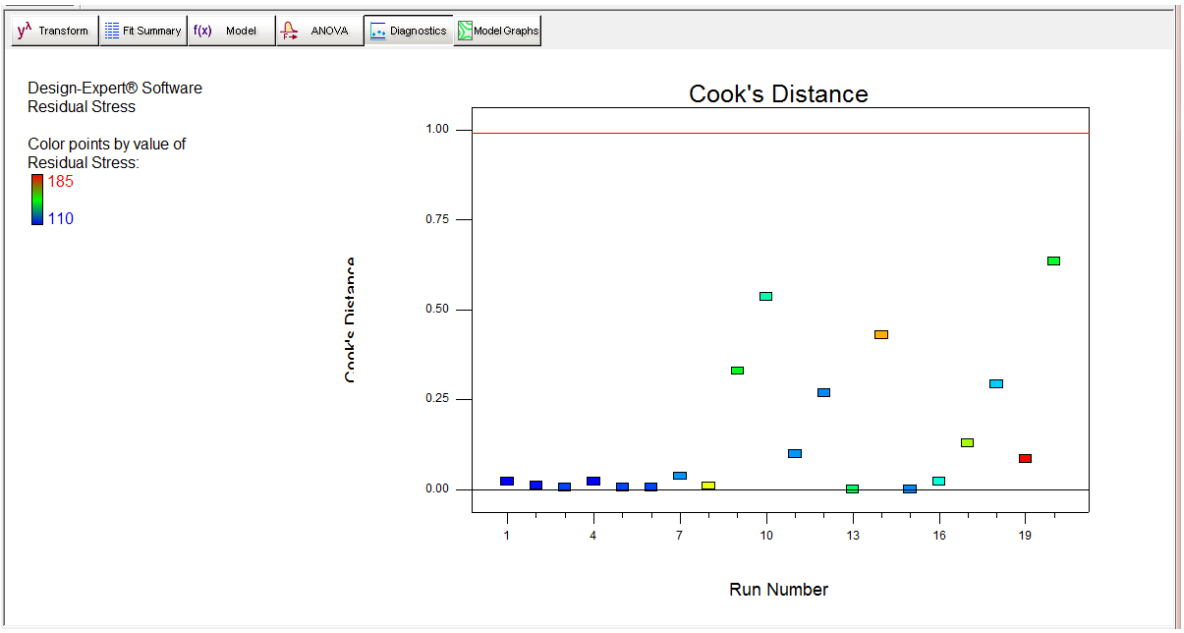

Figure 6: Generated Cook's Distance for Residual Stress.

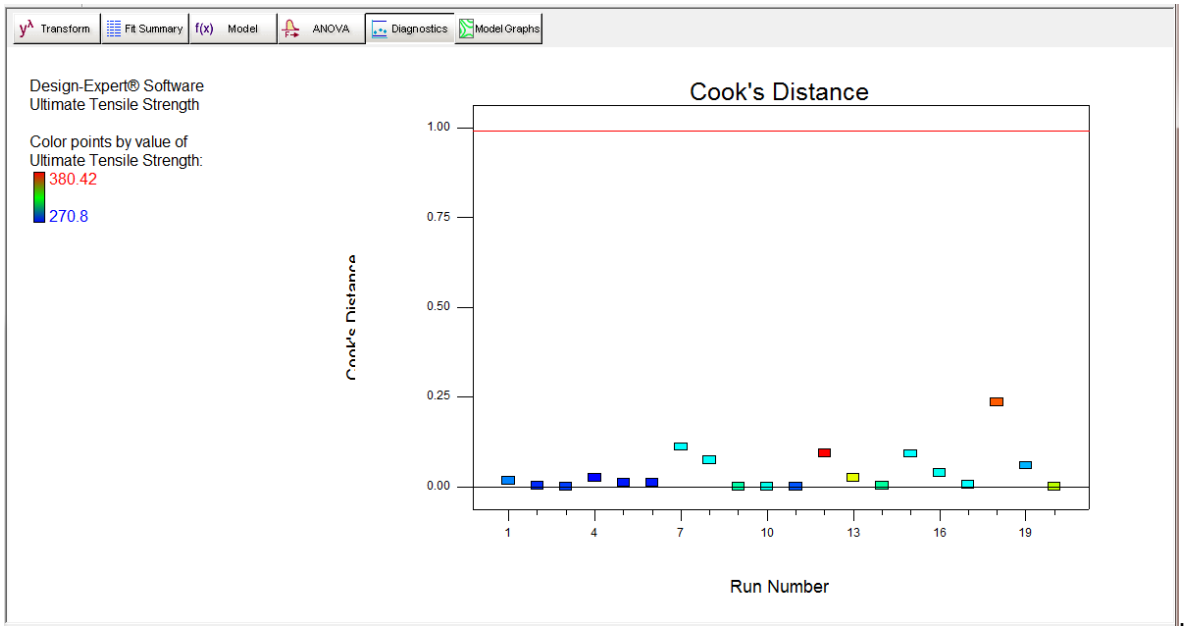

Figure 7: Generated Cook's Distance for Ultimate Tensile Strength.

Typically, a Cook's distance value greater than 1 suggests that a particular data point has a strong influence on the model, potentially skewing the results. However, since all observed values were below this threshold, it suggests that the models for residual stress, ultimate tensile strength, strain, and wear rate were not significantly affected by any individual data point. The results imply that the regression models were stable and robust. There were no extreme outliers or influential points that could potentially distort the interpretations or predictions derived from the models. This enhances the reliability and validity of the models, ensuring that the conclusions drawn from the data are sound and not biased by any anomalous observations. Consequently, the models can be confidently used for further analysis and decision-making processes related to the studied response variables.

The Cook's Distance plot for ultimate tensile strength provided an assessment of the influence of individual data points (run numbers) on the regression model. The points were color-coded according to their ultimate tensile strength values, with blue representing lower tensile strength values and red representing higher tensile strength values. Most data points exhibited low Cook's Distance values, indicating minimal influence on the overall model. However, a few data points, especially towards the right side of the plot, showed relatively higher Cook's Distance values, approaching the threshold of 1.0 marked by the horizontal red line.

These influential points suggested that certain runs had a more significant impact on the regression model's estimates. The plot underscored the necessity to examine these influential data points further, as their high Cook's Distance values indicated potential outliers or leverage points that could distort the model's accuracy and reliability. Overall, the Cook's Distance plot for ultimate tensile strength was a crucial diagnostic tool for identifying and mitigating the effects of influential observations in the dataset.

Numerical Optimization

Numerical optimization was performed to ascertain the desirability of the overall model. The objective of numerical optimization was to determine the actual operating current, drilling angle and the cutting speed that will significantly minimize the residual stress and maximize the ultimate tensile strength. The interphase of the numerical optimization showing the objective function is presented in Figures 8 – 9.

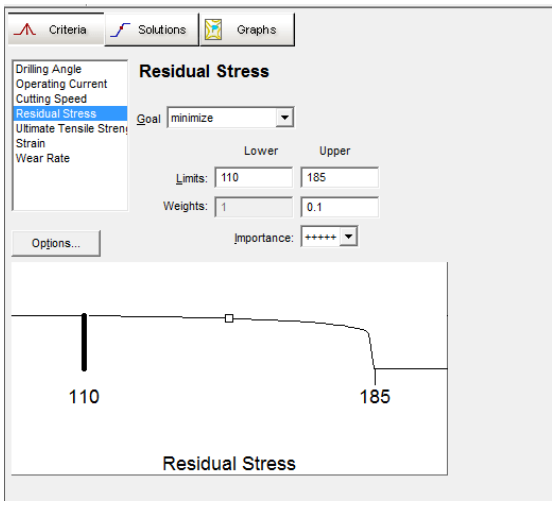

Figure 8: Interphase of Numerical Optimization Model for Optimizing Residual Stress.

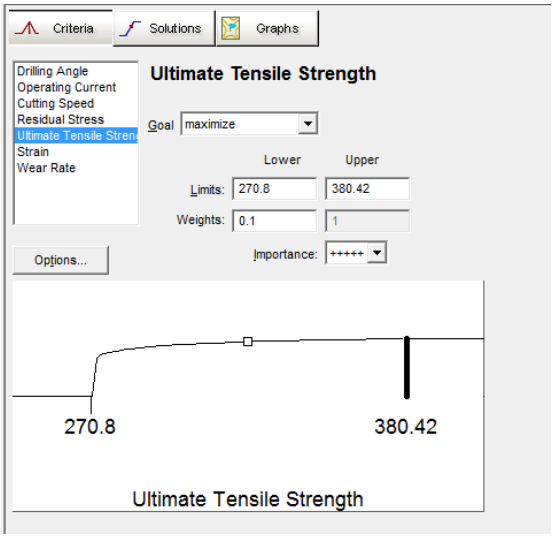

Figure 9: Interphase of Numerical Optimization Model for Optimizing Ultimate Tensile Strength.

The optimization objective was to minimize the residual stress; maximize the ultimate tensile strength; optimize the strain; and minimize the wear rate. The relative importance was set at the optimum value of 5.0 and the lower and upper boundary conditions were set at 0.1 and 1.0 respectively. The final solution of numerical optimization is presented in Table 21.

Table 21: Optimal Solutions of Numerical Optimization.

Run	Factor 1 Drilling Angle (degree)	Factor 2 Operating Current (amp)	Factor 3 Cutting Speed (mm/min)	Response 1 Residual Stress (MPa)	Response 2 Ultimate Tensile Stress (MPa)	Desirability	
1	30	53.74	13.23	111.7848	320.0544	0.949712	Selected
2	30	53.95	13.3	111.8321	320.0266	0.949711	
3	30	53.65	13.29	111.8327	319.8557	0.94971	
4	30.01	53.37	13.17	111.766	319.905	0.949707	
5	30	54	13.15	111.7212	320.3979	0.949707	
6	30	54.45	13.43	111.919	320.0848	0.949705	
7	30	52.95	13.1	111.7473	319.8073	0.949705	
8	30	54.28	13.19	111.741	320.4867	0.949705	
9	30.05	54.19	13.31	111.815	320.0708	0.949694	
10	30	55.73	13.75	112.2191	320.3572	0.94967	
11	30.01	53.43	13.64	112.129	318.973	0.949665	
12	30	56.4	13.43	111.9047	321.4491	0.949638	
13	30	57	13.7	112.186	321.3699	0.949621	
14	30	55.11	14.19	112.7351	319.1695	0.949583	
15	30	54.84	17.69	121.0871	316.9163	0.945383	
16	90	58.28	20	136.4642	292.9314	0.924528	
17	90	57.97	20	136.2105	292.9072	0.924528	
18	88.14	54.56	20	132.1367	290.947	0.923684	
19	90	36.5	20	122.5985	294.1856	0.918512	
20	90	33.43	10.62	137.5659	286.8643	0.912722	

CONCLUSION

Residual stress, a critical factor in material performance, is measured at 111.7848MPa under these conditions. This value is indicative of the internal stresses remaining in the material post-machining, and being within an optimal range, it suggests that the material will maintain its structural integrity and resist deformation or failure during usage. The ultimate tensile stress, recorded at 320.0544MPa, reflects the maximum stress the material can endure while being stretched or pulled before breaking. This high tensile strength is crucial for ensuring that the material can withstand significant mechanical loads without failure.

REFERENCES

1. Etin-Osa, C.E., J.I. Achebo, and K.O. Obahiagbon. 2020. “Design and Fabrication of Portable Weld Residual Stress Measuring Device Using MBN Principle”. *NIPES Journal of Science and Technology Research*. 2: 70-84. https://doi.org/10.37933/nipes/2.3.2020.8

2. Fukumoto, Y. and Y. Itoh. 1981. “Statistical Study of Experiments on Welded Beams”. *Journal of Structural Division, ASCE*. 107 (ST1): 89-103.

3. Gdoutos, E.E. 2022. “Residual Stresses—The Hole-Drilling Method”. In: *Experimental Mechanics. Solid Mechanics and Its Applications*. vol 269. Springer, Cham. https://doi.org/10.1007/978-3-030-89466-5_16

4. Huber, A.W., L.S. Beedle and Fritz Engineering Laboratory. 1954. “Residual Stress and the Compressive Strength of Steel”. *Welding Journal*. 33(12): 589-s,

5. Jawahir, I.S., E. Brinksmeier, R. M'Saoubi, D.K. Aspinwall, J.C. Outeiro, D. Meyer, D. Umbrello, and A.D. Jayal. 2011. “Surface Integrity in Material Removal Processes: Recent Advances”. *CIRP Annals*. 60(2): 603-626, ISSN 0007-8506, https://doi.org/10.1016/j.cirp.2011.05.002.

6. Montgomery, D.C. 2005. *Design and Analysis of experiments, 6th ed.*. John Wiley & Sons, Inc.: New York, NY.

7. Withers, P.J. and H.K.D.H. Bhadeshia. 2001. “Residual Stress Part 1 – Measurement Techniques”. *Materials Science and Technology*. April 2001. 17: 355 – 365.

Yang, X. and C.R. Liu. 2001. “A New Stress-Based Model of Rriction Behavior in Machining and its Significant Impact on Residual Stresses Computed by Finite Element Method”. *International Journal of Mechanical Sciences*. 44.

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