Recycling of Municipal Solid Waste: A Deterministic Approach

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ABSTRACT

Effective waste management aims at minimizing garbage's detrimental effects on the environment, public health, and aesthetics, which also attempts to recover valuable resources and support sustainable development.

The Next Generation matrix was employed to calculate the reproduction number, the model equations were solved using the Differential Transformation Method (D.T.M.), and the obtained result was simulated using the Maple software. The results showed that waste management will be effective if recycling of waste was given the proper attention that it deserved. It also indicated that waste for disposal will be limited and managing waste will become easier.

(Keywords: *waste management, recycling, differential transformation method, simulation*)

INTRODUCTION

Solid Waste Management (SWM) has become a challenge in the major cities in Nigeria. It can be combatted efficiently if the stakeholders consider recycling of Municipal Solid Waste (MSW) as an important way of creating employment and thereby generating wealth. The Differential Transformation Method (DTM) is a powerful semi-analytical method for solving linear and non-linear differential equations. It can be applied directly to non-linear problems without linearization, discretization or perturbation.

Waste management is the collection, storage, treatment, and disposal of wastes in a manner that makes them harmless to human and animal life, the ecosystem, and the environment, as a whole. Solid waste materials must be gathered, processed, recycled, and disposed of in the course of waste management (Adewole, 2009).

Siddiqi, Haraguchi, and Narayanamurti (2020) used a quantitative Waste to Energy Recovery Assessment (WERA) framework to stochastically analyze the feasibility of waste-to-energy in urban regions using emerging technologies such as gasification, pyrolysis and Refuse-Derived Fuel (RDF). Future policy measures of feed-in tariffs, payments for avoided pollution, and higher waste collection fees were used to evaluate if Waste to Energy systems can be made a self-sustaining investment.

O'Connell (2011) researched into what motivates and what hinders participation in waste diversion and minimization practices by drawing out the common factors found in a wide spectrum of studies and suggested ways by which these strategies could be used by government agencies and other organizations seeking to increase the diversion of waste from disposal to recovery thereby reducing the number of materials considered as waste.

Mamhobu-Amadi, Kinigoma, Momoh, Yusuf, and Oji (2019), defined abattoir wastes as wastes generated during the various operations that are aimed at processing meat for consumption categorized as solid, liquid or gaseous have the potential to pollute the environment. This study described the different stages of abattoir operations, from animal arrival and storage, to stunning, slaughtering, skinning, eviscerating splitting, roasting and washing, and the types of wastes produced at each stage.

It further linked the various wastes to their potential impacts on the environment, public health, animal health and economy. They suggested alternative uses for abattoir wastes, such as biogas production, composting, rendering, incineration, blood processing and charred bone purification. The study recommended the adoption of good manufacturing and hygienic practices, proper waste segregation, collection, storage and transportation, to minimize the environmental and health risks of abattoir wastes. It was also recommended that collaboration and coordination among the government, industry, academia and civil society to establish and enforce standards, policies and regulations for abattoir operations and waste management.

Šomplák, *et al*., (2023), in their work titled Comprehensive Review on Waste Generation Modelling gives a systematic review of the methods and approaches for forecasting and projecting the amount and composition of waste in different contexts. The authors provided a general guide for waste generation modelling, as well as specific recommendations for dealing with common challenges, such as data scarcity, uncertainty, and scenario analysis. It also presented two novel methods for waste generation modelling: one based on short time series, and one based on system dynamics. The system dynamics modelling used a causal loop diagram to represent the feedback loops and delays among the variables that affect waste generation. The study aims at helping researchers and stakeholders to choose the most suitable method for their waste management tasks and plans.

The work of Luciana, *et al*. (2021), explored why households generate food wastes and the challenges of reducing it. Reasons for food wastage were categorized into two: understandings and perceptions of food waste, and food-related household practices and routines – planning, shopping, storing, preparing, consuming, and disposing of food. Four main challenges for reducing food waste were identified, these are changing household routines, increasing awareness and knowledge, improving communication and cooperation, and enhancing infrastructure and technology. It was recommended that policymakers, practitioners, and researchers should address these challenges and foster more sustainable food consumption behaviors.

Munganga, Mwambakanab, Maritza, Batubengea, and Moremedia (2014) introduced the differential transform method (DTM), a semi-analytical technique that can solve differential equations using the Taylor series, to undergraduate students. The study explained the basic concepts and formulas of DTM, showing how to apply them to various differential equations, such as linear, non-linear, ordinary, and partial. They compared the DTM with other methods, such as the Laplace transform, the separation of variables, and the numerical methods discussed the advantages and disadvantages of each technique and provided several examples and exercises to help students understand and practice the DTM.

Shashank, Vishal, Prakhar, and Sulekha (2015) focused on the technological and economic aspects of the Waste-To-Energy (WTE) process. Untreated garbage was seen as a helpful resource in the work that can be used to produce both renewable and non-renewable energy sources. The thermal energy produced by conventional combustion can boil water, create steam, drive turbine generators, and create electricity. The other technological uses considered pyrolysis and gasification. When WTE facilities are in use, they can reduce greenhouse gas (GHG) emissions and other emissions, such as methane from landfilling. WTE produces electricity at a slower rate than other energy-generation techniques. He concluded that facilities for WTE are required because of the need for alternative resources, conversation of landfilling space, reduction of GHG and generation of renewable energy.

Ochoa-Barragán, Contreras, Fernández, and Laporte (2021) conducted a systematic study of the optimal planning and operation of the municipal waste management system in the context of the COVID-19 pandemic. They proposed a novel strategy that combines two methods: optimization and machine learning. The optimization method is a mathematical model that determines the best configuration and operation of the waste collection, transportation, treatment, and disposal facilities, considering the economic and environmental objectives. The machine learning method is a set of prediction models that estimate future waste generation and composition, based on the historical and socio-economic data.

The article applies the proposed strategy to a case study of New York City, using extensive data sets to train and test the machine learning models. The article evaluates the performance of the strategy under different scenarios of waste generation, taxation, and pandemic duration. The strategy can provide useful insights and trade-offs for the decision-makers and stakeholders involved in the municipal waste management system. The article also discusses the limitations and challenges of the proposed strategy, as well as some directions for future research in this field.

Khan, *et al*. (2024) use geographic information systems (GIS) and mathematical modelling approaches to investigate ways to improve garbage management in Peshawar, Pakistan. The measures for reducing waste took into account the reduction of waste, raising consumer awareness, encouraging the use of eco-friendly products, and recovering recyclable materials. The methods of treatment and recovery include thermal treatment, which recovers energy from waste while reducing the volume of waste through gasification and incineration, methane capture, and leachate management from landfills; biological treatment, which turns organic waste into valuable resources through composting and anaerobic digestion.

Akinboro, Alao, and Akinpelu (2014) compared the differential transformation method (DTM) and the variational iteration method (VIM) in finding the approximate solution of the susceptible/infected/removed (SIR) model, which is a mathematical model of infectious disease dynamics. The SIR model is a non-linear system of ordinary differential equations which have no analytic solution. The VIM uses a general Lagrange multiplier to construct a correction function for the problem, while the DTM uses a transformed function of the original nonlinear system. It was observed that the SIR model can be solved by both the VIM and the DTM with high accuracy and convergence. The solutions obtained agree perfectly with the numerical solutions obtained by the classical fourth-order Runge-Kutta method. The graphs of the solutions were compared with the numerical solutions. the VIM and the DTM were recommended as alternative methods for solving the SIR model and other nonlinear systems of ODEs.

METHODOLOGY

The total waste $N(t)$ at time t, comprises Manufactured goods $A(t)$ which were produced at the rate Λ, Domestic/Municipal use of Manufactured goods $M(t)$, Industrial use of manufactured goods $I(t)$, Collation center $C(t)$, Recycling center of wastes $K(t)$, waste converted to Energy $E(t)$, Wealth generated from Energy $W(t)$, non-recyclable waste for Disposal $D(t)$. Λ is the rate of production for manufactured goods. Manufactured products were used for domestic and industrial purposes at the rate ξ with v and $1-v$ being the proportion used for domestic and industrial purposes respectively. ω represents the rate of controlling waste production in the industries. Waste generated for municipal use $M(t)$ and domestic use $I(t)$ are collected at the rate α_1 and α_2 respectively. Waste from collation center $C(t)$ is recycled and disposed of at the rates ρ_1 and ρ_2 respectively.

Recycled products generate energy at the rate η the energy generated creates wealth at the rate β. Each of the compartments has a waste natural decomposition rate of μ and δ as the rate at which non-recyclable wastes are disposed of from collation and recycling centers. $\phi = \xi(M + \omega I)$ is the rate at which goods are manufactured where ξ is the rate at which goods are used domestically and industrially and the wastes from the industries are controlled at the rate ω.

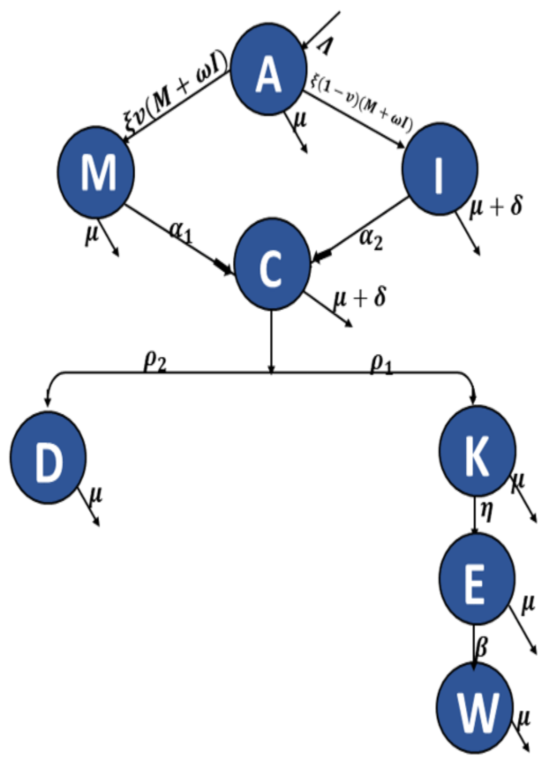

Figure 1: Schematic Model Diagram

$$
\left.\begin{aligned}
\frac{dA}{dt} &= \Lambda - \xi(M+\omega I)A - \mu A \\
\frac{dM}{dt} &= \xi v(M+\omega I)A - (\alpha_1+\mu)M \\
\frac{dI}{dt} &= \xi(1-v)(M+\omega I)A - (\alpha_2+\mu)I \\
\frac{dC}{dt} &= \alpha_1 M + \alpha_2 I - (\rho_1+\rho_2+\mu+\delta)C \\
\frac{dK}{dt} &= \rho_1 C - (\eta+\mu+\delta)K \\
\frac{dE}{dt} &= \eta K - (\beta+\mu)E \\
\frac{dW}{dt} &= \beta E - \mu W \\
\frac{dD}{dt} &= \rho_2 C - \mu D
\end{aligned}\right\} \tag{1}
$$

S/NO	Variables	Description
1	*A*	Manufactured Goods
2	*M*	municipal / Goods used Domestic purpose
3	*I*	goods used in the Industries
4	*C*	Collected waste
5	*K*	Recycled
6	*D*	Disposed waste
7	*E*	Energy generated from Recycled waste
8	*W*	Wealth generated from Energy

x	Description
Λ	the rate at which new products are produced
φ	the rate at which manufactured goods are used
v	Proportion of manufactured goods used domestically
$1-v$	Proportion of manufactured goods used industrially
α_1	Rate at which domestic waste is collated
α_2	Rate at which industrial waste is collated
ρ_1	The proportion of collated waste that can be recycled
ρ_2	The proportion of collated waste that can decompose
η	The rate at which recycled waste generates Energy.
β	The rate at which wealth is created from Energy.
ξ	Force of infection.
μ	The rate at which manufacture material deteriorates/decomposes naturally.
ω	Rate of controlling waste generation in industries
δ	The rate at which non-recyclable waste is disposed.

Model Assumptions

The model assumed that manufactured goods are used for domestic and industrial purposes only. The model considers solid waste only.
The wastes generated in the industries are controlled and minimized. The wastes from the collation center are either recycled or disposed of. It is assumed that wastes are not properly disposed.

Model Reformulation

Existence of Waste-Free Equilibrium at the Waste-Free Equilibrium (W.F.E.) point:

$$\frac{dA}{dt}=\frac{dM}{dt}=\frac{dI}{dt}=\frac{dC}{dt}=\frac{dK}{dt}=\frac{dD}{dt}=\frac{dE}{dt}=\frac{dW}{dt}=0 \tag{2}$$

$$A,M,I,C,D,K,E,W=A_0,M_0,I_0,C_0,D_0,K_0,E_0,W_0 \tag{3}$$

$$\left(A_0,M_0,I_0,C_0,D_0,K_0,E_0,W_0\right)=\left(\frac{\Lambda}{\mu},0,0,0,0,0,0,0\right) \tag{4}$$

$$
\begin{aligned}
&\Lambda - \xi(M_0+\omega I_0)A_0 - \mu A_0 = 0 \\
&v\xi(M_0+\omega I_0)A_0 - (\alpha_1+\mu)M_0 = 0 \\
&(1\text{-}v)\xi(M_0+\omega I_0)A_0 - (\alpha_2+\mu)I_0 = 0 \\
&\alpha_1 M_0 + \alpha_2 I_0 - (\rho_1+\rho_2+\mu+\delta)C_0 = 0 \\
&\rho_1 C_0 - (\omega+\mu+\delta)K_0 = 0 \\
&\omega K_0 - (\beta+\mu)E_0 = 0 \\
&\beta E_0 - \mu W_0 = 0 \\
&\rho_2 C_0 - \mu D_0 = 0
\end{aligned} \tag{5}
$$

$$k_1=(\alpha_1+\mu), k_2=(\alpha_2+\mu), k_3=(\rho_1+\rho_2+\mu+\delta), k_4=(\omega+\mu+\delta), k_5=(\beta+\mu), \tag{6}$$

$$
\left.\begin{aligned}
&\Lambda - \xi(M_0+\omega I_0)A_0 - \mu A_0 = 0 \\
&\xi v(M_0+\omega I_0)A_0 - k_1 M_0 = 0 \\
&\xi(1-v)(M_0+\omega I_0)A_0 - k_2 I_0 = 0 \\
&\alpha_1 M_0 + \alpha_2 I_0 - k_3 C_0 = 0 \\
&\rho_1 C_0 - k_4 K_0 = 0 \\
&\omega K_0 - k_5 E_0 = 0 \\
&\beta E_0 - \mu W_0 = 0 \\
&\rho_0 C_0 - \mu D_0 = 0
\end{aligned}\right\} \tag{7}
$$

Waste Reproduction Number

According to Diekmann and Heesterbeek (2000); Driessche (2002); and Akinboro, Alao, and Akinpelu (2014)), the average number of secondary infections generated by an infectious individual during the entire period of infectiousness. When applied to solid waste management the Waste Reproduction Number refers to the rate at which solid waste spreads in the environment. Ibrahim and Abdurrahman (2023). The spread of waste in the community can be determined by the value of R_0, it can either be wiped out or it can spread at an alarming rate. Whenever the value of the R_0 goes below unity R_0 < 1, the spread of the waste is insignificant and waste management becomes easier. When the value is above unity R_0>1 it means that the spread of waste in the community becomes significant
.

$$\left(FV^{-1}-\lambda\right)=\begin{pmatrix} \frac{vA_0\xi}{k_1}-\lambda & \frac{vA_0\xi\omega}{k_2} & 0 \\ \frac{(1\text{-}v)A_0\xi}{k_1} & \frac{(1\text{-}v)A_0\xi\omega}{k_2}-\lambda & 0 \\ 0 & 0 & -\lambda \end{pmatrix} \quad \textbf{(8)}$$

$$\lambda_1=0,\lambda_2=0 \text{ and } \lambda_3=\frac{-\left(vA_0\xi k_2+(1\text{-}v)A_0\xi\omega k_1\right)}{k_1k_2} \quad \textbf{(9)}$$

$$A_0=\frac{\Lambda}{\mu},k_1=\alpha_1+\mu,k_2=\alpha_2+\mu \quad \textbf{(10)}$$

$$R_0=\frac{\Lambda\xi\left(\mu\omega v+\omega v\alpha_1-\mu\omega-v\mu-\omega\alpha_1-v\alpha_2\right)}{\mu\left(\alpha_2+\mu\right)\left(\alpha_1+\mu\right)} \quad \textbf{(11)}$$

Differential Transformation Method

Let h(t) be an analytic and continuously differentiable function in the domain of interest, then the differential transform of h(t) is given by>

$$H(k)=\sum_{k=0}^{\infty}\frac{1}{k!}\left[\frac{d^kh(t)}{dt^k}\right)_{t=0} \quad \textbf{(12)}$$

where H(k) is the transformed function, also called the T-function, and k is a non-negative integer.

The inverse differential transform of H(k) is given by:

$$h(t)=\sum_{k=0}^{\infty}H(k)t^k \quad \textbf{(13)}$$

where h(t) is the original function. The Equation (13) above was derived from Taylor series expansion which can be written in the form:

$$h(t)=\sum_{k=m+1}^{\infty}H(k)\left(t-t_0\right)^k \quad \textbf{(14)}$$

The values of m are determined by the convergence series coefficients.

The Fundamental Properties of Differential Transform Method.

S/NO	Existing Functions	Transformed Functions
1	$h(t)=f(t)\pm g(t)$	$H(k)=F(k)\pm G(k)$
2	$h(t)=af(t)$	$H(k)=aF(k)$
3	$h(t)=\frac{df(t)}{dt}$	$H(k)=(k+1)F(k+1)$
4	$h(t)=\frac{d^2f(t)}{dt^2}$	H(k)=(k+1)(k+2)...k(k+n)F(k+n) $H(k)=(k+1)(k+2)F(k+2)$
5	$h(t)=\frac{d^nf(t)}{dt^n}$	H(k)=(k+1)(k+2)...k(k+n)F(k+n)
6	$h(t)=1$	$H(k)=\delta(k)$
7	$h(t)=t$	$H(k)=\delta(k-l)$
8	$h(t)=t^r$	$H(k)=\delta(k-r);\delta(k-r)=\begin{cases}1 & k=r\\ 0 & \text{else}\end{cases}$
9	$h(t)=f(t)g(t)$	$H(k)=\sum_{m=0}^{k}F(m)G(k-m)$
10	$h(t)=\frac{f(t)}{g(t)}$	$H(k)=\sum_{m=0}^{k}\frac{F(m)}{G(k-m)}$
11	$h(t)=\exp(rt)$	$H(k)=\frac{r^k}{k!}$
12	$h(t)=(1+t)^m$	$H(k)=\frac{m(m-1)...(m-k+1)}{k!}$
13	$h(t)=sin(\alpha t+\beta)$	$H(k)=\frac{\alpha^k}{k!}sin\left(\frac{\pi}{2}k+\beta\right)$
14	$h(t)=cos(\alpha t+\beta)$	$H(k)=\frac{\alpha^k}{k!}cos\left(\frac{\pi}{2}k+\beta\right)$
15	$h(t)=tan(\alpha t+\beta)$	$H(k)=\sum_{m=0}^{k}\frac{(k-m)!}{m!}\alpha^{2m-k}\frac{sin\left(\frac{\pi}{2}m+\beta\right)}{cos\left(\frac{\pi}{2}(k-m)+\beta\right)}a$
16	$h(t)=sinh(\alpha t)$	$H(k)=\frac{1}{2k!}\left(\alpha^k-\left(-\alpha^k\right)\right)$
17	$h(t)=cosh(\alpha t)$	$H(k)=\frac{1}{2k!}\left(\alpha^k+\left(-\alpha^k\right)\right)$
18	$h(t)=tanh(\alpha t)$	$H(k)=\sum_{m=0}^{k}\frac{(k-m)!}{m!}$

$$
\left.\begin{aligned}
&A(k+1)=\frac{1}{k+1}\left(\Lambda\delta(k)-\xi\sum_{m=0}^{k}\big(M(m)+\omega I(m)\big)A(k-m)-\mu_1 A(k)\right)\\
&M(k+1)=\frac{1}{k+1}\xi v\sum_{m=0}^{k}\big(M(m)+\omega I(m)\big)A(k-m)-k_1 M(k)\\
&I(k+1)=\frac{1}{k+1}\left((1\text{-v})\xi\sum_{m=0}^{k}\big(M(m)+\omega I(m)\big)A(k-m)-k_2 I(k)\right)\\
&C(k+1)=\frac{1}{k+1}\big(\alpha_1 M(k)+\alpha_2 I(k)-k_3 C(k)\big)\\
&K(k+1)=\frac{1}{k+1}\big(\rho_1 C(k)-k_4 K(k)\big)\\
&E(k+1)=\frac{1}{k+1}\big(\eta K(k)-k_5 E(k)\big)\\
&W(k+1)=\frac{1}{k+1}\big(\beta E(k)-\mu W(k)\big)\\
&D(k+1)=\frac{1}{k+1}\big(\rho_2 C(k)-\mu D(k)\big)
\end{aligned}\right\} \qquad \textbf{(15)}
$$

Using the initial conditions, were:
A(0) = 12000,
M(0)= 6000,
I(0)= 6000,
C(0) = 21000,
R(0)=11000,
W(0) =10000,
E(0) = 8000,
D(0) = 10000;

$$A(t)=1.754847056\times10^{14}t^4-2.212036168\times10^{12}t^3-1.937760000\times10^{6}t^2-3.621600000\times10^{6}t+12000$$

$$M(t)=-8.774235282\times10^{13}t^4-5.168124901\times10^{10}t^3+945199.2000t^2+1.817520000\times10^{6}t+6000$$

$$I(t)=-8.774096771\times10^{13}t^4-5.540368577\times10^{10}t^3+927030t^2+1.81740\times10^{6}t+6000$$

$$C(t)=1.413010420\times10^{9}t^4-30719.30800t^3-90868.2t^2-3300t+21000$$

$$R(t)=16905.30450t^3+54509.85000t^2+270.00t+11000$$

$$E(t)=235.0658400t^4-283.9260000t^3+3260.400000t^2-7000.00t+8000$$

$$W(t)=-75.09424916t^4+1010.442333t^3-3243.500000t^2+6540.00t+10000$$

$$D(t)=11754.74167t^3+36339.5t^2+340t+10000$$

RESULTS AND DISCUSSION

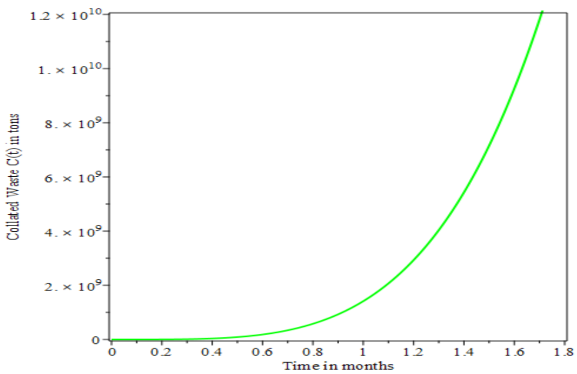

Fig1a:change in Collated wastes with time

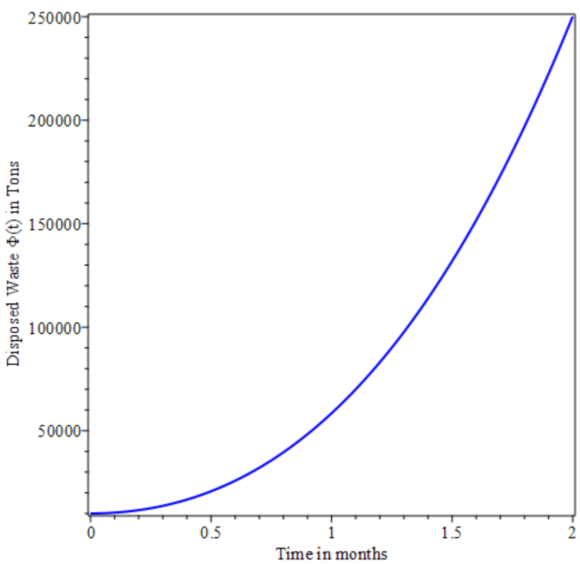

Fig2a:change in Disposed wastes with time

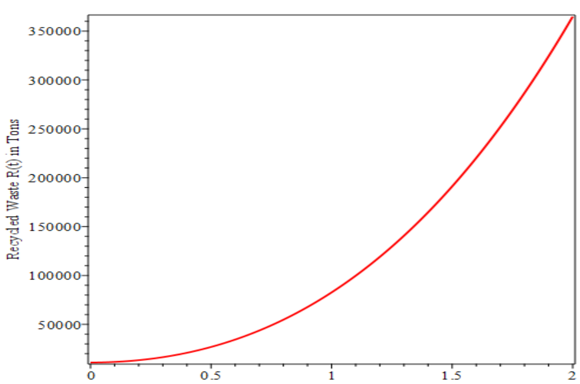

Fig1b:change in Recycled wastes with time

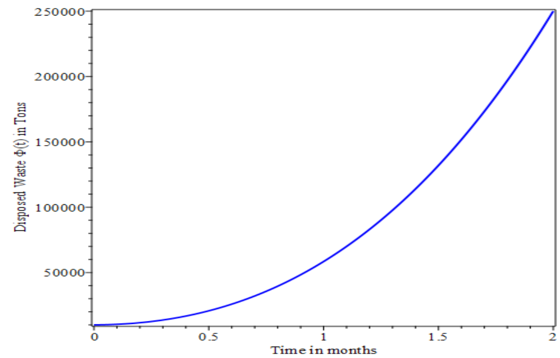

Fig2a:change in Disposed wastes with time

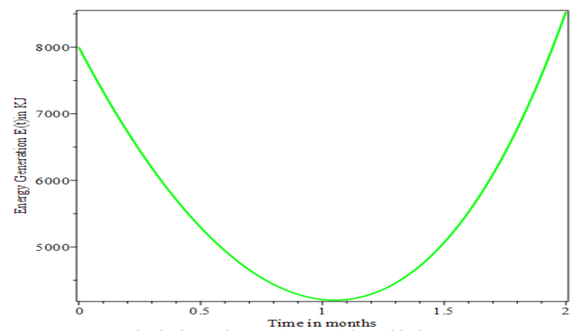

Fig2b:change in Energy Generation with time

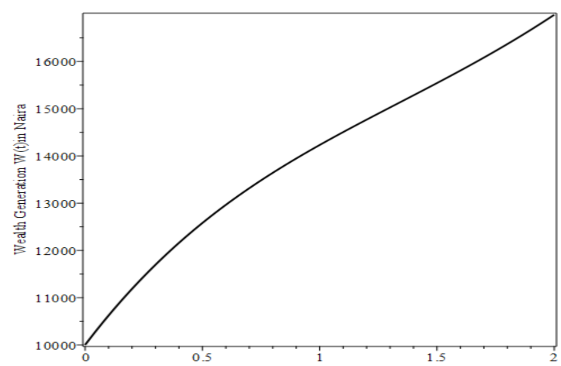

Fig3:change in Wealth Generation with time

DISCUSSION OF RESULTS

The results obtained from the DTM were simulated using the Maple software. They are shown above in Figures 1 to 3. Figure 1a shows that the collated waste increases at an exponential rate with time and it gets to the peak after some time, indicating that some of the wastes decompose at the collation center and some were lost by other means.

The recycled waste increases with time as shown in Figure 1b, this is due to the fact that recycled waste is obtained from the collation center, so it increases at the same rate.

Figure 2a shows that disposed waste increases with time, based on the sources of disposal. Figure 2b gives the energy generation as reducing with time till a minimum point is achieved and it then increases till a maximum point is achieved. This implies that energy generation from waste will take some time before it can get to the optimal point.

Wealth creation as shown in Figure 3 increases with time as the quantity of waste generated increases.

CONCLUSION

A novel model for recycling of waste using a mathematical approach was formulated, confirmed to be epidemiologically well posed and that its unique solution lies in the positive invariant region. The model equation was solved using the DTM. The results from the DTM showed that energy generation and wealth creation from waste will take some time before it can be maximized as it relied solely on the quantity of waste recycled for that purpose. This study recommended that concerted effort should be put into waste recycling and energy generation to increase wealth creation and facilitate proper waste management.

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