Comparative Analysis of Sudoku Square Design Models Using Variance Components and Heritability Estimates

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ABSTRACT

One of the attributes of selection of traits in plants and animal is heritability. Heritability is a function of variance components estimates from random effect and mixed models. This study focuses on the variance components and heritability estimation of Sudoku square design models. It considered the Sudoku models of balanced order with equal number of rows and column, equal number of rowblock, and colunmblock.

This study used ANOVA estimators, obtain estimates of variance components for the Sudoku square design models, treatment (trait) effect was subjected to test of significance. Modification for heritability for the four models of Sudoku square design models was carried out and heritability estimators for the models were obtained. The variances components estimate for treatments (trait) showed that there is no significant difference across all the four models and heritability results show that Sudoku square design model I and II recorded high heritability trait and models III and IV have medium heritability.

(Keywords: heritability, variance components, Sudoku models)

INTRODUCTION

Variance components are extensively used in animal and plant breeding for the estimation of heritability with the aim to help increasing the production of economically important products from farm Animals or crops that will be disease resistance, drought resistance crops, and provide high quality yields. Variance component estimates at times are usually obtained from the linear functions of mean squares with the aid of mean squares gotten from expectation (Comstock and Robinson, 1952a).

The ratio of variances of these traits can be used to assess what is known to be the Heritability of a trait. Heritability in the broad and narrow sense provide two measures of the importance of genetic factors to a trait, with the broad-sense heritability, *VCEg* /*VCEt*, also called the "coefficient of genetic determination," expressing the extent to which the phenotype is explained by genotype in a particular population. The VCEg, is refer to as genetic variance and *VCEt* is the total variance of the quantitative trait.

In accordance with assertions of Panse (1957) and Tsegaye, *et al.* (2007) that in the selection of traits or new improved variety in plant and animal attention must be gear towards the use of wide genetic diversity that is inherited character (high heritability value and high value of genetic advance). Therefore, with the use of high heritability value of a trait that is a function of components of variance, it shows that the character is modifiable and inheritable.

Valencia, *et al.* (2007) used variance components estimates to obtain heritability for total milk and cumulated milk production and for lactation length in Saanen goats in Mexico using data from one herd. Torres-Vazquez, *et al.* (2009) used a population of 4007 lactation records from 5200 Saanen goats kidding from 1999-2006 obtained from 10 herds. The analysis was carried out the estimation of heritability for milk, fat and proteins were obtained with the aid of a five-trait model.

Tippett (1931) used variance components to evaluate a method of optimal sampling design. Daniel (1938) explained the application of variance components to investigate factors that cause unevenness in wool. Searle, *et al.* (1992), Raudaubush, *et al.* (2002), and Verbeke and Molenbergh (2002) explained the application of variance components in areas of genetic and sampling design, which have importance in the fields of research such as in animal breeding studies in biological sciences, educational research like psychology, as well as medical research.

Ewa, *et al.* (2017) used Analysis of variance (ANOVA) method to obtain the estimates of variance components, some traits of BC2 (a particular variety of Cassava) population of Cassava, the phenotypic and genotypic coefficients of variation were computed. The results show that the phenotypic coefficient of variation (PCV) in the BC2 population was higher than the corresponding genotypic coefficient of variation (GCV) for all the traits measured. This indicates the effect of environment on the expression of these traits, high heritability was recorded in some of the traits evaluated.

Rahajeng and Rahayuningsih (2017) calculated the phenotypic and genotypic variances and the heritability of traits from ANOVA estimator and the result of analysis of variance showed high significant genotype in all the traits observed. All the traits showed a wide range of phenotypic variance and genotypic variance and heritability.

As the world is suffering from food deficits, there is need for yield improvement, which has been the major thrust in crop improvement programs. This can be achieved by utilizing variability within a gene pool. Some studies on the genetic variability among cassava genotypes and heritability showed remarkable intra-varietal variation (Asante and Offei, 2003; Zaldivar, *et al.*, 2004). Shokrollah and Zandich (2012), heritability estimates were obtained for body weight in birth of Kurdish sheep, the study considered ages of the sheep from three months to twelve months.

Comstock and Moll (1963), make the comparison of two random effect models, they obtain heritability for each of the models.

Many authors have carried out research involving Sudoku square models such Subramani and Ponnuswamy (2009), Hui-Dong and Ru-Gen (2008), Shehu and Dauran (2020), and Shehu and Danbaba (2018). These authors have worked on different areas such as construction of linear models, analysis of variance, derivation of variance components, and so on. However, our attention would be on four models proposed by Subramani and Ponnuswamy in 2009 and from the literature cited above, none as ever compares the models using the estimates of variance components and heritability of traits. The aim of this research is to make comparison of the Sudoku square models proposed by Subramani and Ponnuswamy using component of variance and heritability estimates.

METHODOLOGY

This research makes use of Equations 1 – 4 proposed by Subramani and Ponnuswamy in (2009).

Type I Model

$$Y_{ij(k,l,p,q)} = \mu + \alpha_i + \beta_j + \tau_k + C_p + \gamma_l + s_q + e_{i,j(k,l,p,q)} \quad \textbf{(1)}$$

where $i, j = 1 \cdots m \quad k, l, p, q = 1 \cdots m^2$

Type II Model

$$Y_{ij(k,l,p,q)} = \mu + \alpha_i + \beta_j + \tau_k + \gamma(\alpha)_{l(i)} + C(\beta)_{p(j)} + s_q + e_{i,j(k,l,p,q)} \quad \textbf{(2)}$$

where $i, j\ l, p = 1 \cdots m \quad k, q = 1 \cdots m^2$

Type III Model

$$Y_{ij(k,l,p,q,r)} = \mu + \alpha_i + \beta_j + \tau_k + \gamma_l + c_p + s(\alpha)_{q(i)} + \pi(\beta)_{r(j)} + e_{i,j(k,l,p,q,r)} \quad \textbf{(3)}$$

where $i, j, q, r = 1 \cdots m \quad k, l, p = 1 \cdots m^2$

Type IV Model

$$Y_{ij(k,l,p,q,r)} = \mu + \alpha_i + \beta_j + \tau_k + \gamma(\alpha)_{l(i)} + c(\beta)_{p(j)} + s(\alpha)_{q(i)} + \pi(\beta)_{r(j)} + e_{i,j(k,l,p,q,r)} \quad \textbf{(4)}$$

where $i, j, l, p, q, r = 1 \cdots m \ \ and \ \ k = 1 \cdots m^2$

Definition of Terms

μ = Grand mean
α_i = ith Row block effect

$\beta_j = jth$ Column block effect
$\tau_k = kth$ treatment effect
s_q = qth square effect
$C_p = p$th Column effect
$\gamma_l = l$th Row effect
s_q =qth square effect
$\gamma(\alpha)_{l(i)}$ = lth Row effect nested in ith row block effect
$c(\beta)_{p(j)}$= pth Column effect nested in jth column block effect
$s(\alpha)_{q(i)} = qth$ Horizontal square effect nested in ith Row block effect
$\pi(\beta)_{r(j)} = rth$ vertical square effect nested in the jth column block effect
$e_{i,j(k,l,p,q,r)}$ = is the error component assumed to have mean zero and constant variance σ^2.

All effects are random, normal, independent derivation with expected values equal to zero.

$$Heritability\ of\ a\ trait\ (HT) = 4\,VC_{trait}\,(VC_{total})^{-1} \quad \textbf{(5)}$$

Equation (5) is the heritability of a trait from one-way random effect model which has a random effect error and with a random effect treatment. The variance components for equations 1-4 were derived and obtained earlier in the previous research (see Shehu and Danbaba, 2018; for more details). ANOVA method was used for the derivation of the variance component of the above trait models (Equations 1-4) and ANOVA estimators for Sudoku square design models were obtained and the estimators were subjected to properties of ANOVA.

The estimators satisfy all the properties (see Shehu and Dauran, 2020 for details). As defined earlier, heritability of a trait is a function of variance components, the equation (5) is modified to get the heritability of Models 1-4 and presented as follows:

For trait in model I, Heritability $(HT1)$ can be computed as:

$$VC_{total} = VCE_\alpha + VCE_\beta + VCE_\tau + VCE_\gamma + VCE_C + VCE_s + VCE_e$$

$$HT1 = 4VCE_\tau\,(VCE_\alpha + VCE_\beta + VCE_\tau + VCE_\gamma + VCE_C + VCE_s + VCE_e)^{-1} \quad \textbf{(6)}$$

For trait in Model II, Heritability $(HT2)$ can be computed:

$$HT2 = 4VCE_\tau\,(VCE_\alpha + VCE_\beta + VCE_\tau + VCE_{C(\beta)} + VCE_{\gamma(\alpha)} + VCE_s + VCE_e)^{-1} \quad \textbf{(7)}$$

For trait in model III, Heritability $(HT3)$ can be computed as:

$$HT3 = 4VCE_\tau\,(VCE_\alpha + VCE_\beta + VCE_\tau + VCE_\gamma + VCE_C + VCE_{s(\alpha)} + VCE_{\pi(\beta)} + VCE_e)^{-1} \quad \textbf{(8)}$$

For trait in Model IV, Heritability $(HT4)$ can be computed as:

$$HT3 = 4VCE_\tau\,(VCE_\alpha + VCE_\beta + VCE_\tau + VCE_{\gamma(\alpha)} + VCE_{C(\beta)} + VCE_{s(\alpha)} + VCE_{\pi(\beta)} + VCE_e)^{-1} \quad \textbf{(9)}$$

The Equations 6-9 can also be expressed in terms of percentage, this study is considering.

<u>Classification of Heritability Estimates</u>

Bhateria, *et al.* (2006) method of classification of heritability estimates as follows:

If $HT > 50\%$ is considered high
$30 \le HT \le 50\%$ is considered medium and
$HT < 30\%$ is considered low heritability.

<u>Test of Significance of Variance Components</u>

$$F_0 > F_{dfeffect,dferror,0.95} \quad \textbf{(10)}$$

SIMULATION

Computer programs were developed using R version 3.3.1 (2016) Statistical Software for the simulations. The program was written with the sums of squares for all sources of variation in the models and the mean sums of squares was obtained. Also in the program, the estimators for variance components were incorporated in order to generate variance component estimates for all the sources of variation we have in the sudoku square models.
The program also used Normal distribution *N(0, 1)* and some of the parameters as follows:

nrowblock=ncolumnblock =m =3, nrow = ncolumn = k = 9 as observed in the models.

Tables 1-4 revealed the results of simulations which includes the sums of squares (SS), mean sums of squares (MSS), Variance components estimates, F-ratio, and p-value.

RESULTS AND DISCUSSION OF VARIANCE COMPONENT AND HERITABILITY ESTIMATES

In this section the comparison is made on the variance component estimates of effect in all the four models. Only the report of test of significance at alpha level of 0.05 of variance components of treatment (trait) effects are presented for the four models as the procedure for other effects are the same and also the most important. The variance components for the treatment for all the models are not significant at the 5% level.

Because from Table 5, none of the values of F_0 is greater than $F_{dfeffect,dferror,0.95}$ this is an indication that the estimate of variance components across the four models has no significant difference. This is an indication that the variance component of treatment effect is similar across the four models.

Table 1: Variance Components for Sudoku Square Design Model I.

Source	DF	SS	MSS	VCE	F_{ratio}	P_value
treatment	8	24.8315	3.1039	0.1334	1.6307	0.1436
row block	2	16.3143	8.1572	0.1587	4.2854	0.0199
column block	2	4.0978	2.0489	0.0054	1.0764	0.3496
row	8	27.7225	3.4653	0.1735	1.8205	0.0987
column	8	21.7167	2.7146	0.0901	1.4261	0.2127
square	8	49.6646	6.2081	0.4783	3.2615	0.0053
Error	44	83.7527	1.9035	---	---	---
Total	80	228.	---	---	---	---

Table 2: Variance Components for Sudoku Square Design Model II.

Source	DF	SS	MSS	VCE	F_{ratio}	P_value
treatment	8	24.8315	3.1039	0.1037	1.4303	0.211
row block	2	16.3143	8.1572	0.2217	3.7589	0.0311
column block	2	4.0978	2.0489	-0.0045	0.9441	0.3967
rows within RB	6	11.4081	1.9014	-0.0223	0.8762	0.5202
column within RB	6	17.6189	2.9365	0.0638	1.3532	0.2547
square	8	49.6646	6.2081	6.2081	2.8607	0.0119
Error	48	104.1648	2.1701	---	---	---
Total	80	228.	---	---	---	---

Table 3: Variance Components for Sudoku Square Design Model III.

Source	DF	SS	MSS	VCE	F_{ratio}	P_value
treatment	8	24.8315	3.1039	0.1008	1.4133	0.2209
row block	2	16.3143	8.1572	0.2208	3.7141	0.0331
column block	2	4.0978	2.0489	-0.0055	0.9329	0.4018
rows	8	27.7225	3.4653	0.1410	1.5778	0.1623
column	8	21.7167	2.7146	0.0576	1.236	0.3039
square within RB	6	33.3503	5.5584	0.2801	2.5308	0.5283
square within CB	6	45.5668	7.5945	0.4499	3.4579	0.2636
Error	40	87.8505	2.1963	---	---	---
Total	80	228	---	---	---	---

Table 4: Variance Components for Sudoku Square Design Model IV.

Source	*DF*	*SS*	*MSS*	*VCE*	F_{ratio}	*P_value*
treatment	*8*	24.8315	3.1039	0.0715	1.2615	0.2879
row block	*2*	16.3143	8.1572	0.2109	3.3152	0.0456
column block	*2*	4.0978	2.0489	-0.0152	0.8327	0.4416
rows within RB	6	11.4081	1.9014	-0.0466	0.7727	0.5955
column within CB	6	17.6189	2.9365	0.0396	1.1934	0.3276
square within RB	6	33.3503	5.5584	0.2581	2.259	0.0549
square within CB	6	45.5668	7.5945	0.4278	3.0835	0.0131
Error	*44*	108.2626	2.4605	---	---	---
Total	*80*	228	---	---	---	---

Table 5: Test for the Variance Components of Treatment Effects for Sudoku Square Models I-IV.

Model		$df_{treatment}$	$\hat{\sigma}_\tau^2$	F_0	$F_{df effect, df error 0.95}$
I	44	8	0.1334	1.6307	2.1572
II	48	8	0.1037	1.4303	2.1382
III	40	8	0.1008	1.4303	2.1802
IV	44	8	0.0715	1.2616	2.1572

Heritability

Table 6 revealed the heritability of a trait for the four models, Model I recorded highest heritability followed by Model II while the smallest heritability came from Model IV. The heritability of trait reduces as more effects are introduced into the source of variation and more effects are nested into another effect in the models.

Table 6: Heritability for Single Traits of Sudoku Square Models I-IV.

Model	*Trait VCE*	*Total VCE*	*HT*
I	0.1334	1.0394	51.34
II	0.1037	0.8111	51.14
III	0.1008	1.2428	32.44
IV	0.0715	0.9461	30. 03

CONCLUSION

This study has revealed the variance components estimates for the four Sudoku square design models proposed by Subramani and Ponnuswamy. However, the variance components estimates were obtained using ANOVA method, it was observed that some estimates were negatives, which variance ought not.
Though, the negativity of variance component estimates using ANOVA has been discussed by Shehu and Dauran, (2020). Actually, the negative variance component is one of the challenges of ANOVA method.

Test of significance was carried out on the variance components of treatment (trait) effect, the result shows that there was no significant difference on the variance components of treatment effect across the four models, that is, to say variance components estimates are similar in all the models.

The research also made some modification and obtained estimators for heritability for the Sudoku models, the comparison of the models was carried out, the result shows that models I and II have high heritability, model III and IV are medium heritability of one trait. From Table 6, it was observed that the more factors are added to the sources of variation the heritability of a trait decreases.

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