

Fast Fourier Transform and its Application in Determining the Depth of Magnetic Sources in Parts of Lower Benue Trough

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ABSTRACT

This paper seeks to look at the concept of Fast Fourier Transform (FFT) and its various applications with special focus on how it can be used to determine the depth to magnetic sources in parts of Lower Benue Trough. The study area covers between latitude 6° 0' to 7° 0' North and longitude 7° 0' to 8° 0' East with an undulating topography and an elevation ranging from 200 – 500 m above sea level. Four aeromagnetic data sheets were acquired from the Nigeria Geological Survey Agency (NGSA). The data sheets cover Igumale, Nsukka, Nkalagu and Udi with sheet numbers 288, 287, 302 and 301, respectively.

The total magnetic intensity obtained from the sheets was windowed and the data smoothed. FFT was carried out on the data set to extract the frequency and energy components which was thereafter used to obtain a spectral log. The depth of the magnetic source for the study area was determined using the slope of the graph between energy and frequency. Results of the operation show that magnetic sources are found at depths ranging from 272 m to 4,4043 m with an average of 3,704m. It further reveals that Igumale area has the deepest depth while the Nkalagu area has the least depth to the magnetic sources. This paper has further corroborated that FFT can be used in determining the depth where magnetic sources lie in the subsurface.

(Keywords: Fast Fourier Transform, FFT, applications, aeromagnetic data)

INTRODUCTION

Fast Fourier Transform (FFT) is an invaluable problem-solving tool that is widely used in signal processing and analysis concept. It is an algorithm for efficiently computing the Discrete

Fourier transform (DFT) of a sequence. The DFT is a mathematical operation that transforms a sequence of P complex numbers into another sequence of P complex numbers, and it is widely used in signal processing and other fields for analyzing the frequency content of signals, filtering signals, and performing other operations on signals in recent times especially in the areas of geophysical analysis, biomedical engineering, stock market data and nonlinear system analysis (Bouchaleun, 2018; Nyakundi, *et al.*, 2019; and Ozegin, *et al.*, 2021).

The FFT algorithm works by decomposing a sequence of P complex numbers into P/2 pairs of complex numbers, and then recursively applying the FFT to each pair. This operation is done again until each pair consists of a single complex number, at which point the FFT is complete. The computational complexity of the FFT is (P log₂P), which means that the time required to compute the FFT grows only logarithmically with the size of the input sequence. This makes the FFT an efficient algorithm for computing the DFT of large sequences. FFT algorithm was introduced in response to a problem in digital signal processing. In 1965, Cooley and Tukey first published it in their paper titled "An Algorithm for the Machine Calculation of Complex Fourier Series" (Cooley and Tukey, 1965). Since then, the algorithm has been widely used for computing the DFT due to its computational efficiency and ease of implementation.

There are some basic concepts related to the FFT, these include:

Complex Numbers: The FFT algorithm operates on complex numbers, which are numbers that have both a real and imaginary component. That is: $z = x + iy$

where x is the real component,

$y \in \Re$

and i is the imaginary unit.

The represents the horizontal position of the number on the complex plane, and the imaginary component represents the vertical position.

Frequency Spectrum: The frequency spectrum of a signal is a represents the distribution of frequencies in the signal. The frequency spectrum of a signal DFT can be computed using DFT through the decomposing of a signal into its frequency components.

Discrete Fourier Transform: The discrete Fourier transform (DFT) is a mathematical operation that transforms a sequence of P complex numbers into another sequence of P complex numbers. It is widely used in signal processing and other fields for analysing the frequency content of signals, filtering signals, and performing other operations on signals.

Fast Fourier Transform: This is an algorithm for efficiently computing the DFT of a sequence and it is widely used due to its computational efficiency and ease of implementation.

However, the ability of one to apply the FFT algorithm relies on the understanding of the Fourier Transform (FT) which is a mathematical operation that decomposes a function, signal, or sequence of values into the various frequency constituents. It is a fundamental tool in many areas of mathematics and physics, and it has a wide range of applications in science and engineering. It is defined as the integral of a function $f(t)$ multiplied by a complex exponential over a given range. For example, the Continuous Fourier Transform, (CFT) of a function $f(t)$ is given in equation (1) according to Brace (1986):

$$F(\omega) = \int f(t) e^{(-i\omega t)} dt \quad (1)$$

Where $F(\omega)$ is the Fourier transform of $f(t)$, ω is the frequency, t is time and i is the imaginary unit.

Also, the inverse Fourier transform is given in equation (2)

$$f(t) = \frac{1}{2\pi} \int F(\omega) e^{(i\omega t)} d\omega \quad (2)$$

FT has many useful properties, such as the convolution theorem, which states that the Fourier transform of the convolution of two functions is equal to the product of their Fourier transforms. This property makes the Fourier transform a useful tool for analysing and manipulating signals and functions. There are different forms of FT such as the continuous Fourier transform, (CFT), the Discrete Fourier transform (DFT), and the Fast Fourier transform (FFT). While (FFT) is quite efficient to be used as an algorithm for computing the DFT, the DFT is a discrete version of the Fourier transform that is used to analyse discrete sequences of data.

Derivation of Fourier and Fast Fourier Transforms

With the definition of the CFT in equation (1), we can rewrite the expression as:

$$F(\omega) = \int f(t) \cos(-\omega t) - i \int f(t) \sin(-\omega t) dt \quad (3)$$

Now, using the identities:

$$\cos(-x) = \cos(x) \text{ and } \sin(-x) = -\sin(x),$$

we can simplify the equation to:

$$F(\omega) = \int f(t) \cos(\omega t) + i \int f(t) \sin(\omega t) dt \quad (4)$$

Equation (4) is the standard form of the CFT.

To obtain the expression for the FFT algorithm, if P is even, the FFT of a sequence

$X_{(0)}, X_{(1)}, \dots, X_{(P-1)}$ is given in equation (5):

$$X(k) = X(k) + WP^k * X(k + \frac{P}{2}), \text{ for } k = 0 \text{ to } \frac{P}{2} - 1 \quad (5)$$

Where $X(k)$ and $X(k + \frac{P}{2})$, are the DFT coefficients for the even and odd indices of the input sequence, respectively, and WP^k is a complex exponential term.

However, if P is odd, the FFT of a sequence $X_{(0)}, X_{(1)}, \dots, X_{(P-1)}$ is given in equation (6):

$$X(k) = X(k) + WP^k * X(k + \frac{(P-1)}{2}), \text{ for } k = 0 \text{ to } \frac{(P-1)}{2} \quad (6)$$

Where $X(k)$ and $X(k + \frac{(P-1)}{2})$, are the DFT coefficients for the even and odd indices of the input sequence, respectively, and WP^k is a complex exponential term.

The FFT algorithm can be derived from the definition of the DFT, given in equation (7):

$$X(k) = \sum xp e^{i2\pi kp/P}, \text{ for } p = 0 \text{ to } P-1 \quad (7)$$

However, if we express the sum as a pairwise sum, and using the identity via convolution, we now obtain (8):

$$e^{i(x+y)} = e^{ix} * e^{iy} \quad (8)$$

Equation (8) shows that the FFT algorithm is equivalent to the DFT.

Areas of Application of FFT

The FFT has many applications in different fields of study such as:

1. **Signal Processing:** The FFT is widely used in signal processing for analyzing the frequency contained in signals, filtering signals, and performing other operations on signals.
2. **Geophysics:** FFT is applied in geophysics for tasks such as seismic data analysis and mineral exploration.
3. **Image Processing:** In image processing, FFT is useful for tasks such as image compression, image de-noising, and image restoration.

4. **Data Analysis:** FFT has been very useful in data analysis for tasks such as spectral analysis, which involves decomposing a time series into its frequency components.

5. **Communications:** In communication systems it can be used for tasks such as channel estimation, equalisation, and modulation.

REVIEW OF RELATED LITERATURE

Since the birth of FFT in the 1960s, FFT has been the subject of numerous research studies by researchers in various fields. There have been improved algorithms for improving the efficiency of the FFT such as the Cooley-Tukey algorithm and the Bluestein's algorithm. These algorithms have proven to minimise the computational complexity of the FFT, making it more efficient for certain types of input data, (Cooley and Turkey, 1965), (Bluestein, 1968). Various techniques for parallelising the FFT, such as using multi-core processors and graphics processing units (GPUs) have been explored by researchers. These techniques have been beneficial in speeding up the FFT by distributing the computation across multiple processors, (Duhamel and Vetterli, 2000).

Lia, *et al.* (2005) in their study, present a pixel-based modelling approach of concrete which combines an experimental characterisation of concrete and the Fast-Fourier Transform simulations. Here, they created high-resolution phase maps from experimental characterisation by energy dispersive X-ray analysis, micro X-ray fluorescence and X-ray diffraction containing nine phases, including 54.21 vol.% of minerals, 22.17 vol.% of hydrated cement paste and 23.62 vol.% of interfaces. These phases afforded the input for determining the effective elastic properties, alongside the Fast-Fourier Transform-based simulation which shows that the effective range of Young's modulus of concrete is comparable with the range of experimental values $\approx 37 \pm 4$ GPU with the assumption of realistic properties of interfaces.

Nyakundi, *et al.* (2019) used FFT to convert the space domain grid data to the Fourier wave number domain where filters are applied. And their results from spectral analysis revealed a curie point isotherm depth of approximately

2150m. This shows that there is a shallow isotherm depth indicating high heat flux within the subsurface of Eburru area. Also, they showed that there is increased tendency for the availability of geothermal resource in the study area since at Curie point isotherm depth of approximately 2150m, the temperatures must be beyond 130°C.

Ozegin and Alile (2021) used FFT to study the analysis of aeromagnetic data using Oasis Montaj software. In order to reduce aliasing error, the aeromagnetic data was gridded at a space of 2km. It was evident that the depth to the magnetic sources in the area ranges from shallow 50 m to 1500 m. The change in the depth values as they inferred shows common inclinations in the magnetic basement surfaces. The FFT decomposes the residual into its frequency components and energy spectral segments. Logarithm of the energy spectral is thus plotted against the radial frequency component. The average radial power spectrum is calculated using FFT.

FFT In Mineralogy

FFT as a mathematical operation, decomposes a function into its constituent frequencies, and it has been applied in the field of mineralogy for works like mineral identification and mineral analysis. One application of the FFT in mineralogy is the analysis of X-ray diffraction patterns. X-ray diffraction is a technique often employed in studying the crystal structure of minerals. By analyzing the diffraction pattern of a mineral, the arrangement of atoms in the crystal lattice can be determined and this can be used to identify the type of mineral under study. FFT can be used to analyse the diffraction pattern and extract information about the crystal structure of the mineral (Wiedenbeck, 2011).

Smeds and Selroos (2007) applied the FFT in mineralogy in the study of the analysis of infrared spectra. Where infrared spectroscopy is a technique used to study the vibrational modes of molecules, and it is commonly used in mineralogy to identify minerals. By analysing the infrared spectrum of a mineral, one can determine the types of bonds present in the mineral, which can be used to identify the mineral. FFT is useful in analysing the infrared spectrum and extract information about the vibration modes of the mineral.

FFT when operated on the Total Magnetic Intensity (TMI) of an area, energy and frequency components of the data are separated and this is used in spectrum analysis. Nyakundi, *et al.* (2019) showed that there is a variation between the radially averaged energy spectrum of a magnetic data and wave number after analysing magnetic data using spectral analysis. This is in line with the findings of other scholars who used FFT and spectral analysis to determine the magnetic sources, geothermal isotherms and heat flow of a given area as seen in the works of Ikeh, *et al.* (2017), Mono, *et al.* (2018), Udegbe, *et al.* (2017), and Bello *et al.* (2017).

The Study Area

The Benue Trough is the major sedimentary basin in Africa covering a length of 1000 km and a width of up to 250 km. It is a portion of the Cretaceous West African rift system which stretches up to 4000 km from Nigeria to Niger Republic and then terminates in Libya (Bings and Fairhead, 1992). The Benue Trough is divided into three geographical and structural regions namely, Lower Benue Trough, Middle Benue Trough and Upper Benue Trough (Figure 1).

The major area of focus of this study lies in the Lower Benue Trough lies between latitude 6° 0' to 7° 0' North and longitude 7° 0' to 8° 0' East with an undulating topography and an elevation ranging from 200 – 500 m above sea level. The area is geologically composed of the Cretaceous shale, sandstone, and siltstone of the Asu River group, the Eze-Aku formation, Agwu, Nkporo and Nsukka formations. The Tertiary age is composed of the Imo group, Agbada and the Ogwashi-Asaba formations. The youngest of them all is the Benin formation comprising of alluvium deposits, clays and intercalation of sand and clays (Azunna, *et al.*, 2021). The major towns covered in the study are Udi, Eha-Amufu, Amagunze, Awgu, and Agulu (Figure 2).

MATERIALS AND METHOD

The aeromagnetic data sheets were obtained from the Nigeria Geological Survey Agency (NGSA). The sheets are four in number and covers Igumale, Nsukka, Nkalagu and Udi with sheet numbers 288, 287, 302 and 301 respectively (Figure 3).

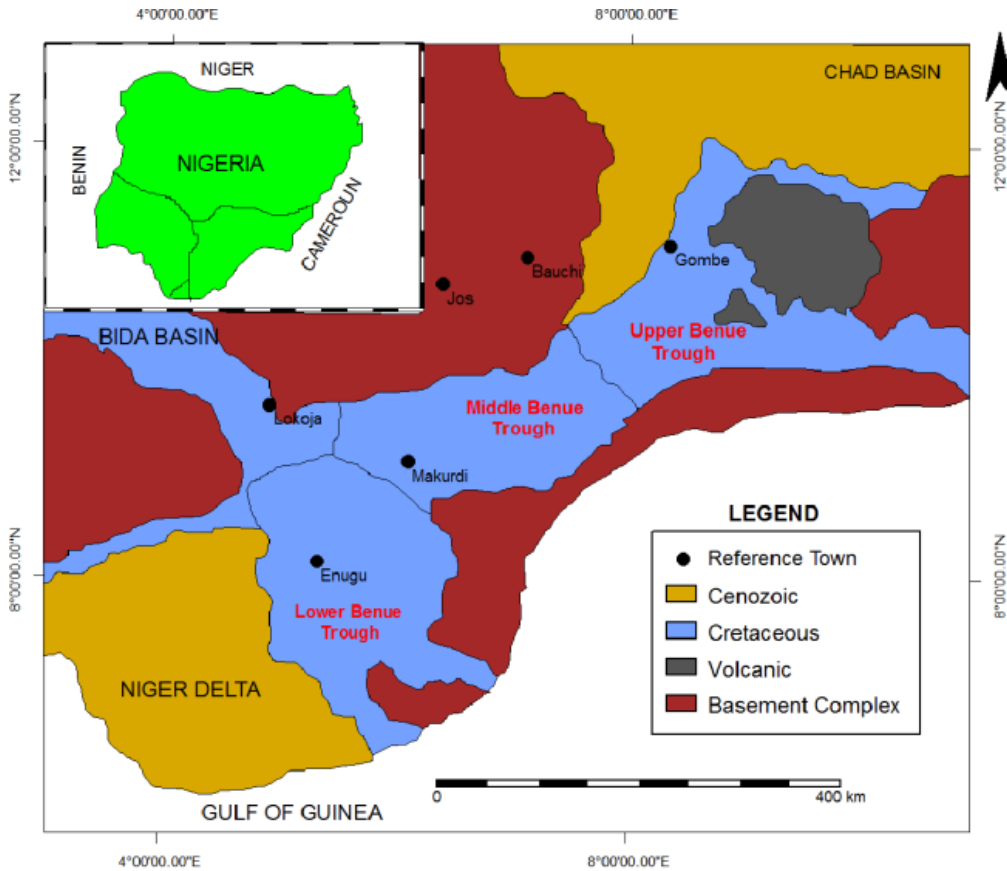


Figure 1: Map of the Benue Trough of Nigeria. Source. Obaje *et al.*, 1999)

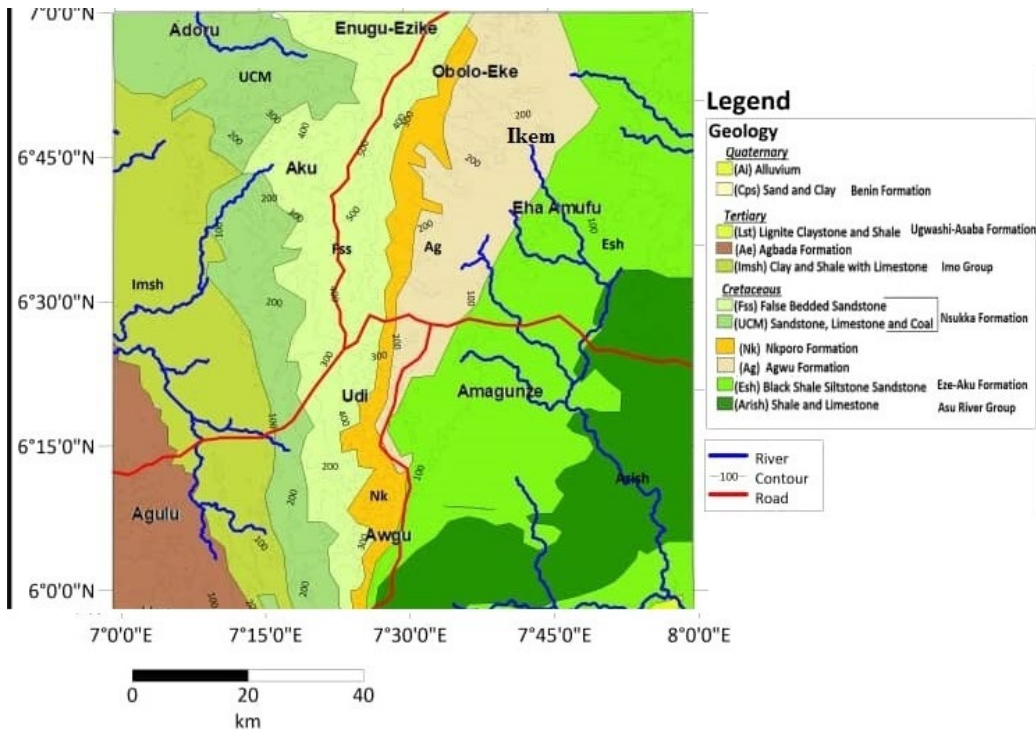


Figure 2: Location map of the Study Area Showing its Geology.

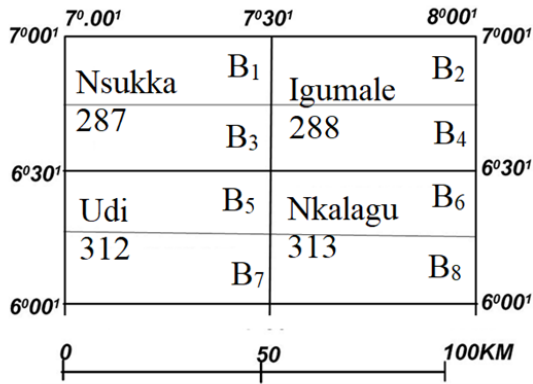


Figure 3: Sheet Map of the Study Area Showing the Spectral Blocks B₁ – B₈.

For a given space domain function $f(x,y)$, the Fourier transform and its inverse is defined in equations 9 and 10 according to (Nyakundi, 2019).

$$\bar{f}(\mu, \nu) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) e^{-i(\mu x + \nu y)} dx dy \quad (9)$$

$$f(x, y) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \bar{f}(\mu, \nu) e^{i(\mu x + \nu y)} d\mu d\nu \quad (10)$$

where μ and ν are measured in cycles per metre which are the wave numbers in the x and y directions.

Similarly, the power spectrum $\left| \bar{f}(\mu, \nu) \right|^2$ and its

total energy E_T are related as stated in equation (11).

$$E_T = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left| \bar{f}(\mu, \nu) \right|^2 d\mu d\nu \quad (11)$$

Spector and Grant (1970) shows that there is a linear gradient in the log of the energy spectrum of a magnetic source have with magnitude dependent on the depth of the source as expressed in equation 12

$$E(r) = e^{-2Zr} \quad (12)$$

where $E(r)$, Z and r are spectral energy, depth and frequency, respectively.

Therefore, the radial frequency and spectral energy as well as their logs were obtained and plotted for each windowed portion (See Appendix). The gradient of their linear segments were used to determine the depths to basement using equations 13 and 14. The gradients are usually negative which implies that the depth is with reference from the subsurface.

$$M_1 = \frac{\Delta(\log E)_1}{\Delta r_1} \quad (13)$$

$$M_2 = \frac{\Delta(\log E)_2}{\Delta r_2} \quad (14)$$

The deep depth (Z_d) and shallow depth (Z_s) were obtained using the two gradients M_1 and M_2 as given in equations (15) and (16)

$$Z_d = \frac{-M_1}{\pi} \quad (15)$$

$$Z_s = \frac{-M_2}{\pi} \quad (16)$$

These mathematical operations were performed using Geosoft Oasis Montaj and Microsoft excel software.

RESULTS AND DISCUSSION

Table 1 shows two different depths Z_d and Z_s as a result of the noticed presence of the magnetic body in the study area. From the table, magnetic bodies can be found at depths ranging from 272 m to 4,4043 m at an average depth of 3,704m. Comparing the results obtained with the sheet map in Figure 3, the Igumale area has the deepest depth while the Nkalagu area has the shallowest depth to the magnetic sources. Therefore, the study area has magnetic bodies which gave rise to the magnetic anomalies and most likely to be unfit for hydrocarbon exploration.

Table 1: Depth Estimates Obtained

Spectral Block	Depth	
	Shallow (Z_s)	Deep (Z_d)
B ₁	276.385	3998.927
B ₂	272.317	4043.490
B ₃	272.83	3867.783
B ₄	272.798	3540.879
B ₅	272.604	3621.093
B ₆	272.696	3319.017
B ₇	272.741	3729.637
B ₈	271.012	3512.230
Average	272.9229	3704.132

This result corroborates what other researchers have inferred in the area using other methods of depth determination such as Nwokoma, *et al.* (2021), Nyakundi, *et al.* (2017), and Ikeh, *et al.* (2017).

The aeromagnetic data consists of the Total Magnetic Intensity (TMI), latitude and longitude of the study area. It is given on a scale of 1:100,000 produced during an aero survey done at a flying altitude of 80m above the terrain with a series of NW-SE, a flight line spacing of 500m and direction of 45° azimuth. The average magnetic declination and inclination across the survey is 1.30° and 9.75°, respectively.

The acquired TMI was thereafter windowed into eight (8) portions and the data smoothed. A FFT was carried out on the smoothed data in order to extract the frequency components as well as the energy components of the signals for each of the spectral blocks B₁ – B₈ (Figure 3).

CONCLUSION

Fast Fourier Transform is a mathematical operation defined as the product of a complex exponential over a given range and the integral function $f(t)$. It can be used for various operations in science and engineering such as geophysical analysis, stock market data, non-linear system analysis. This paper seeks to apply FFT in determining the depth of magnetic sources in the sub-surface. Findings show that the depth for the study area ranges from 272 m to 4,4043 m with an average depth of 3,704m indicating the presence of magnetic ore bodies. This research further establishes that FFT is applicable in geophysical analysis especially in the area of depth estimation.

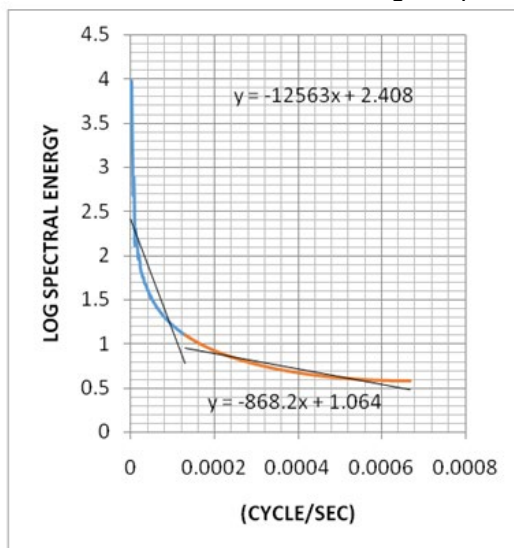
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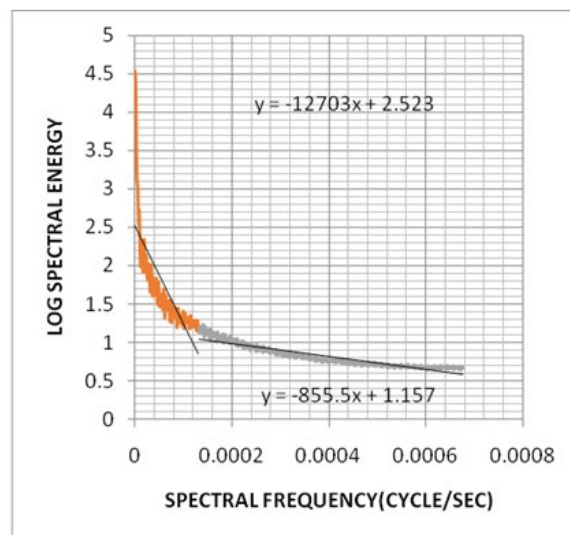
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Appendix

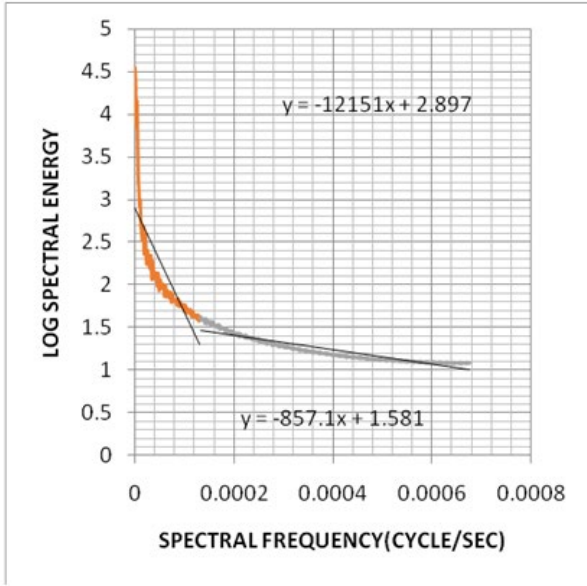
Plot of the Log of Spectral Energy Against Spectral Frequency



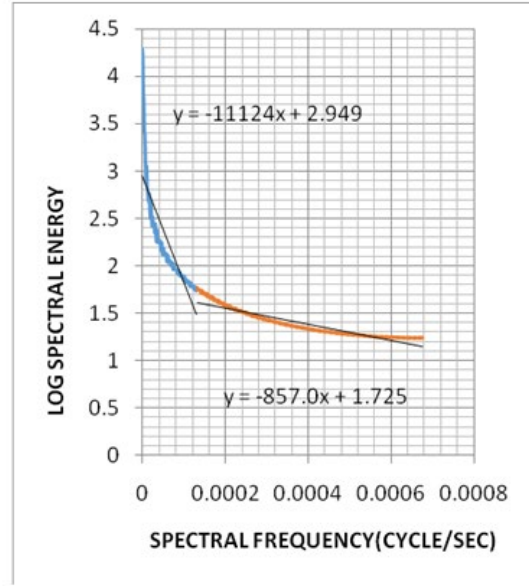
B₁



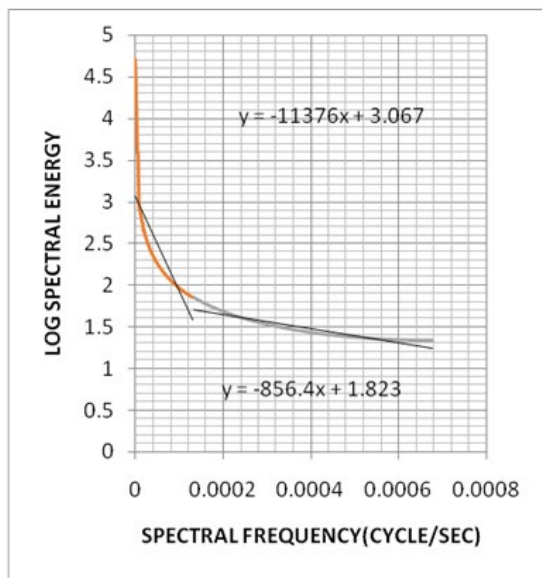
B₂



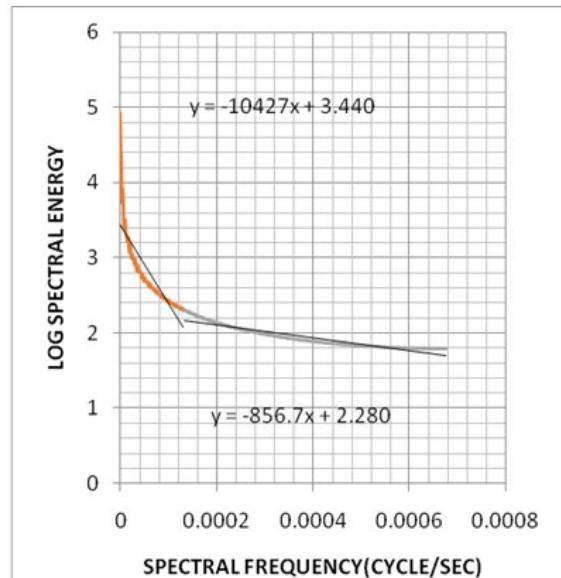
B₃



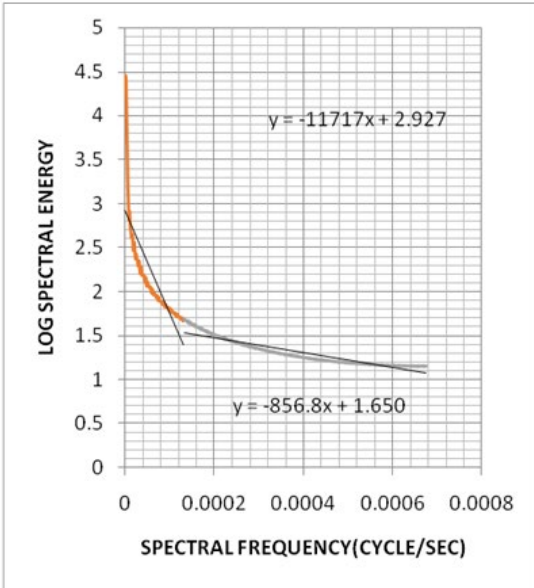
B₄



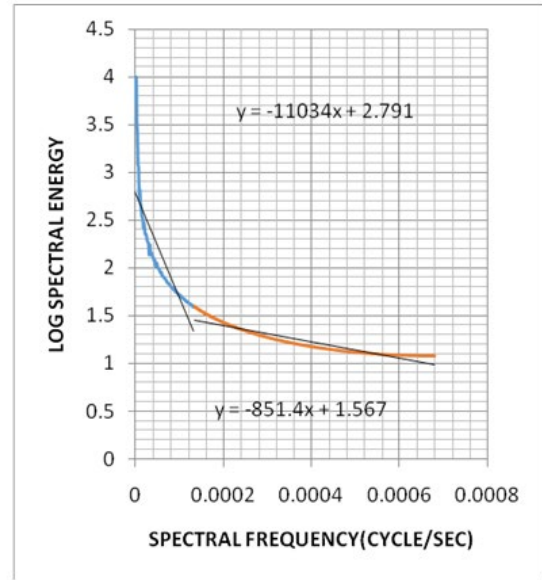
B₅



B₆



B₇



B₈

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