Standard Collocation Method by Legendre Basis Polynomial Function for Solving Integro-Differential Equations

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ABSTRACT

In this study, a new Gauss-Legendre Polynomials basis function was constructed and used for solving integro-differential difference equations using standard collocation method. An assumed approximate solution in terms of the constructed polynomial was substituted into the general class of integro-differential difference equation considered. The resulted equation was collocated at appropriate points within the interval of consideration to obtain a system of algebraic linear equations. Solving the system of equations, the unknown constant coefficients involved in the equations are obtained. The required approximate solution is obtained when the values of the constant coefficients are substituted back into the assumed approximate solution. Some numerical examples were solved to demonstrate the method.

(Keywords: integro-differential difference equations, Gauss-Legendre polynomials)

INTRODUCTION

Integro-differential equations (IDE) have been given considerable attention in recent times due to their applications in the field of mechanics, engineering, science and technology. However, most of the problems of IDEs are difficult to solve analytically. Therefore, numerical methods are needed to solve them.

Many numerical methods have been proposed for the determination of approximate solutions to this class of problems. They include Collocation Method, Variational Iteration Method, Homotopy Analysis Method, Adomian Decomposition Method, Laplace Decomposition Method, Iterative Decomposition Method, Galerkin Method, and Successive Approximation Method to mention but a few.

While solving higher order integro-differential equations Nada, et al. (2012) applied Variational Iteration method, described the method as a simple and yet powerful method for solving a wide class of linear and nonlinear problems. In the paper, it was shown that higher order integro-differential equations can be transformed into a system of integral equations which makes them easier to be solved using the method.

Taiwo and Adebisi (2013) also solved higher order linear Fredholm and Volterra integro-differential equations using multiple perturbed collocation methods and observed that the results obtained as N increases are closer to the exact solutions. However, many new methods such as variational method, variational iterations method and others.

Afrouzi, et al. (2011) are proposed to eliminate the shortcomings arising in the small parameter assumed in perturbation method. In the paper, fourth order Volterra integro-differential equations were solved using modified homotopy-perturbation method. The study concluded that the method is efficient and easy to use.

Jafari, et al. (2010) applied Legendre Wavelets method for solving Fredholm and Volterra system of linear integral equations. The method, like many other methods reduced the system of integral equations to a set of linear algebraic equations from which the values of the unknown coefficients are determined. The procedure consists of reducing the problem to linear algebraic equations and then using collocation method for the solution.

Biazar, and Salehi (2016) solved integro-differential equations of the second kind using Chebyshev Galekin method and said that it is a powerful tool for solving the kind of equations in various fields of science and engineering.

Avipsita, et al. (2017) solved the Volterra type of fractional order integro-differential equations using Bernstein polynomial as basis functions with collocation method. In the work, a simple algorithm for solving fractional integro-differential of Volterra type was introduced. The properties of Bernstein polynomials were used to reduce fractional order integro-differential equations with weakly singular to a system of algebraic equations.

In the books by Sastry (2010) and Wazwaz (2011), the authors introduced various methods for solving different Fredholm and Volterra differential and integro-differential equations with practicable algorithms. Collocation method was implemented by [9] to solve multi-order fractional integro-differential equations and paper described the method as a good method is capable of converting such problems into a system of algebraic linear equations to be able to get the solution easily.

Other researchers that have solved integro-differential equations includes and not limited to Yousefi, et al. (2017) who implemented operational matrix and Yanxin, et al. (2018) fractional-order Euler functions. Several numerical methods to solve the FIDEs have be found in Ibrahim et al. (2017), Ayoade, et al. (2018), Peter, et al. (2018), Peter (2020), and Peter, et al. (2021).

In this study, we proposed the application of standard Collocation Method for solving Integro-differential Difference Equations using constructed Gauss-Legendre Polynomials as basis functions.

Definition of Relevant Terms

Here, we present some relevant definitions which are very useful in this work.

Integro-differential equation: Integro-differential equation is an equation where the differential and the integral appear together in same equation. The general form is given as:

$$y^{(n)}(x) = f(x) + \lambda \int_0^x k\,(x;t)y(t)dt \qquad \textbf{(1)}$$

subject to condition:

$$y^k(0) = \Phi_k; k = 1,2,\dots n$$

where $y^{(n)}(x) = \frac{d^n y}{dx^n}$, and $k(x;t)$ is the kernel of the function and $f(x)$ is the smooth function.

Differential Equation: An equation is called a differential equation if it has a derivative of the unknown function say $y(x)$. The general form of a differential equation is given as:

$$y^{(n)}(x) = f(x; y(x)) \qquad \textbf{(2)}$$

subject to the conditions: $y(0) = \phi_k$; $k = 0,1,2,\dots n$, n is the order derivative of the function.

COLLOCATION METHOD

Collocation Method is one of the methods of solving differential equations. It is one of the most commonly used methods to handle ordinary, fractional and partial differential equations. It involves the use of an assumed or trial solution to determine the approximate solution of a given function. The assumed solution is substituted into the problem and the resulting equation is collocated (evaluated) at various points within the interval of consideration.

Legendre Polynomial: The well known Legendre polynomials $P(x)$ is defined in the interval [-1,1] by the Rodrigues' formula:

$$L_n(x) = \frac{1}{2^n!}\frac{d^n}{dx^n}[(x^2-1)^n] \quad \textbf{(3)}$$

where, $L_0(x) = 1$ and $L_1(x) = x$.

Few numbers of the Legendre polynomials are given as follows:

$$L_0(x) = 1, L_1(x) = x, L_2(x) = \frac{1}{2}(3x^2-1), L_3(x) = \frac{1}{2}(5x^3-3x), L_4(x) = \frac{1}{8}(35x^4-30x^2+3)\cdots$$

The orthogonality conditions of the Legendre polynomials is stated as follows:

$$\int_{-1}^{1} L_i\, L_j(x)dx = \begin{cases} 0 & \text{if } i \neq j \\ \frac{2}{2n+1}; & \text{if } i = j \end{cases}$$

Derivation of the Legendre polynomial basis function: In this section, we construct a new polynomial using the existing Legendre polynomials functions as basis. The procedure is presented below.
We considered the Rodrigue's formula of the form in (3). The roots of the L_i (x) are given as x_i,(i=0,1,2...) so that the roots:

$$x_i = \sqrt{\frac{1}{2^n!}\frac{d^n}{dx^n}[(x^2-1)^n]} = 0 \quad \textbf{(4)}$$

Solving Equation (4), for instance n=5, gives:

x_0=0.000000,x_1=0.90618,x_2=-0.90618,x_3=0.90618,x_4=-0.90618

Now, using the formula:

$$L_n^*(x) = \frac{\prod_{i=0}^{n}(x-x_i)}{\prod_{i=0}^{n}(x_n-x_i)}, i = 0,1,2,\ldots n; i \neq n$$

$$L_5^*(x) = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_5-x_1)(x_5-x_2)(x_5-x_3)(x_5-x_4)}$$

Substituting successive terms, we obtain

$$L_1(x) = 4.200000003x^4 + 0.9999999999 - 4.666666670x^2,$$
$$L_2(x) = -3.246232571x^4 - 1.747996612x^3 + 2.665682548x^2 + 1.435388242x,$$
$$L_3(x) = -3.246232570x^4 + 1.747996612x^3 + 2.665682549x^2 - 1.435388242x,$$
$$L_4(x) = 1.146232569x^4 + 1.038692853x^3 - 0.3323492138x^2 - 0.3011681593x,$$
$$L_5(x) = 1.146232569x^4 - 1.038692853x^3 - 0.3323492138x^2 + 0.3011681593x$$

and now we write:

$$L_5^*(x) = a_1L_1(x) + a_2L_2(x) + a_3L_3(x) + a_4L_4(x) + a_5L_5(x)$$

where $a_i, i = 1,2,\cdots,n-1$ are constant coefficients to be determined. Then we compute the required polynomials for n=6 using to obtain:

$$L_5^*(x) = 1.146232569\, a_4x^4 + 1.038692853\, a_4x^3 - 0.3323492138\, a_4x^2 - 0.3011681593\, a_4x + 1.146232569\, a_5x^4 - 1.038692853\, a_5x^3 - 0.3323492138\, a_5x^2 + 0.3011681593\, a_5x + 4.200000003\, a_1x^4 + 0.9999999999\, a_1 - 4.666666670\, a_1x^2 - 3.246232571\, a_2x^4 - 1.747996612\, a_2x^3 + 2.665682548\, a_2x^2 + 1.435388242\, a_2x - 3.246232570\, a_3x^4 + 1.747996612\, a_3x^3 + 2.665682549\, a_3x^2 - 1.435388242\, a_3x$$

being our derived polynomial for $n = 5$.

DEMONSTRATION OF THE STANDARD COLLOCATION METHOD

Consider the general class of integro-differential difference equation:

$$y^{(n)}(x) = f(t) + \int_0^x (x-t)y(t)dt \quad \textbf{(5)}$$

subject to conditions:

$$y^k(0) = \phi_k; k = 1; 2.\dots n$$

To determine a single polynomial approximation $y_N(x)$ of the unique solution y(x) of problem (5) , we use an assumed approximate solution of the form:

$$y(x) \equiv y_N(x) = \sum_{j=0}^{N} c_j L_j(x) \quad \textbf{(6)}$$

where c_j are unknown constants to be determined and $L_j(x)$ are the Legendre polynomials functions derived in the previous section. Equation (5) is re-written as:

$$y^{(n)}(x) - \int_0^x (x-t)y(t)dt = f(x) \quad \textbf{(7)}$$

Substituting the above,

$$y_N^{(n)}(x) - \int_0^x (x-t)y_N(t)dt = f(x) \quad \textbf{(8)}$$

or

$$\sum_{j=0}^{N} c_j L_j^{(n)}(t) - \int_0^x (x-t)\sum_{j=0}^{N} c_j L_j(t)dt = f(t) \quad \textbf{(9)}$$

with the initial conditions,

$$\sum_{j=0}^{N} c_j L^{(j)}(0) = \Phi_j; j = 1,2,\ldots n$$

$$c_1 L_1^{(n)}(x) + c_2 L_2^{(n)}(x) + c_3 L_3^{(n)}(x) + \cdots + c_N L_N^{(n)}(x)$$
$$- \int_0^x (x-t)(c_1 L_1(x) + c_2 L_2(x) + c_3 L_3(x) + \cdots + c_N L_N(x))dt = f(x)$$

The necessary differentiation and integration is carried out on the LHS in the above equation and the initial conditions are applied to get k equations. To obtain the remaining $N-k$ equations, is collocated at equally spaced interior points $x = x_j$, where $x_j = a + \frac{(b-a)j}{N-k}, (j = 1,2,\ldots,(N-k))$, thus resulting to $(N-k)$ linear system of algebraic equations. Altogether, we have N algebraic system of linear equations with N unknown constants, $c_j; (j = 0,1,2,\ldots.N)$ to be determined. The N algebraic equations are put in matrix form:

$$Ax = b$$

The N *by* N matrix equations are then solved using Gaussian elimination method or any suitable computer package like Maple 18 to obtain the unknown constants, which are then substituted back into the assumed approximate solution to give the required approximate solution.

Numerical Examples

Example 1

Consider the fourth order initial value integro-differential equation:

$$y^{(iv)}(x) = 1 + x + \int_0^x (x-t)y(t)dt \qquad x \in [0,1] \tag{10}$$

subject to conditions:

$$y(0) = y'(0) = 1, y''(0) = y'''(0) = -1 \tag{11}$$

The exact solution is:

$$y(x) = sinx + cosx$$

Taking the assumed approximate solution for $N = 4$

$$\begin{aligned} y_4(x) = {} & 0.25000000(-0.9275530713\, a_4 - 2.349411882\, a_1 + 2.349411882\, a_2 + 0.9275530713\, a_3)x^4 + \\ & 0.33333333(1.429452038\, a_4 - 0.7987530516\, a_1 - 0.7987530516\, a_2 + 0.7987530518\, a_3)x^3 + \\ & 0.5000000(-0.6503326458\, a_4 + 1.742234808\, a_1 - 1.742234809\, a_2 - 0.1072125207\, a_3)x^2 + \\ & 0.09232499007\, a_4 x + 0.5923249900\, a_1 x + 0.5923249900\, a_2 x - 0.09232499007\, a_3 x \end{aligned}$$

Substituting and applying the conditions in (11) and after some simplifications, gives:

$$\begin{gathered} 0.9275530713\, a_4 x^4 + 2.349411882\, a_1 x^4 - 2.349411882\, a_2 x^4 - 0.9275530713\, a_3 x^4 - x^2\cos(x) + x\sin(x) \\ -1.429452038\, a_4 x^3 + 0.6503326458\, a_4 x^2 + 5.472993438\, a_4 x + 0.7987530516\, a_1 x^3 - 1.742234808\, a_1 x^2 + \\ 13.50414630\, a_1 x + 0.7987530516\, a_2 x^3 + 1.742234809\, a_2 x^2 - 14.68879628\, a_2 x - 0.7987530518\, a_3 x^3 + \\ 0.107212521\, a_3 x^2 - 5.472993438\, a_3 x + 2.706414352\, a_4 - 7.1628245\, a_3 + 15.6939771\, a_1 - 12.4989652\, a_2 = 0 \end{gathered}$$

Equation above is collocated to get four linear algebraic equations which are solved to obtain the approximate solutions:

$$y_4(x) = -.16666667x^3 - .499999990x^2 + 1.00000000x + .9999999999000$$

Solving the problem for $N = 5$ and $N = 6$, we obtained:

$$y_5(x) = 0.1091794108x^4 - 0.1666666679x^3 - 0.4999999991x^2 + 1.000000000x + 0.9999999999$$

and

$$y_6(x) = 0.999999999x + 0.9999999998 + 0.16249931e^{-1}x^5 - 0.16666667x^3 + 0.27832951e^{-1}x^4 - 0.500000001x^2,$$

respectively.

Example 2

Consider a fifth order initial value integro-differential equation:

$$y^{(v)}(x) - x^2y^{(iv)}(x) - y'(x) - xy(x) = x^2cosx - xsinx + \int_0^x y(x)dx \qquad x \in [0,1] \tag{12}$$

subject to conditions:

$$y(0) = 1, y'(0) = -1, y''(0) = 1, y'''(0) = 0, y^{(iv)}(0) = 1 \tag{13}$$

The exact solution is:

$$y(x) = cosx$$

Example 2 was solved for $N = 4{,}5$ and 6 and the approximate solutions obtained following the same method are:

$$y_4(x) = 0.1588999990x^3 + 1.\ times10^{-10}x^2 + 0.9999999998x + 2.3563411 \times 10^{-11}$$

$$y_5(x) = 0.0046975375e - 1x^4 + 0.1666666655x^3 + 1.236545 \times 10^{-10}x^2 + 1.000000000x$$

and

$$y_6(x) = 0.9999999995x + 1. \times 10^{-6}x^4 - 0.1749688e^{-2}x^5 + .1566666663x^3$$

The tabular and graphical representations of the examples are shown in Tables 1 and 2 and Figures 1 and 2, respectively.

Table 1

		$N=4$		$N=5$		$N=6$	
	Exact	Appx	Error	Appx	Error	Appx	Error
0.0	1.0000000	1.0000000	1.000e-10	1.0000000	1.000e-10	1.0000000	2.000e-10
0.1	1.0948376	1.0948443	6.668e-06	1.0948391	1.506e-06	1.0948363	1.305e-06
0.2	1.1787359	1.1788414	1.054e-04	1.1787588	2.276e-05	1.1787164	1.960e-05
0.3	1.2508567	1.2513844	5.266e-04	1.2509662	1.085e-04	1.2507649	9.282e-05
0.4	1.3104793	1.3121283	1.643e-03	1.3108068	3.215e-04	1.3102123	2.731e-04
0.5	1.3570080	1.3609904	3.959e-03	1.3577641	7.328e-04	1.3564140	6.172e-04
0.6	1.3899776	1.3981497	8.102e-03	1.3914596	1.412e-03	1.3888707	1.177e-03
0.7	1.4090583	1.4240473	1.481e-02	1.4116532	2.415e-03	1.4072471	1.991e-03
0.8	1.4140583	1.4393866	2.492e-02	1.4182428	3.779e-03	1.4113918	3.072e-03
0.9	1.4049252	1.4451326	3.937e-02	1.4012644	5.506e-02	1.4013566	4.402e-03
1.0	1.3817460	1.3925127	5.918e-02	1.3908923	7.559e-02	1.3774162	5.917e-03

Table 2

		$N=4$		$N=5$		$N=6$	
	Exact	Appx	Error	Appx	Error	Appx	Error
0.0	0.0000000	0.0000000	2.000e-11	0.0000000	0.000e-11	0.0000000	5.000e-11
0.1	0.0998334	0.0998316	1.804e-06	0.0998001	3.330e-05	0.0997800	2.010e-05
0.2	0.1986693	0.1986116	5.771e-05	0.1980000	6.693e-04	0.1979630	3.700e-05
0.3	0.2955202	0.2950820	4.382e-04	0.3035420	8.022e-03	0.3034201	1.219e-04
0.4	0.3894183	0.3875718	1.847e-03	0.4066295	1.721e-03	0.4065251	1.045e-04
0.5	0.4794255	0.4737908	5.635e-03	0.5085130	2.909e-02	0.5076111	8.902e-04
0.6	0.5646424	0.5506231	1.402e-03	0.6053431	4.070e-02	0.6042015	1.142e-03
0.7	0.6442176	0.6439207	3.030e-02	0.6909051	4.669e-02	0.6891045	1.801e-03
0.8	0.7173557	0.7162967	2.906e-02	0.7561456	3.879e-02	0.7454284	1.072e-02
0.9	0.7833258	0.7886995	5.374e-02	0.6769195	1.064e-02	0.6632515	1.367e-02
1.0	0.8414683	0.7724170	6.905e-02	0.6613059	1.802e-01	0.6402993	1.111e-02

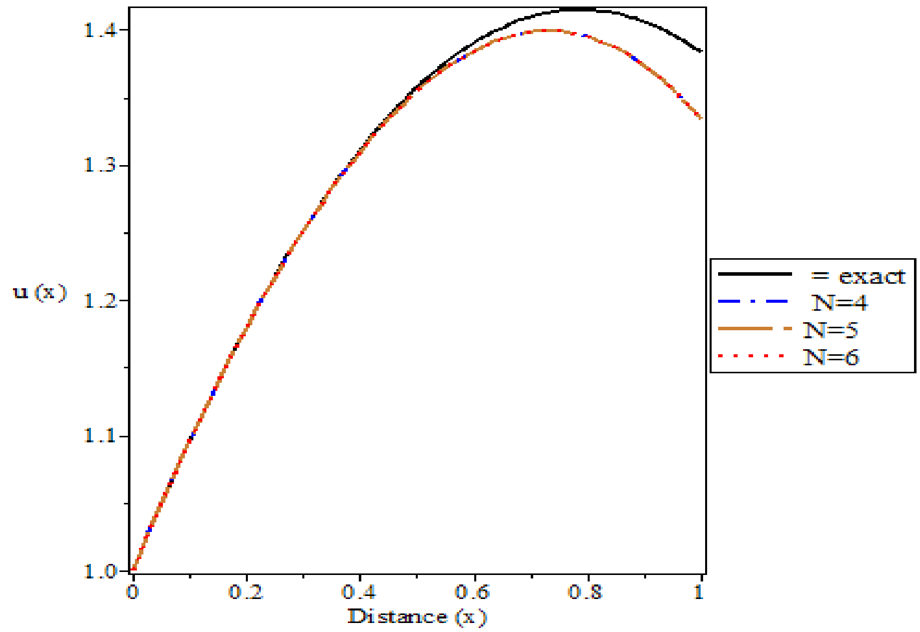

Figure 1: Graphical Representation of Error in Table 1.

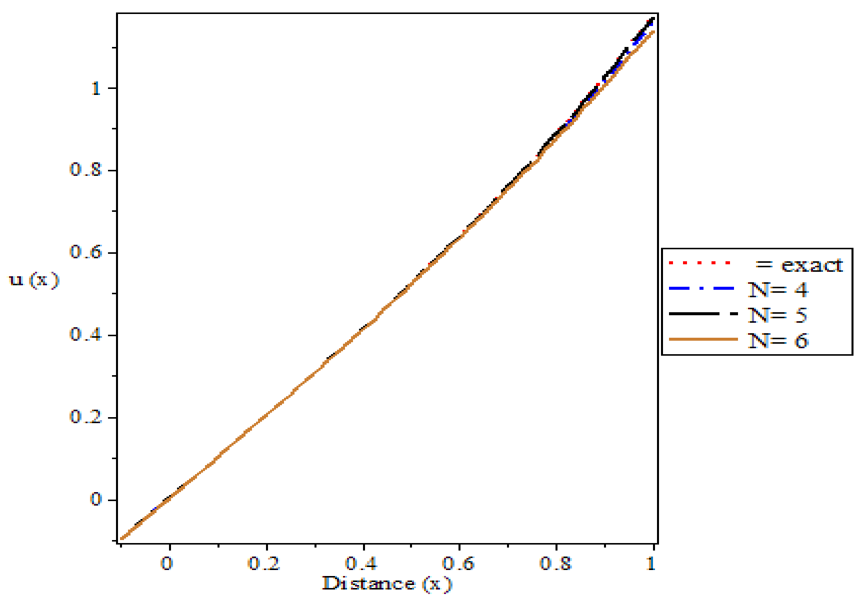

Figure 2: Graphical Representation of Error in Table 2.

CONCLUSION

In this study Gauss-Legendre formula was used to construct a new Legendre polynomials. Collocation method was applied using the polynomials as basis functions to solve integro-differential difference equations. Examples were given to illustrate the method and the results obtained show that the method is good and closely agrees with the exact solutions even at lower values of N. It is however observed that the convergence occurred at the lower part of the interval [0,1].

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