

Least Squares Technique for Solving Volterra Fractional Integro-Differential Equations Based on Constructed Orthogonal Polynomials

T. Oyedepo, Ph.D.^{1*}; A.F. Adebisi, Ph.D.²; O.A. Uwaheren, Ph.D.¹; C.Y. Ishola, M.Sc.³;
S. Amadiogwu, M.Sc.⁴; and T. Latunde, Ph.D.⁵

¹Department of Mathematics, University of Ilorin, Ilorin, Nigeria.

²Department of Mathematics, Osun State University, Oshogbo, Nigeria.

³Department of Mathematics, National Open University of Nigeria, Jabi, Abuja, Nigeria.

⁴Department of Mathematics, School of General Studies, Maritime Academy of Nigeria, Oron, Akwa Ibom State, Nigeria.

⁵Department of Mathematics, Federal University Oye-Ekiti, Ekiti, Nigeria.

E-mail: oyedepotaiye@yahoo.com*

ABSTRACT

A numerical method is presented in this paper to solve fractional integro-differential equations. In the sense of Caputo, the fractional derivative is considered. The method proposed is the Standard Least Squares Method (SLSM) with the aid of orthogonal polynomials constructed as basic functions. The suggested method reduces this type of problem to the solution of system of linear algebraic equations and then solved using Maple18. Some numerical examples are provided to show the accuracy and applicability of the presented method. Numerical results show that when applied to fractional integro-differential equations, the method is easy to implement and accurate.

(Keywords: orthogonal polynomial; standard least squares, numerical method)

INTRODUCTION

Fractional calculus is a field dealing with integral and derivatives of arbitrary orders, and their applications in science, engineering and other fields. In recent years, the assistance of fractional differentiation for mathematical modeling of real-world physical problems has increased dramatically (e.g., earthquake modeling, reducing the spread of virus, control the memory behavior of electric socket, etc.). There are many fascinating or exciting books about fractional calculus and fractional differential equations (Caputo, 1967; Munkhammar, 2005; Podlubny, 1999).

Many fractional integro-differential equations (FIDEs) are often difficult to solve and hence may not have analytical or exact solutions in the interval of consideration, so approximate and numerical methods must be used. Several numerical methods to solve the FIDEs have been given such as Adomian Decomposition Method (Mittal & Nigam, 2008), Standard Least Squares Method (Mohammed, 2014; Oyedepo et al., 2016; Oyedepo & Taiwo, 2019), homotopy analysis transform method (Mohamed et al., 2016) and collocation method (Rawashdeh, 2006).

Rawashdeh (2006) proposed a numerical solution of integro-differential fractional equations using the method of collocation in which polynomial spline functions was used to find the approximate solution. Momani and Qaralleh (2006) suggested an efficient method for the solution of the systems of fractional integro-differential equations solution using Adomian decomposition method (ADM). Also, Mittal and Nigam (2008), employed Adomian Decomposition Method for the solution of fractional integro-differential equations. ADM requires the construction of Adomian polynomials which was reported demanding to construct.

Mohammed (2014) applied least squares method and shifted Chebyshev polynomial for the solution of fractional integro-differential equations. In the work, the author employed shifted Chebyshev polynomial of the first kind as basis function and the result was presented graphically. Taiwo and Fesojaye, (2015) applied perturbation least-Squares Chebyshev method for solving fractional order integro-differential

equations. In their work, an approximate solution taken together with the Least - Squares method is utilized to reduce the fractional integro-differential equations to system of algebraic equations, which are solved for the unknown constants associated with the approximate solution.

Mohamed et al., (2016) applied homotopy analysis transform method for the solving fractional integro-differential equation in the work, Laplace transforms was used to reduce a differential equation to an algebraic equation. Oyedepo, et al., (2016) suggested a numerical method called Numerical Studies for the resolution of fractional Integro differential

equations using the least square method and polynomials of Berntein. Also, Oyedepo and Taiwo (2019), applied standard least squares method for solving fractional integro-differential equations using constructed orthogonal as basic functions.

The main objective of this work is to find the numerical solution of the Volttera type fractional-integro differential equation using the standard less square method based on the orthogonals constructed as basic functions. The general form of the problem class considered in this work is as follows:

$$D^\alpha u(x) = p(x)u(x) + f(x) + \int_0^x k(x,t)u(x)dt, \quad 0 \leq x, t \leq 1, \quad (1)$$

With the following supplementary conditions:

$$u^{(i)}(0) = \delta_i, \quad i = 0, 1, 2, \dots, n-1, \quad n-1 < \alpha \leq n, \quad n \in \mathbb{N} \quad (2)$$

Where $D^\alpha u(x)$ indicates the α th Caputo fractional derivative of $u(x)$; $p(x), f(x),$

$K(x, t)$ are given smooth functions, δ_i are real constant, x and t are real variables varying $[0, 1]$ and $u(x)$ is the unknown function to be determined.

Some Relevant Basic Definitions

Definition 1:

Fraction Calculus involves differentiation and integration of arbitrary order (all real numbers and complex values). Example $D^{\frac{1}{2}}, D^\pi, D^{2+i}$ etc.

Definition 2:

Gamma function is defined as:

$$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt \quad (3)$$

This integral converges when real part of z is positive ($Re(z) > 0$).

$$\Gamma(1+z) = z\Gamma(z) \quad (4)$$

Where z is a positive integer.

$$\Gamma(z) = (z-1)! \quad (5)$$

Definition 3:

Beta function is defined as:

$$B(v, m) = \int_0^1 (1-u)^{v-1} u^{m-1} du = \frac{\Gamma(v)\Gamma(m)}{\Gamma(v+m)} = B(v, m), \quad v, m \in R_+ \quad (6)$$

Definition 4:

Riemann – Liouville fractional integral is defined as:

$$J^\alpha f(x) = \frac{1}{\Gamma(\alpha)} \int_0^x \frac{f(t)}{(x-t)^{1-\alpha}} dt, \quad \alpha > 0, x > 0, \quad (7)$$

J^α denotes the fractional integral of order α

Definition 5:

Riemann – Liouville fractional derivative denoted D^α is defined as:

$$D^\alpha J^\alpha f(x) = f(x) \quad (8)$$

Definition 6:

Riemann-Liouville fractional derivative defined as:

$$D^\alpha f(x) = \frac{1}{\Gamma(m-\alpha)} \int_0^x (x-s)^{m-\alpha-1} f^m(s) ds, \quad (9)$$

m is positive integer with the property that $m - 1 < \alpha < m$.

Definition 7:

The Caputo Fractional Derivative is defined as:

$$D^\alpha f(x) = \frac{1}{\Gamma(m-\alpha)} \int_0^x (x-s)^{m-\alpha-1} f^m(s) ds \quad (10)$$

Where m is a positive integer with the property that $m - 1 < \alpha < m$

For example, if $0 < \alpha < 1$ the caputo fractional derivative is:

$$D^\alpha f(x) = \frac{1}{\Gamma(1-\alpha)} \int_0^x (x-s)^{-\alpha} f^1(s) ds \quad (11)$$

Hence, we have the following properties:

- (1) $J^\alpha J^\nu f = J^{\alpha+\nu} f, \alpha, \nu > 0, f \in C_\mu, \mu > 0$
- (2) $J^\alpha x^\gamma = \frac{\Gamma(\lambda+1)}{\Gamma(\alpha+\gamma+1)} x^{\alpha+\gamma}, \alpha > 0, \gamma > -1, x > 0$
- (3) $J^\alpha D^\alpha f(x) = f(x) - \sum_{k=0}^{m-1} f^k(0) \frac{x^k}{k!}, \quad x > 0, m - 1 < \alpha \leq m$
- (4) $D^\alpha J^\alpha f(x) = f(x), \quad x > 0, m - 1 < \alpha \leq m,$
- (5) $D^\alpha C = 0, C$ is the constant,

$$(6) \begin{cases} 0, & \beta \in N_0, \beta < [\alpha], \\ D^\alpha x^\beta = \frac{\Gamma(\beta+1)}{\Gamma(\beta-\alpha+1)} x^{\beta-\alpha}, & \beta \in N_0, \beta \geq [\alpha], \end{cases}$$

Where $[\alpha]$ denoted the smallest integer greater than or equal to α and $N_0 = \{0, 1, 2, \dots\}$

Definition 8:

Orthogonality: Two functions say $u_p(x)$ and $u_q(x)$ defined on the interval $a \leq x \leq b$ are said to be orthogonal if:

$$\langle u_p(x), u_q(x) \rangle = \int_a^b u_p(x) u_q(x) dx = 0 \tag{12}$$

If, on the other hand, a third function $w(x) > 0$ exists such that

$$\langle u_p(x), u_q(x) \rangle = \int_a^b w(x) u_p(x) u_q(x) dx = 0 \tag{13}$$

Then, we say that $u_p(x)$ and $u_q(x)$ are mutually orthogonal with respect to the weight function $w(x)$. Generally, we write:

$$\int_a^b w(x) u_p(x) u_q(x) dx = \begin{cases} 0 & p \neq q \\ \int_a^b w(x) u_p^2(x) dx & p = q, \end{cases} \tag{14}$$

Definition 9:

We defined absolute error as:

$$\text{Absolute Error} = |Y(x) - y_n(x)|; \quad 0 \leq x \leq 1, \tag{15}$$

where $Y(x)$ is the exact solution and $y_n(x)$ is the approximate solution.

MATERIALS AND METHODS

In this section, we constructed our orthogonal polynomials using the general weight function of the form:

$$w(x) = (a + bx^i)^k.$$

This corresponds to quartic functions for $a = 1, b = -1, k = 1$ and $i = 4$, respectively, satisfying the orthogonality conditions in the interval $[a, b]$ under consideration. According to Gram-Schmidt orthogonalization process, the orthogonal polynomial $u_j(x)$ Valid in the interval $[a, b]$ with the leading term x^j , is given as:

$$u_j(x) = x^j - \sum_{i=0}^{j-1} a_{j,i} u_i(x) \quad i = 0, 1, 2, \dots, j-1 \text{ and } j \geq 1 \tag{16}$$

Where $u_j(x)$ is an increasing polynomial of degree j and $u_i(x)$ are the corresponding values of the approximating functions in x . Then, starting with $u_0(x) = 1$, we find that the linear polynomial $u_j(x)$ with leading term x , is written as:

$$u_1(x) = x + a_{1,0}u_0(x) \tag{17}$$

Where $a_{1,0}$ is a constant to be determined. Since $u_1(x)$ and $u_0(x)$ are orthogonal, we have:

$$\int_a^b w(x)u_1(x)u_0(x) dx = 0 = \int_a^b xw(x)u_0(x) dx + a_{1,0} \int_a^b w(x)u_0^2(x) dx \tag{18}$$

Using (14) and (18) from the above, we have:

$$a_{1,0} = \frac{\int_a^b w(x)xu_0(x) dx}{\int_a^b w(x)u_0^2(x) dx} \tag{19}$$

Hence, substituting (19) into (16) gives:

$$u_1(x) = x + \frac{\int_a^b w(x)xu_0(x) dx}{\int_a^b w(x)u_0^2(x) dx} \tag{20}$$

Proceeding in this way, the method is generalized and is written as:

$$u_j(x) = x^j + a_{j,0}u_0(x) + a_{j,1}u_1(x) + a_{j,2}u_2(x) + \dots + a_{j,j-1}u_{j-1}(x) \tag{21}$$

Where the constants $a_{j,0}$ are so chosen such that $u_j(x)$ is orthogonal to $u_0(x), u_1(x), u_2(x) \dots u_{j-1}(x)$. These conditions yield:

$$a_{j,i} = -\frac{\int_a^b x^j w(x)u_i(x) dx}{\int_a^b w(x)u_i^2(x) dx} \tag{22}$$

For $k = 1, a = 1, b = -1$ and $i = 4$ valid in $[0, 1]$

$$w(x) = 1 - x^4 \tag{23}$$

$$u_0(x) = 1 \tag{24}$$

We have $k = 1, j = 1$ and $u_0(x) = 1$, we write Equation (16) as:

$$u_1(x) = x - a_{1,0}u_0(x) \tag{25}$$

Simplifying the above equation, we have:

$$u_1(x) = x, u_2(x) = x^2 - \frac{5}{21} \tag{26}$$

The shifted equivalent of the (26) that is valid in $[0, 1]$ are given as:

$$u_0^*(x) = 1, u_1^*(x) = 2x - 1, u_2^*(x) = 4x^2 - 4x + \frac{16}{21} \tag{27}$$

In this work the method assumed an approximate solution with the orthogonal polynomial as basis function as:

$$u(x) \cong u_n(x) = \sum_{i=0}^n a_i u_i^*(x) \quad (28)$$

Where $u_i^*(x)$ denotes the orthogonal polynomial of degree N where $a_i, i = 0, 1, 2, \dots$ are constants.

Demonstration of the Proposed Method

In this section, we demonstrated the two proposed methods mentioned above.

Standard Least Squares Method (SLSM): The standard least square method with the orthogonal polynomials constructed as the basis is applied in order to find the numerical solution of the fractional integro-differential equation of the type and this method is based on approximating the unknown function $u(x)$ by assuming an approximation solution from the defined in (28).

Consider Equation (1) operating with J^α on both sides as follows:

$$J^\alpha D^\alpha u(x) = J^\alpha f(x) + J^\alpha \left(\int_0^x k(x,t) u(t) dt \right) \quad (29)$$

$$u(x) = \sum_{k=0}^{m-1} u^k(0) \frac{x^k}{k!} + J^\alpha f(x) + J^\alpha \left[\int_0^x k(x,t) u(t) dt \right] \quad (30)$$

Substituting (28) into (30):

$$\sum_{i=0}^n a_i u_i^*(x) = \sum_{k=0}^{m-1} u^k(0) \frac{x^k}{k!} + J^\alpha f(x) + J^\alpha \left[\int_0^x k(x,t) \sum_{i=0}^n a_i u_i^*(t) dt \right] \quad (31)$$

Hence, the residual equation is obtained as:

$$R(a_0, a_1, \dots, a_n) = \sum_{i=0}^n a_i u_i^*(x) - \left\{ \sum_{k=0}^{m-1} u^k(0) \frac{x^k}{k!} + J^\alpha f(x) + J^\alpha \left[\int_0^x k(x,t) \sum_{i=0}^n a_i u_i^*(t) dt \right] \right\} \quad (32)$$

Let,

$$S(a_0, a_1, \dots, a_n) = \int_0^1 \left[R(a_0, a_1, \dots, a_n) \right]^2 w(x) dx \quad (33)$$

Where $w(x)$ is the positive weight function defined in the interval, $[a, b]$. In this work, we take $w(x) = 1$ for simplicity. Thus:

$$S(a_0, a_1, \dots, a_n) = \int_0^1 \left\{ \sum_{i=0}^n a_i u_i^*(x) - \left\{ \sum_{k=0}^{m-1} u^k(0) \frac{x^k}{k!} + J^\alpha f(x) + \left[\int_0^x k(x,t) \sum_{i=0}^n a_i u_i^*(t) dt \right] \right\} \right\}^2 dx \quad (34)$$

In order to minimize equation (34), we obtained the values of a_i ($i \geq 0$) by finding the minimum value of S as:

$$\frac{\partial S}{\partial a_i} = 0, i = 0, 1, 2, \dots, n \quad (35)$$

Applying (35) on (36), we have:

$$\int_0^1 \left\{ \sum_{i=0}^n a_i u_i^*(x) - \left[\sum_{k=0}^{m-1} u^k(0) \frac{x^k}{k!} + J^\alpha f(x) + J^\alpha \left[\int_0^x k(x,t) \sum_{i=0}^n a_i u_i^*(t) dt \right] \right] \right\} dx \times \int_0^1 \{u_i^*(x) - J^\alpha (\int_0^x k(x,t) u_i^*(t) dt)\} dx \quad (36)$$

Thus, (36) are then simplified for $i = 0, 1, \dots, n$ to obtain $(n + 1)$ Algebraic System of equations in $(n + 1)$ unknown a'_i s which are put in matrix form as follow:

$$A = \begin{pmatrix} \int_0^1 R(x, a_0) h_0 dx & \int_0^1 R(x, a_1) h_0 dx & \dots & \int_0^1 R(x, a_n) h_0 dx \\ \int_0^1 R(x, a_0) h_1 dx & \int_0^1 R(x, a_1) h_1 dx & \dots & \int_0^1 R(x, a_n) h_1 dx \\ \vdots & \vdots & \ddots & \vdots \\ \int_0^1 R(x, a_0) h_n dx & \int_0^1 R(x, a_1) h_n dx & \dots & \int_0^1 R(x, a_n) h_n dx \end{pmatrix}$$

$$B = \begin{pmatrix} \int_0^1 \left[J^\alpha f(x) + \sum_{k=0}^{m-1} u^k(0) \frac{x^k}{k!} \right] h_0 dx \\ \int_0^1 \left[J^\alpha f(x) + \sum_{k=0}^{m-1} u^k(0) \frac{x^k}{k!} \right] h_1 dx \\ \vdots \\ \int_0^1 \left[J^\alpha f(x) + \sum_{k=0}^{m-1} u^k(0) \frac{x^k}{k!} \right] h_n dx \end{pmatrix} \quad (37)$$

Where,

$$h_i = u_i^*(x) - J^\alpha \left[\int_0^x k(x,t) u_i^*(t) dt \right], i = 0, 1, \dots, n$$

$$R(x, a_i) = \sum_{i=0}^n a_i u_i^*(x) - J^\alpha \left[\int_0^x k(x,t) \sum_{i=0}^n a_i u_i^*(t) dt \right], i = 0, 1, \dots, n$$

The $(n + 1)$ linear equation is then solved using maple 18 to obtain the unknown constants a_i ($i = 0(1)n$), which are then substituted back into the assumed approximate solution to give the required approximation solution.

Numerical Examples

In this section, we have shown on some examples the method discussed above on the general integro-differential equations. The problems are solved using the constructed orthogonal polynomials as basic function. The examples are solved to illustrate the computational cost accuracy and efficiency of the proposed methods using Maple 18.

Example 1: Consider the following fractional Integra-differential (Khosrow *et al.*, 2013):

$$D^{\frac{3}{4}}u(x) = -\frac{x^2 e^x}{5}u(x) + \frac{6x^{2.25}}{\Gamma(3.25)} + e^x \int_0^x tu(t)dt \quad (38)$$

$$\text{Subject to } u(0) = 0. \text{ The exact solution is } u(x) = x^3 \quad (39)$$

Applying the above method on (38) to have the required approximate solution as:

$$u(x) = 3 \times 10^{-11} + x^3 \quad (40)$$

Example 2: Consider the following fractional Integra-differential (Mohamed *et al.*, 2016):

$$D^{\frac{1}{2}}u(x) = u(x) + \frac{8x^{2.25}}{3\Gamma(0.5)} - x^2 - \frac{1}{2}x^3 + \int_0^x tu(t)dt \quad (41)$$

$$\text{Subject to } u(0) = 0. \text{ The exact solution is } U(x) = x^2 \quad (42)$$

Applying the above method on (41) to have the required approximate solution as:

$$u(x) = 6.428498356 \times 10^{-8} + 3.20 \times 10^{-7}x^2 + 5.057252466 \times 10^{-7}x^3 \quad (43)$$

Example 3: Consider the following fractional Integra-differential (Khosrow *et al.*, 2013):

$$D^{\frac{1}{2}}u(x) = (\cos(x) - \sin(x))u(x) + f(x) + \int_0^x x \sin(t)u(t)dt \quad (44)$$

$$f(x) = \frac{2x^{1.5}}{\Gamma(2.5)} + \frac{1}{\Gamma(1.5)}x^{0.5} + x(\cos(x) - x \sin(x) + x^2 \cos(x)) \quad (45)$$

$$\text{Subject to } u(0) = 0. \text{ The exact solution is } U(x) = x^2 + x \quad (46)$$

Applying the above method on (44) to have the required approximate solution as:

$$u(x) = 6.6633966302 \times 10^{-10} + 0.99999999987x^2 + 3.504905097 \times 10^{-9}x^3 \quad (47)$$

RESULTS

Tables of Results

Table 1: Numerical Results of Example 1.

x	Exact Solution	Approximate Solution	Absolute Error
0.0	0.000	0.00000000003000	3.000E-11
0.1	0.001	0.00100000003000	3.000E-11
0.2	0.008	0.00800000003000	3.000E-11
0.3	0.273	0.02700000003000	3.000E-11
0.4	0.064	0.06400000003000	3.000E-11
0.5	0.125	0.12500000000000	0.00E+00
0.6	0.216	0.21600000000000	0.00E+00
0.7	0.343	0.34300000000000	0.00E+00
0.8	0.512	0.51200000000000	0.00E+00
0.9	0.729	0.72900000000000	0.00E+00
1.0	1.000	1.00000000000000	0.00E+00

Table 2: Numerical Results of Example 2.

x	Exact Solution	Approximate Solution	Absolute Error
0.0	0.00	0.00000006428498	6.428E-08
0.1	0.01	0.01000009461000	9.460E-08
0.2	0.04	0.04000012359000	1.236E-07
0.3	0.09	0.09000015426000	1.543E-07
0.4	0.16	0.16000018970000	1.897E-07
0.5	0.25	0.25000023290000	2.329E-07
0.6	0.36	0.36000028680000	2.868E-07
0.7	0.49	0.49000035470000	3.546E-07
0.8	0.64	0.64000043930000	4.393E-07
0.9	0.81	0.81000054390000	5.439E-07
1.0	1.00	1.00000067200000	6.714E-07

Table 3: Numerical Results of Example 1.

e	Exact Solution	Approximate Solution	Absolute Error
0.0	0.000	0.00000000046634	4.663E-10
0.1	0.110	0.10999999950000	5.432E-10
0.2	0.240	0.23999999840000	1.558E-09
0.3	0.390	0.38999999750000	2.556E-09
0.4	0.560	0.55999999650000	3.517E-09
0.5	0.750	0.74999999560000	4.421E-09
0.6	0.960	0.95999999480000	5.245E-09
0.7	1.190	1.18999999400000	5.968E-09
0.8	1.440	1.43999999400000	6.571E-09
0.9	1.710	1.70999999300000	7.032E-09
1.0	2.000	1.99999999300000	7.329E-09

Graphical Representation of the Method

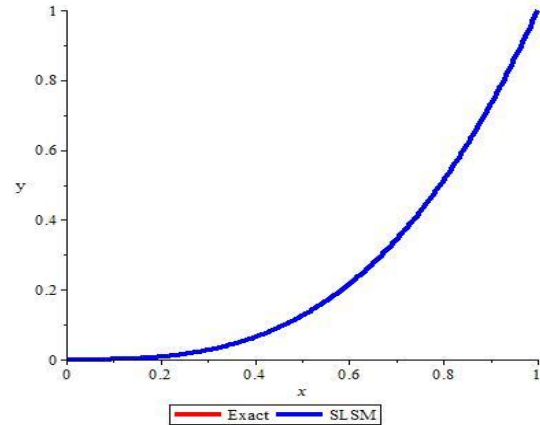


Figure 1: The Graph of Approximation Solution and Exact of Sample 1.

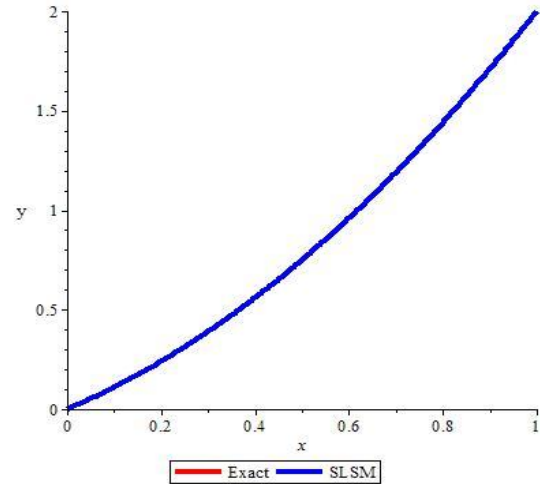


Figure 2: The Graph of Approximation Solution and Exact of Example 2.

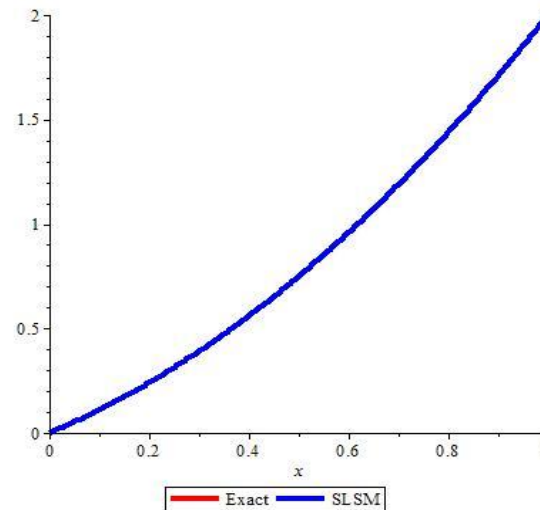


Figure 3: The Graph of Approximation Solution and Exact of Example 3.

DISCUSSION OF RESULTS

All the three numerical examples presented in this study were solved using Maple 18. The Tables of error for the examples shows that the method with the constructed orthogonal polynomials is accurate and converges at the lower numbers of the approximate. Also, for the graphs of the three examples when compared the approximate solution with the exact equations, we have exact equation graphs.

CONCLUSION

The study showed that the method with the constructed orthogonal polynomials is successfully used for solving FIDEs in a wide range with three examples. The method gives more realistic series solutions that converge very rapidly in fractional equations. The results obtained showed that the method is powerful when compared with the exact solutions and also show shown that there is a similarity between the exact and the approximate solution. Calculation showed that SLSM is a powerful and efficient technique to find a very good solution for this type of equation as well as analytical solutions to numerous physical problems in science and engineering. Also, the results were presented in graphical forms to further demonstrate the method.

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ABOUT THE AUTHORS

T. Oyedepo, is a Researcher in the Department of Mathematics, University of Ilorin, Nigeria. He holds a Ph.D. in Mathematics from University of Ilorin, Nigeria. His research interest is in numerical analysis.

A.F. Adebisi, is a Lecturer in the Department of Mathematical Sciences Osun State University Oshogbo, Nigeria. She holds Ph.D., M.Sc., and B.Sc. from University of Ilorin, Ilorin, Nigeria. Her

research interests are in numerical analysis and mathematical modelling

O.A. Uwaheren, is a Lecturer in the Department of Mathematics University of Ilorin, Nigeria. He holds a Ph.D., M.Sc., and B.Sc. from the University of Ilorin, Ilorin, Nigeria. His research interest is in numerical analysis.

S. Amadiogwu, is a Lecturer in the Department of Mathematics, School of General Studies, Maritime Academy of Nigeria, Oron, Akwa Ibom State, Nigeria. He holds a Master of Science (M.Sc.) in Mathematics from the University of Ibadan, Ibadan, Nigeria. His research interests are in biomathematics.

C.Y. Ishola, is a Ph.D. student in the Department of Mathematics, University of Ilorin, Nigeria. She holds a Master of Science (M.Sc.) in Applied Mathematics from the University of Ilorin, Nigeria. Her research interests are in mathematical modelling and numerical analysis.

T. Latunde, is a Lecturer in the Department of Mathematics, Federal University, Oye-Ekiti, Ekiti State, Nigeria. He holds a Ph.D., M.Sc., and B.Sc. in Mathematics from the University of Ilorin, Ilorin, Nigeria. His research interest lies in optimization and its applications.

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