Cylindrical Laplacian on 2D FD for Computational Electromagnetics

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ABSTRACT

When finite difference (FD) is applied to higher order partial derivatives, the derivation or computation of the expression is easily maneuverable for rectangular coordinates. But cylindrical counterparts suffer from expressional as well as computational complexity. Even with the rectangular derivatives, mathematical manipulations become clumsy with unequal resolutions. In this paper we mainly present 2D FD Laplacian expression in cylindrical system, application of which is also implemented and verified with standard computational electromagnetic problems.

(Keywords: Laplace equation, 2D FD, cylindrical FD)

INTRODUCTION

Laplace equation has importance in determining capacitance of an electromagnetic system. System property determines voltage-current-power-frequency requirements of the system which is vital for design context. Analysis for this sort of system in not new. Well established empirical mathematical expression of the system exists in rectangular coordinate or other for continuous case [1]-[9].

The traditional more appropriately continuous approach uses variable separation method in conjunction with sinusoidal basis functions. The problems are two-fold using such technique: a) the solution is highly boundary condition dependent and every new boundary condition results in different analytics (i.e., involvement of noncoherent computing) and b) presence of sinusoidal basis results undesired ringing phenomenon with finite harmonic components.

Advent of powerful computer processors force us to think about all-too-familiar electromagnetic problems or derived complicated ones to solve by engaging the neoteric tool. Focus on Laplacian is vital for electromagnetic system property mainly for inductance and capacitance. But the computing is not one shot, instead two steps are essential: a) potential to field and b) field to capacitance or inductance. Given the intricacy of discrete mathematics, concentration has been on potential distribution.

To date, cylindrical system Laplacian analytics involving finite difference are not available for robust computational electromagnetics. It is imperative that we engage the didactic tool for practical implication in the field.

LAPLACIAN IN CYLINDRICAL SYSTEM

Two-dimensional Laplacian is derived from its three-dimensional counterpart which in cylindrical system is given by [3]-[9]:

$$\rho^2\frac{\partial^2 f}{\partial \rho^2}+\rho\frac{\partial f}{\partial \rho}+\frac{\partial^2 f}{\partial \varphi^2}+\rho^2\frac{\partial^2 f}{\partial z^2}=0$$

Certainly the above equation is for charge, flux, or source free region. Problem geometry dictates the type of component's presence and three possibilities are associated with as follows:

Presence of ρ and φ components:

$$\rho^2\frac{\partial^2 f}{\partial \rho^2}+\rho\frac{\partial f}{\partial \rho}+\frac{\partial^2 f}{\partial \varphi^2}=0,$$

Presence of φ and z components:

$$\frac{\partial^2 f}{\partial \varphi^2}+\rho^2\frac{\partial^2 f}{\partial z^2}=0, \text{ and}$$

Presence of z and ρ components:

$$\rho\frac{\partial^2 f}{\partial z^2}+\frac{\partial f}{\partial \rho}+\rho\frac{\partial^2 f}{\partial \rho^2}=0.$$

DERIVATION OF THE FINITE DIFFERENCE EQUATIONS

There are three 2D equations for Laplacian in cylindrical system. Since each equation is different from the other two, unified derivation does not hold true instead treatment on each is required separately which we pay attention to in this section.

Treating ρ and φ components: In cylindrical coordinate system continuous function $f(\rho,\varphi)$ we seek in a source free region from $\rho^2\frac{\partial^2 f}{\partial\rho^2}+\rho\frac{\partial f}{\partial\rho}+\frac{\partial^2 f}{\partial\varphi^2}=0$ over certain domain $\rho_1\le\rho\le\rho_2$ and $\varphi_1\le\varphi\le\varphi_2$. The $f(\rho,\varphi)$ may represent any physical quantity, flux or potential depending on the problem.

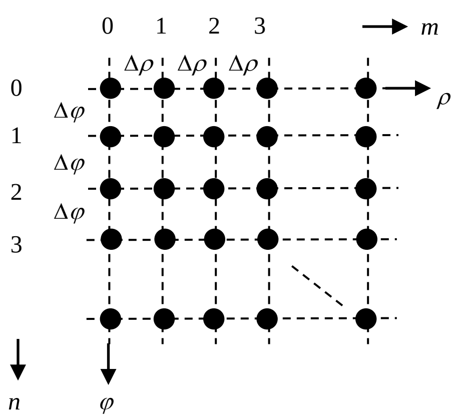

Figure 1: Link Between the Continuous and Discrete Base Coordinates of $f(\rho,\varphi)$ due to FD.

Figure 1 depicts the linkage between the continuous and discrete base coordinates of $f(\rho,\varphi)$ due to 2D finite difference. The discretization of base coordinate variables happens by $\rho=m\Delta\rho$ and $\varphi=n\Delta\varphi$ where m or n is purely integers. Although 0 starting of ρ or φ is used, either one can be negative too causing m or n to be likewise. The φ direction is chosen downward despite upward being in conventional coordinate system. This convention is just to be consistent with widely followed computer coordinate system.

Discrete function $f[m,n]$ or sampled $f(m,n)$ is the discrete counterpart of $f(\rho,\varphi)$. Finite difference links of the related derivatives (considering forward divided difference) are as follows [3]-[6]:

for the first order ρ component:
$\frac{\partial f}{\partial\rho}=\frac{f(m+1,n)-f(m,n)}{\Delta\rho}$,

for the second order ρ component:
$\frac{\partial^2 f}{\partial\rho^2}=\frac{f(m+1,n)-2f(m,n)+f(m-1,n)}{(\Delta\rho)^2}$, and

for the second order φ component:
$\frac{\partial^2 f}{\partial\varphi^2}=\frac{f(m,n+1)-2f(m,n)+f(m,n-1)}{(\Delta\varphi)^2}$.

Inserting the last three derivatives into Laplace equation and simplifying for $f(m,n)$ yield FD equation of the Laplacian in 2D cylindrical system:

$$f(m,n)=\frac{m^2(\Delta\varphi)^2 f(m-1,n)+(m^2+m)(\Delta\varphi)^2 f(m+1,n)+f(m,n-1)+f(m,n+1)}{(2m^2+m)(\Delta\varphi)^2+2}.$$

The FD counterpart of $f(\rho,\varphi)$ along with the neighboring samples takes following rhombic shape:

$f(m,n-1)$

$f(m-1,n)$ $f(m,n)$ $f(m+1,n)$

$f(m,n+1)$

Alternately, the lattice of FD is arranged by relative nodes as follows:

Top

Left *Center* *Right*

Bottom

Let us assign the nodes with weight factors likewise by:

w_3

w_1 a w_2

w_4

(i.e., center, left, right, top, and bottom by a, w_1, w_2, w_3, and w_4, respectively). The weight factors are then obtained from the FD equation as:

$$w_1 = \frac{m^2(\Delta\varphi)^2}{(2m^2+m)(\Delta\varphi)^2+2},$$

$$w_2 = \frac{(m^2+m)(\Delta\varphi)^2}{(2m^2+m)(\Delta\varphi)^2+2},$$

$$w_3 = \frac{1}{(2m^2+m)(\Delta\varphi)^2+2}, \text{ and}$$

$$w_4 = \frac{1}{(2m^2+m)(\Delta\varphi)^2+2}.$$

The a is 1 for all above. If denominators of weight factor are pushed to be with $f(m,n)$, the a will be non-unity. The three coordinates are chosen in cyclic order and any two concern discrete variables m and n respectively.

Treating φ and z components: The resolution modification is needed which is $\varphi = m\Delta\varphi$ and $z = n\Delta z$ in conjunction with:

$$\frac{\partial^2 f}{\partial\varphi^2} = \frac{f(m+1,n)-2f(m,n)+f(m-1,n)}{(\Delta\varphi)^2} \text{ and}$$
$$\frac{\partial^2 f}{\partial z^2} = \frac{f(m,n+1)-2f(m,n)+f(m,n-1)}{(\Delta z)^2}.$$

Since ρ is frozen (say $\rho = c$), consequently we obtain $f(m,n) = \frac{(\Delta z)^2 f(m-1,n)+(\Delta z)^2 f(m+1,n)+}{2[(\Delta z)^2+c^2(\Delta\varphi)^2]}$

$\underline{c^2(\Delta\varphi)^2 f(m,n-1)+c^2(\Delta\varphi)^2 f(m,n+1)}$

where the weight factors are derived as follows:

$$w_1 = \frac{(\Delta z)^2}{2[(\Delta z)^2+c^2(\Delta\varphi)^2]},$$

$$w_2 = \frac{(\Delta z)^2}{2[(\Delta z)^2+c^2(\Delta\varphi)^2]},$$

$$w_3 = \frac{c^2(\Delta\varphi)^2}{2[(\Delta z)^2+c^2(\Delta\varphi)^2]}, \text{ and}$$

$$w_4 = \frac{c^2(\Delta\varphi)^2}{2[(\Delta z)^2+c^2(\Delta\varphi)^2]}.$$

Treating z and ρ components: Discretization now requires subject to $z = m\Delta z$ and $\rho = n\Delta\rho$ along with

$$\frac{\partial^2 f}{\partial z^2} = \frac{f(m+1,n)-2f(m,n)+f(m-1,n)}{(\Delta z)^2},$$
$$\frac{\partial f}{\partial\rho} = \frac{f(m,n+1)-f(m,n)}{\Delta\rho}, \text{ and}$$
$$\frac{\partial^2 f}{\partial\rho^2} = \frac{f(m,n+1)-2f(m,n)+f(m,n-1)}{(\Delta\rho)^2}$$

thereby resulting:

$$f(m,n) = \frac{n(\Delta\rho)^2 f(m-1,n)+n(\Delta\rho)^2 f(m+1,n)+}{2n(\Delta\rho)^2+(\Delta z)^2+2n(\Delta z)^2} \text{ and}$$

$\underline{+(\Delta z)^2 f(m,n+1)+n(\Delta z)^2 f(m,n-1)+n(\Delta z)^2 f(m,n+1)}$

the necessary weight factors are the following:

$$w_1 = \frac{n(\Delta\rho)^2}{2n(\Delta\rho)^2+(\Delta z)^2+2n(\Delta z)^2},$$

$$w_2 = \frac{n(\Delta\rho)^2}{2n(\Delta\rho)^2+(\Delta z)^2+2n(\Delta z)^2},$$

$$w_3 = \frac{n(\Delta z)^2}{2n(\Delta\rho)^2+(\Delta z)^2+2n(\Delta z)^2}, \text{ and}$$

$$w_4 = \frac{(n+1)(\Delta z)^2}{2n(\Delta\rho)^2+(\Delta z)^2+2n(\Delta z)^2}.$$

EMPERICAL SOLUTION

By taking some specific finite difference lattice now we introduce how the solution will be obtained. Two examples are demonstrated by considering band matrix approach [3]. Of the examples, one is symmetric and the other is asymmetric.

Solution obtaining for the potential function is not coherent because asymmetric or irregular geometry of electromagnetic structure poses a challenge owing to lattice complexity. In the following we assume that boundary samples are b's and unknown functionals are f's with integer subscripts. If B and C are band and column matrices respectively, the solution for $f(\rho,\varphi)$ samples or $f(m,n)$ is simply $B^{-1}C$. Question marks are put where FD rhombic lattice does not hold true. Also when many coefficients are to be handled in matrix form, manual typing becomes clumsy that is why computer assistance is sought for some matrices with different font symbols for example –w2 for w_2, a for a, etc.

Symmetric 3x3 Unknowns

$f(m,n)$ sample lattice structure:

$$\begin{bmatrix} ? & b_1 & b_2 & b_3 & ? \\ b_{12} & f_1 & f_2 & f_3 & b_4 \\ b_{11} & f_4 & f_5 & f_6 & b_5 \\ b_{10} & f_7 & f_8 & f_9 & b_6 \\ ? & b_9 & b_8 & b_7 & ? \end{bmatrix}$$

Band matrix, B:

```
[   a, -w2,   0, -w4,   0,   0,   0,   0,   0]
[ -w1,   a, -w2,   0, -w4,   0,   0,   0,   0]
[   0, -w1,   a,   0,   0, -w4,   0,   0,   0]
[ -w3,   0,   0,   a, -w2,   0, -w4,   0,   0]
[   0, -w3,   0, -w1,   a, -w2,   0, -w4,   0]
[   0,   0, -w3,   0, -w1,   a,   0,   0, -w4]
[   0,   0,   0, -w3,   0,   0,   a, -w2,   0]
[   0,   0,   0,   0, -w3,   0, -w1,   a, -w2]
[   0,   0,   0,   0,   0, -w3,   0, -w1,   a]
```

Column matrix, C:

$$\begin{bmatrix} w_1 b_{12} + w_3 b_1 \\ w_3 b_2 \\ w_2 b_4 + w_3 b_3 \\ w_1 b_{11} \\ 0 \\ w_2 b_5 \\ w_1 b_{10} + w_4 b_9 \\ w_4 b_8 \\ w_2 b_6 + w_4 b_7 \end{bmatrix}$$

Asymmetric 6x4 unknowns:

$f(m,n)$ sample lattice structure:

$$\begin{bmatrix} ? & b_1 & b_2 & b_3 & b_4 & ? \\ b_{20} & f_1 & f_2 & f_3 & f_4 & b_5 \\ b_{19} & f_5 & f_6 & f_7 & f_8 & b_6 \\ b_{18} & f_9 & f_{10} & f_{11} & f_{12} & b_7 \\ b_{17} & f_{13} & f_{14} & f_{15} & f_{16} & b_8 \\ b_{16} & f_{17} & f_{18} & f_{19} & f_{20} & b_9 \\ b_{15} & f_{21} & f_{22} & f_{23} & f_{24} & b_{10} \\ ? & b_{14} & b_{13} & b_{12} & b_{11} & ? \end{bmatrix}$$

Band matrix, B: It is shown in figure 6.

Column matrix, C:

$$[w_1 b_{20} + w_3 b_1 \quad w_3 b_2 \quad w_3 b_3 \quad w_2 b_5 + w_3 b_4$$
$$w_1 b_{19} \quad 0 \quad 0 \quad w_2 b_6 \quad w_1 b_{18} \quad 0 \quad 0 \quad w_2 b_7$$
$$w_1 b_{17} \quad 0 \quad 0 \quad w_2 b_8 \quad w_1 b_{16} \quad 0 \quad 0 \quad w_2 b_9$$
$$w_1 b_{15} + w_4 b_{14} \quad w_4 b_{13} \quad w_4 b_{12} \quad w_2 b_{10} + w_4 b_{11}]^T$$

where T is the transpose operator. In order to determine a generic band we may inspect the matrix how the entries evolve.

For a symmetric band a assumes the diagonal position. The w_1 and w_2 occupy first sub and super diagonals respectively. Every after two w_1 or w_2's one 0 takes place. The second sub or super diagonal is all zeroes. The third sub and super diagonals are filled by w_3 and w_4 respectively. Of course all w's are negative regardless of the diagonal type. The rest entries are simply zeroes.

In the other counterpart a assumes similar diagonal position too. The w_1 and w_2 occupy first sub and super diagonals respectively. The 0 appearance is every after three w_1 or w_2. The second and third sub or super diagonals are all zeroes. The fourth sub and super diagonals are filled by w_3 and w_4 respectively. Similar negativity of the weight factor holds true besides zero appearance.

The boundary entries are labeled as b's which can assume any type of boundary values whether constant or variable. For function based b's, the sample values must be computed first. For example $f(\rho = c, \varphi)$ indicates constancy on ρ but changing on φ more appropriately $b(\varphi)$. For instance, sinusoidal variation on φ makes the $b(\varphi)$ available as $\sin\varphi$.

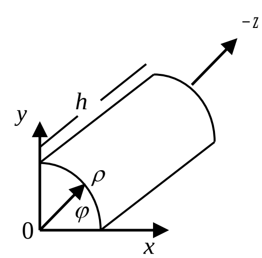

Figure 2: An Electromagnetic System in Cylindrical Coordinate.

RESULTS ON APPLICATION

To test suitability of the finite difference we have chosen Figure 2 shown electromagnetic system. The geometric dimension of ρ, φ, or z indicates which two components are to be considered. Let us see the following examples.

Components ρ - φ:

If $z >> \rho$, we consider ρ - φ based finite difference. For instance, cylindrical surface in figure 2 contains $f(10cm,\varphi) = 10V$ while all other boundary potentials are at 0 V. The $f(\rho,\varphi)$ solution is needed subject to radius $10cm$, first cylindrical quarter i.e. φ variation 90^0, $\Delta\rho = 2.5cm$, and $\Delta\varphi = 22.5^0$.

Geometry of the structure stretches over $0 \le \rho \le 10cm$ and $0 \le \varphi \le 90^0$ translating to discrete intervals $0 \le m \le 4$ and $0 \le n \le 4$ respectively. The $f(m,n)$ samples take the shape of 3x3 symmetric lattice as addressed earlier.

It has to be pointed out that each weight factor is two-dimensional function on m and n too thereby yielding w_1 counterpart as:

$$\begin{matrix} & \rho \rightarrow \\ \varphi \\ \downarrow \end{matrix} \begin{bmatrix} 0 & 0.0626 & 0.1741 & 0.2649 & 0.3267 \\ 0 & 0.0626 & 0.1741 & 0.2649 & 0.3267 \\ 0 & 0.0626 & 0.1741 & 0.2649 & 0.3267 \\ 0 & 0.0626 & 0.1741 & 0.2649 & 0.3267 \\ 0 & 0.0626 & 0.1741 & 0.2649 & 0.3267 \end{bmatrix}.$$

The other three weight factors follow similar trend and are computed as $w_2 =$

$$\begin{matrix} & \rho \rightarrow \\ \varphi \\ \downarrow \end{matrix} \begin{bmatrix} 0 & 0.1252 & 0.2612 & 0.3533 & 0.4084 \\ 0 & 0.1252 & 0.2612 & 0.3533 & 0.4084 \\ 0 & 0.1252 & 0.2612 & 0.3533 & 0.4084 \\ 0 & 0.1252 & 0.2612 & 0.3533 & 0.4084 \\ 0 & 0.1252 & 0.2612 & 0.3533 & 0.4084 \end{bmatrix}, \quad \text{and}$$

$$w_3 = w_4 = \begin{matrix} & \rho \rightarrow \\ \varphi \\ \downarrow \end{matrix} \begin{bmatrix} 0.5 & 0.4061 & 0.2823 & 0.1909 & 0.1324 \\ 0.5 & 0.4061 & 0.2823 & 0.1909 & 0.1324 \\ 0.5 & 0.4061 & 0.2823 & 0.1909 & 0.1324 \\ 0.5 & 0.4061 & 0.2823 & 0.1909 & 0.1324 \\ 0.5 & 0.4061 & 0.2823 & 0.1909 & 0.1324 \end{bmatrix}$$

hence the $f(\rho,\varphi)$ sample solution is computed as:

$$\begin{matrix} & \rho \rightarrow \\ \varphi \\ \downarrow \end{matrix} \begin{bmatrix} 0 & 0 & 0 & 0 & 10 \\ 0 & 2.1857 & 3.6528 & 6.105 & 10 \\ 0 & 3.0329 & 4.8148 & 7.2535 & 10 \\ 0 & 2.1857 & 3.6528 & 6.105 & 10 \\ 0 & 0 & 0 & 0 & 10 \end{bmatrix}.$$

How do we read the solution out from last matrix? Figure 1 mentioned resolution on ρ - φ is the answer. For instance fourth element in the third row refers to $f(7.5cm,45^0)$ =7.2535 V, so does $f(7.5cm,67.5^0)$ =6.105 V for the fourth element in the fourth row, and so on.

Components φ - z:

In Figure 2 when ρ and z are comparable i.e. ρ >0.1| z | or | z |>0.1 ρ, the analysis is conducted for a particular ρ. For example some strip on the cylindrical surface defined by $0 \le z \le 10cm$ and $30^0 \le \varphi \le 45^0$ on $\rho = 5cm$.

Components z - ρ:

With φ constant, z - ρ variations evolve as a plane in figure 2. The plane may assume any φ between 0 and 360 degrees depending on the problem. For example, some plane is defined by $0 \le z \le 10cm$ and $0 \le \rho \le 5cm$ on $\varphi = 45^0$.

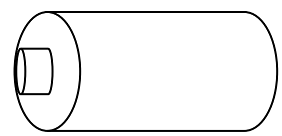

Figure 3: Isometric View of a Battery.

PRACTICAL APPLICATIONS

We cite two applications of the 2D FD in cylindrical system in the following.

<u>Application 1</u>

Suppose a manufacturer intends to add doping into battery electrode. The objective of such study can be for cost saving or performance improvement. This can be best conducted by determining the electrical potential distribution through the carbon rod of the battery. In figure 3 tip of the carbon rod holds a voltage $1.5V$, the length of the rod is $5cm$ and the diameter is $1.5mm$. The question is how far along the rod from the battery cap significant voltage persists and doping can be applied accordingly in order to improve the battery performance.

We create a potential profile for every section within the rod by making the use of ρ - φ based finite difference. Existence of significant equi-potential lines indicates the zone for doping.

Considering battery axis as the z axis, tip of the battery is on $\rho - \varphi$ plane, $\Delta z = 1.25cm$, and $\Delta\rho = 0.375mm$, earlier quoted 3×3 lattice structure is chosen. Figure 4 depicts the potential profile over $0 \le \rho \le 1.5mm$ and $0 \le z \le 5cm$. As pronounced in the graph, battery tip potential 1.5 V reduces to 0.2 V within 1.1 mm from the tip. The rod over $1.1cm \le z \le 5cm$ holds less than 0.2 V which facilitates the manufacturer's doping decision.

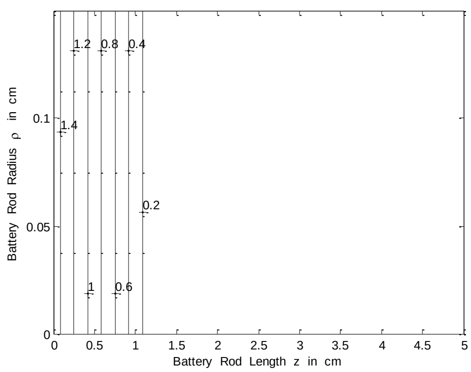

Figure 4: Potential Profile Along the Battery Rod.

Application 2

The FD can be applied to compute the resistance and capacitance of any cylindrical electromagnetic object. Generalized expressions for resistance and capacitance are given by $R = \frac{V_0}{\sigma \oint \bar{E} \circ d\bar{S}}$ and $C = \frac{\varepsilon \oint \bar{E} \circ d\bar{S}}{V_0}$ respectively [3]-[4]. For a given electromagnetic system, the related symbology is as follows: R is resistance, C is capacitance, σ is the electrode conductivity, ε is the dielectric permittivity, V_0 is applied potential difference across the plate, $\bar{E}$ is the electric field developed across dielectric, and $d\bar{S}$ is the elementary surface area mostly for electrode.

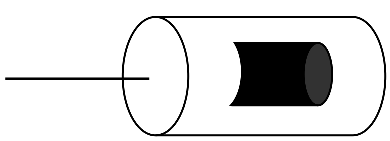

Figure 5: Isometric View of a Capacitor.

Figure 5 shows the isometric view of a capacitor. Expression for elementary surface in cylindrical system is given by $d\bar{S} = \rho\, d\varphi\, dz\, \bar{a}_\rho + d\rho\, dz\, \bar{a}_\varphi + \rho\, d\varphi\, d\rho\, \bar{a}_z$. The electrode lies on the cylindrical surface defined by $\rho = c$, $\varphi_1 \le \varphi \le \varphi_2$, and $z_1 \le z \le z_2$ where c is the cylinder radius and V_0 is the voltage applied on the electrode with axial voltage at 0 V.

Earlier quoted ρ-φ based FD provides potential distribution for a fixed V_0 from which electric field is obtained by $\bar{E} = -\frac{\partial f}{\partial \rho}\bar{a}_\rho - \frac{1}{\rho}\frac{\partial f}{\partial \varphi}\bar{a}_\varphi = -\frac{f(m+1,n) - f(m,n)}{\Delta\rho}\bar{a}_\rho - \frac{f(m,n+1) - f(m,n)}{m\Delta\rho\Delta\varphi}\bar{a}_\varphi$ where $\rho = m\Delta\rho$ and $\varphi = n\Delta\varphi$. With all these $\oint \bar{E} \circ d\bar{S}$ reduces to $-(z_2 - z_1)(\varphi_2 - \varphi_1)\ \sum\sum m[f(m+1,n) - f(m,n)]$. The rationale for selecting ρ-φ basis is elementary surface and electric flux's perpendicularity to each other.

By inspection the $\oint \bar{E} \circ d\bar{S}$ is only m directed difference. For the computation we may choose any lattice e.g. 3x3, 5x5, etc. and calculate the m directed difference. The double summation indicates sum of all elements. Knowing ε, σ, and V_0, one can easily get the resistance or capacitance calculated.

CONCLUSION

Finite difference computing theory on 2D cylindrical system is developed for Laplacian. Although handling 2D finite difference, three distinct Laplacian components are related with the formulation, each of which is analyzed. Application of the theory is presented by taking some electromagnetic example. As far as higher processor speed evolvement is concerned, the tool is useful in spite of being computationally intensive.

ACKNOWLEDGEMENT

The author acknowledges King Fahd University of Petroleum and Minerals (KFUPM) for providing the library and computer facilities necessary for conducting this research.

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[  a, -w2,   0,   0, -w4,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0]
[-w1,   a, -w2,   0,   0, -w4,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0]
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[-w3,   0,   0,   0,   a, -w2,   0,   0, -w4,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0]
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[  0,   0, -w3,   0,   0, -w1,   a, -w2,   0,   0, -w4,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0]
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[  0,   0,   0,   0,   0,   0,   0,   0, -w3,   0,   0,   0,   a, -w2,   0,   0, -w4,   0,   0,   0,   0,   0,   0,   0]
[  0,   0,   0,   0,   0,   0,   0,   0,   0, -w3,   0,   0, -w1,   a, -w2,   0,   0, -w4,   0,   0,   0,   0,   0,   0]
[  0,   0,   0,   0,   0,   0,   0,   0,   0,   0, -w3,   0,   0, -w1,   a, -w2,   0,   0, -w4,   0,   0,   0,   0,   0]
[  0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0, -w3,   0,   0, -w1,   a,   0,   0,   0, -w4,   0,   0,   0,   0]
[  0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0, -w3,   0,   0,   0,   a, -w2,   0,   0, -w4,   0,   0,   0]
[  0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0, -w3,   0,   0, -w1,   a, -w2,   0,   0, -w4,   0,   0]
[  0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0, -w3,   0,   0, -w1,   a, -w2,   0,   0, -w4,   0]
[  0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0, -w3,   0,   0, -w1,   a,   0,   0,   0, -w4]
[  0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0, -w3,   0,   0,   0,   a, -w2,   0,   0]
[  0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0, -w3,   0,   0, -w1,   a, -w2,   0]
[  0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0, -w3,   0,   0, -w1,   a, -w2]
[  0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0, -w3,   0,   0, -w1,   a]
```

Figure 6: Band Matrix for 6x4 Asymmetric Lattice.

REFERENCES

1. Rothwell, E.J. and M.J. Cloud. 2001. *Electromagnetics*. CRC Press LLC: Boca Raton, FL.

2. Jones, D.S. 1987. *Methods in Electromagnetic Wave Propagation*, Volume 2. Clarendon Press: Oxford, UK.

3. Sadiku, M.N. 1995. *Elements of Electromagnetics*. Second Edition. Oxford University Press: New York, NY.

4. Nuruzzaman, M. 2016. "3D FD on Laplacian for Computational Electromagnetics in MATLAB", CreateSpace, South Carolina.

5. Monk, P. 2003. *Finite Element Methods for Maxwell's Equations*. Oxford University Press: Oxford, UK.

6. Nuruzzaman, M. 2013. "Finite Difference Fundamentals in MATLAB". CreateSpace: South Carolina.

7. Kong, J.A. 1986. *Electromagnetic Wave Theory*. John Wiley & Sons, New York, NY.

8. Schwarz, S.E. 1990. *Electromagnetics for Engineers*. Saunders College Publishing: Philadelphia, PA.

9. Vanderlinde, J. 1993. *Classical Electromagnetic Theory*. John Wiley & Sons, Inc., New York, NY.

SUGGESTED CITATION

Nuruzzaman, M. 2018. **"**Cylindrical Laplacian on 2D FD for Computational Electromagnetics". *Pacific Journal of Science and Technology*. 19(2):5-11.

Pacific Journal of Science and Technology