Stochastic Signal Modeling of Economic Indicators.

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ABSTRACT

An objective of every economy is to achieve a high and sustainable economic growth rate coupled with the economic indicators. These economic indicators give the picture of growth or performance of the economy. This study used stochastic signal model for the analysis of Nigerian inflationary rate which is an important economic indicator. Analysis and prediction of these indicators will go a long way to evaluate the performance and hence adjust some factors to enhance better economic growth. The study shows that the stochastic signal model is a random walk plus noise model which satisfies the Markovian and Martingale properties. The model was further seen to be a structural model with ARIMA $(0,1,1)$ as its reduced form.The diagnostic check indicates that the fitted model is adequate and can therefore be used for predictions

(Keywords: stochastic processes, random walk, structural model, economic indicators)

INTRODUCTION

Policymakers in most advanced and several developing nations use economic indicators to predict the direction of aggregate economic activity (Ikoku, 2010). One of the key objectives of every good economy, whether or not developing or developed is to achieve a high and sustainable economic growth rate coupled with the economic indicators Onwukwe and Nwafor (2014).

These economic indicators give the picture of growth or performance of the economy. They facilitate the conduct of macroeconomic policies which must anticipate the future and take corrective action in order to keep the economy growing at, or close to, capacity with price stability (Ikoku, 2010). These economic indicators are dynamical in nature as they are subject to change with time. A dynamic system is a concept in mathematics where a fixed rule describes how a part in a geometrical space depends on time.

Dynamical systems are mathematical objects used to model physical phenomena whose state (or instantaneous description) changes over time. The realizations from these systems are usually corrupted by random noise. This study therefore considered economic indicators as corrupted stochastic process that can be analyzed with the use of signal model.

During the last few decades, interest in the study of stochastic phenomena has increased dramatically and intense research activity in this area has been stimulated by the need to take into account stochastic processes. Stochastic processes are perhaps the most useful objects that can be used to model physical and financial processes directly without any intervening need to sample the data. Stochastic processes are commonly used as a model for noise in physical systems, this modeling of noise being the necessary first step in deciding the best way to lessen its effect (Stark and Woods, 1986).

According to Cunningham (1995), a stochastic process is a set of finite or infinite random variable ordered in time. More precisely, a stochastic process is a function of the domain of which is the sample space and range of which is set of real functions of time. Each time, function is called the realization of the stochastic process and this may be either continuous or discrete. A stochastic process is a collection or ensemble of time functions, continuous or discrete. The discrete time function is referred to as a stochastic sequence and can be thought of as an infinite-dimensional vector of random variables. It is often desirable to partially characterize a stochastic sequence based on the knowledge of its first two moment functions which are its mean and variance functions. The observations of economic variable are considered as realizations from the stochastic process.

The stochastic signal model in this study is seen as a Markovian model which satisfies the martingale property. Markov processes are stochastic process whose future behavior cannot be accurately predicted from its past behavior and which involves random chance or probability. Markov processes are probabilistic models for describing data with a sequential structure. A Markov process is useful for analyzing dependent random events; that is, events whose likelihood depends on what happened last. Markov processes are continuous time process with a denumerable state space and the theory of Markov processes has developed rapidly in recent years (Dynkin, 2006). A Markov process is the probabilistic analog of causality and can be specified by defining the conditional distribution of the stochastic process (Agwuegbo *et al*, 2014). The conditional distribution is seen to satisfy the martingale.

The economic variables are dynamical systems which are described as processes with stochastic components and random noise. The stochastic aspects of the models are used to capture the uncertainty about the environment in with the system is operating and the structure and parameters of the models of the processes under study (Milstein, 1988; Kloeden and Platen, 1992; Shali *et al*, 2012; Akintunde *et al.*, 2015). According to Akintunde *et al.*, 2015, applications of dynamical system are broadly categorized into three main areas which are predictive (also referred to as generative), in which the objective is to predict future states of the system from observations of the past and present states of the systems.

The second is diagnostics, in which the objective is to infer what possibly past states of the system might have led to the present state of the system (or observations leading up to the present state), and finally, applications in which the objectives is neither to predict the future nor explain the past but rather to provide a theory for the physical phenomena.

These three categories correspond roughly to the need to predict, explain and understand physical phenomena. Predictive and diagnostic reasoning are often described in terms of causes and effects. Prediction is reasoning forward in time from causes to effects while diagnosis is reasoning backward from effects to causes. Economic indicators such as exchange rate, inflation rate, interest rate, gross domestic products are brought about on continuous time bases and can therefore be seen as continuous time stochastic processes.

The modeling of continuous time dynamical systems from uncertain observation is an important task especially in finance. Basically in finance, the assumptions for dynamical models are formulated by systems of differential equations. In Bayesian approach, the dynamics are then incorporated by a priori knowledge of the probability distributions on the unknown functions which corresponds for example to driving forces and appear as coefficients or parameters in the differential equations (Akintunde *et al.*, 2015).

MATERIALS AND METHODS

Basic Theory

Let X_t denote economic indicator observation at time, t. The collection of the random variables $\{X_t, t = 0,1,2 \ldots\}$ is a stochastic process in discrete time and continuous state space. The state vector of the system represents noisy observations. That is the observations are considered as being signal plus noise and then the distribution can be classified to follow a random walk plus noise given as:

$$X_t = \mu_t + \varepsilon_t, \qquad t = 1,2, \ldots T \qquad (1)$$

where μ_t, the trend component is simply a level which fluctuates up and down according to a random walk,

$$\mu_t = \mu_{t-1} + \epsilon_t, \qquad t = \cdots, -1,0,1, \ldots \qquad (2)$$

where μ_t is assumed to have started at some point in the remote past (Harvey, 1989). Equation (2) is a non-stationary process and can be written as:

$$\mu_t = \frac{\epsilon_t}{\Delta} \qquad (3)$$

where Δ is the first difference operator Substituting Equation (3) into (1) gives:

$$X_t = \frac{\epsilon_t}{\Delta} + \varepsilon_t, \qquad t = 1,2, \ldots T \qquad (4)$$

If a drift term is added to Equation (4) we have:

$$X_t = \frac{\epsilon_t}{\Delta} + \beta_t + \varepsilon_t, \qquad t = 1,2,\ldots T \qquad (5)$$

X_t is a Markov process where with a probability density function dependent upon the martingale hypothesis. As a Markov process, the conditional distribution of X_t given information up until $\tau < t$ depends only on X_τ while the conditional expectation of the state of X_t at any time in the future is dependent on the present state which is a martingale property. That is the conditional expectation of X_t is given as:

$$E[X_t | X_\tau, \ \tau < T] = X_t \qquad (6)$$

From Equation (5), we have:

$$\Delta X_t = \epsilon_t + \Delta\varepsilon_t \qquad (7)$$

Then,

$$E(\Delta X_t) = E(\epsilon_t) + E(\varepsilon_t) - E(\varepsilon_{t-1}) = 0 \qquad (8)$$

And the auto-covariance is:

$$\gamma = \begin{cases} \sigma_\epsilon^2 + \sigma_\varepsilon^2, & \tau = 0 \\ -\sigma_\varepsilon^2, & \tau = 1 \\ 0 & \tau \geq 2 \end{cases} \qquad (9)$$

Hence, the auto-correlation is:

$$\gamma = \begin{cases} 1, & \tau = 0 \\ -\dfrac{\sigma_\varepsilon^2}{\sigma_\epsilon^2 + 2\sigma_\varepsilon^2}, & \tau = 1 \\ 0 & \tau \geq 2 \end{cases} \qquad (10)$$

The signal model is therefore a structural form of the random walk plus noise model. Equation (10) shows that the reduced form of the model is ARIMA (0,1,1) (Harvey, 1989).

RESULTS AND DISCUSSION

The study applied the model using the Nigerian Inflationary Rate from January 2011 to June 2014 (Source: Central Bank of Nigeria (CBN) "FULL REPORT", 2014). Inflation is an important economic indicator, that is, it is an important indicator of the state of an economy of any country. Figure 1 shows the plot of the realizations from the Nigerian Inflationary rate. The plot indicates that the variable follow a non-stationary process.

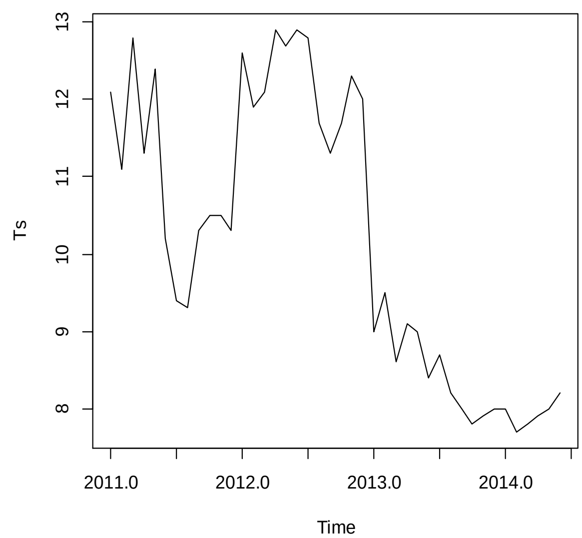

Figure 1: Realization Plot of Nigerian Inflationary Rate.

Figures 2 and 3 show that Nigerian inflationary rate can be modeled with a random walk plus noise model which is a stochastic signal model.

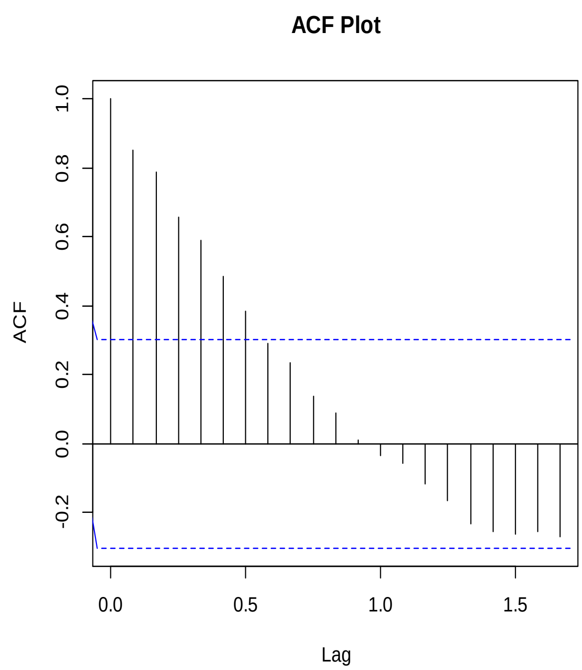

Figure 2: Auto-Correlation Plot of the Nigerian Inflationary Rate.

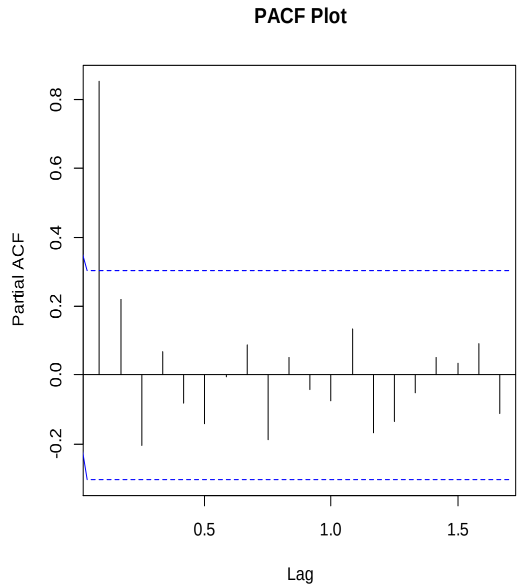

Figure 3: Partial Auto-correlation Plot of the Nigerian Inflationary Rate.

The stochastic signal model from the inflationary rate is:

$$\varepsilon_t = 0.2531\varepsilon_{t-1} + X_t$$

With mean equals zero and variance equals 0.7617. The standard error of the estimate is 0.1386.

Figure 4 is the diagnostics of the fitted model with plots of the standardized residuals, ACF of residuals and the p-values of Ljung-Box Chi-squared statistics. The plots are based on the examination of the residuals, $\hat{e}_t = X_t - \hat{X}_t$, where $\hat{X}_t$ is the fitted value. The residuals are the estimates of the error components while the Ljung-Box Chi-squared statistics contain the portmanteau statistics with their associated p-values (Agwuegbo *et. al.*, 2010). Figure 4 shows that the model is satisfactory because the residual are approximately normally distributed and none of the chi-square values is significant at the 5% level. Therefore the model adequacy or diagnostic check (Figure 4) shows that the fitted model is adequate for the Nigerian inflationary rate.

CONCLUSION

The study revealed that economic indicator are corrupted variables and are dynamical. As a result analysis prediction of such variables can be achieved with the use of a stochastic signal model.

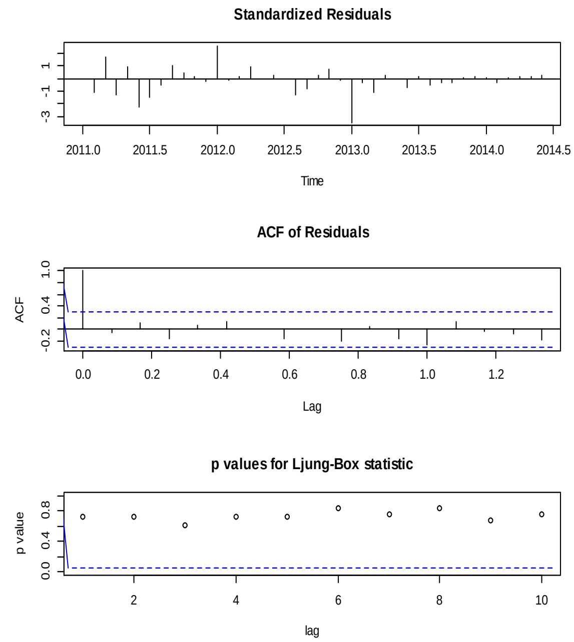

Figure 4: Diagnostics for the Fitted Model.

REFERENCES

1. Agwuegbo, S.O.N., A.P. Adewole, O.M. Olayiwola, and F.S. Apantaku. 2014. "Signal Model for the Prediction of Wind Speed in Nigeria". *Journal of Energy Technologies and Policy.* 4 (6):38-44.

2. Agwuegbo, S.O.N., A.R.T. Solarin, and O.E. Asiribo. 2010. "An Application of Deconvolution of Chaotic Time Series Data". *Journal of Nigeria Statistical Association*. 22:12-24.

3. Akintunde, A.A., S.O.N. Agwuegbo, O.E. Asiribo, M.A. Waheed, and O.M. Olayiwola. 2015. "Modeling Nonlinear Dynamical Systems". *Annals Computer Science Series*. 13(1):30-38

4. Cunningham E.P. 1995 *Digital Filtering: An Introduction*. John Wiley and Sons, New York.

5. Dynkin, E.B. 2006. *Theory of Markov Processes*. Dover Publications: Mineola, NY.

6. Harvey, A.C. 1989 *Forecasting, Structural Time Series Models and the Kalman Filter*. Cambridge University Press: Cambridge, UK.

7. Ikoku, A.E. 2010. "Is the Stock Market a Leading Indicator of Economic Activity in Nigeria?". *Central*

*Bank of Nigeria (CBN) Journal of Applied Statistics 1(1):*17-38

8. Kloeden, P.E. and E. Platen. 1992. *Numerical Solution of Stochastic Differential Equations*. Springer-Verlag" Berlin, Germany.

9. Milstein, G.N. 1988. *Numerical Integration of Stochastic Differential Equations* .Kluwer Academic Publishers.

10. Onwukwe, C.E. and G.O. Nwafor. 2014. "A Multivariate Time Series Modeling of Major Economic Indicators in Nigeria".. *American Journal of Applied Mathematics and Statistics.* 2(6):376-385.

11. Shali, J.A., J. Akbarfam, and H. Bevrani. 2012. "Approximate Solution of the Nonlinear Stochastic Differential Equations". *International Journal of Mathematical Engineering and Science.* 1(4):53-71

12. Stark, H. and J.W. Woods. 1986 *Probability Random Processes and Estimation Theory for Engineers*. Prentice-Hall: Englewood Cliffs, NJ.

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