Estimation of P[Y<X] for Bivariate Exponentiated Gamma Distribution.

Gafar Matanmi Oyeyemi, Ph.D.* **and Kamil Damola Ajagbe, M.Sc.**

Department of Statistics, University of Ilorin, Ilorin, Nigeria.

E-mail: gmoeyeyemi@gmail.com*

ABSTRACT

Exponentiated distributions are the distributions that can be considered when dealing with either monotonic or non-monotonic event. Bivariate exponentiated gamma (EG) distribution is observed by getting the maximum likelihood estimate when scale parameter is known and when it is unknown. Also percentile estimate of the parameters are also observed, which is discovered to be a better estimator of the parameter of EG.

(Keywords: non-monotone event, increasing failure events, decreasing failure events, proportion reversed hazard rate model, percentile estimator, PCE)

INTRODUCTION

In real life situations, most events don't occur in a regular pattern, which is known to be monotonic. Most researchers assumed this fact. In recent days, Gupta, Gupta, and Gupta (1998) gave exponentiated gamma (EG) distribution with:

Pdf:

$$f(x) = \alpha\lambda^2 x e^{-\lambda x}(1 - e^{-\lambda x}(\lambda x + 1))^{\alpha-1} \quad \alpha,\lambda > 0$$

Cdf:

$$F(x) = (1 - e^{-\lambda x}(\lambda x + 1))^{\alpha} \qquad \alpha,\lambda > 0$$

Survival function:

$$R(x) = 1 - (1 - e^{-\lambda x}(\lambda x + 1))^{\alpha} \quad \alpha,\lambda > 0$$

Hazard function:

$$h(x) = \frac{\alpha\lambda^2 x e^{-\lambda x}(1 - e^{-\lambda x}(\lambda x + 1))^{\alpha-1}}{1 - (1 - e^{-\lambda x}(\lambda x + 1))^{\alpha}} \qquad \alpha,\lambda > 0$$

The distribution is flexible to accommodate both monotonic and non-monotonic failure events.

Monotonic failure event is an event that has a continuing rate at an interval of time example are sharpness of object which is decreasing failure event, strength of human among others. Non-monotonic event are the event which are not regular in nature of increase or decrease failure. i.e., some are increasing failure rate model that sometimes having decreasing rate due to some conditions like rate of spread of disease in the body or environment.

Increasing failure event are the occurrence that are increasing as the time increases, although such are not much but we can have the spread of disease as one. Decreasing failure events are the occurrence that are decreasing as the time increases like strength, stress, sharpness, etc.

The major aim of the research is to observe some properties of a bivariate exponentiated gamma distribution (EG) for P[Y < X], where, $X \sim EG(\alpha;\lambda)$, $Y \sim EG(\beta;\lambda)$ and they are independently distributed, where α and β are the shape parameters and λ is scale parameter of the distributions. In real life situation the measuring of Typhoid and malaria in human body is an example of such occasion.

Many researchers have worked this distribution. (Shawky & Bakoban, 2009) worked on EG considering the order statistics using moment estimator, MLE and best linear unbiased estimator (BLUE's).Also (Shawky & Bakoban, 2011) worked on the use of different estimators on distribution considering the use of Maximum Likelihood Estimators (MLE), least square estimator under Type 1 and Type 2 censoring and some other estimators. Considering the Bayesian analysis of EG Nasir et al (2013) using MLE and Bayesian estimator using MCMC to generate the sample for the posterior distributions. In this research, we shall consider the use of maximum Likelihood estimator (MLE) and Percentile Estimator (PCE).

In comparing two occasions or events, the estimation of P[Y < X], when X and Y are normally distributed are considered by (Church & Harris, 1970). The MLE of P[Y < X], when X and Y have bivariate exponential distribution, has been considered by (Awad, Azzam, & Hamadan, 1981).

MAXIMUM LIKELIHOOD ESTIMATOR

For independent random sample $X \sim EG(\alpha;\lambda)$ and $Y \sim EG(\beta;\lambda)$. The bivariate exponentiated gamma distribution of X and Y is:

$$P = f(x,y) = \alpha\beta\lambda^4 xy e^{-\lambda(x+y)}(1-e^{-\lambda x}(\lambda x+1))^{\alpha-1}(1-e^{-\lambda y}(\lambda y+1))^{\beta-1} \qquad \alpha,\beta,\lambda > 0$$

$$P(Y<X) = \int_0^{\infty}\int_0^{x} \alpha\beta\lambda^4 xy e^{-\lambda(x+y)}(1-e^{-\lambda x}(\lambda x+1))^{\alpha-1}(1-e^{-\lambda y}(\lambda y+1))^{\beta-1}\,dy\,dx$$

$$= \int_0^{\infty} \alpha\lambda^2 x e^{-\lambda x}(1-e^{-\lambda x}(\lambda x+1))^{\alpha-1}|(1-e^{-\lambda y}(\lambda y+1))^{\beta}|_0^x\,dx$$

$$= \int_0^{\infty} \alpha\lambda^2 x e^{-\lambda x}(1-e^{-\lambda x}(\lambda x+1))^{\alpha+\beta-1}\,dx$$

$$= \frac{\alpha}{\alpha+\beta} \qquad (1)$$

Let the sample size of X and Y be n and m. Then, the likelihood of the distribution is:

$$F(X_1,X_2,\dots,X_n,Y_1,Y_2,\dots,Y_n) = L$$

$$= \alpha^n\beta^m\lambda^{2(m+n)}e^{-\lambda(\sum_{i=1}^n X_i+\sum_{j=1}^m Y_j)}\prod_{i=1}^{n}(1-e^{-\lambda x}(\lambda x+1))\prod_{j=1}^{m}(1-e^{-\lambda y}(\lambda y+1))$$

$$InF(X_1,X_2,\dots,X_n,Y_1,Y_2,\dots,Y_n) = \Lambda$$

$$n\,ln\alpha + m\,ln\,\beta + 2(m+n)\ln\lambda - \lambda\left(\sum_{i=1}^{n} X_i + \sum_{j=1}^{m} Y_j\right) +$$

$$(\alpha-1)\sum_{i=1}^{n}\left(1-e^{-\lambda x}(\lambda x+1)\right) + (\beta-1)\sum_{j=1}^{m}(1-e^{-\lambda y}(\lambda y+1))$$

$$\frac{d(\Lambda)}{d\alpha} = \frac{n}{\alpha} + \sum_{i=1}^{n} \left(1 - e^{-\lambda x_i}(\lambda x_i + 1)\right)$$

$$\frac{d(\Lambda)}{d\beta} = \frac{m}{\beta} + \sum_{j=1}^{m} \left(1 - e^{-\lambda y_j}(\lambda y_j + 1)\right)$$

$$\frac{d(\Lambda)}{d\lambda} = \frac{2(n+m)}{\lambda} - \left(\sum_{i=1}^{n} X_i + \sum_{j=1}^{m} Y_j\right) + (\alpha - 1)\sum_{i=1}^{n} \frac{\lambda x_i^2 e^{-\lambda x_i}}{\left(1 - e^{-\lambda x_i}(\lambda x_i + 1)\right)}$$
$$+(\beta - 1)\sum_{j=1}^{m} \frac{\lambda y_j^2 e^{-\lambda y_j}}{\left(1 - e^{-\lambda y_j}(\lambda y_j + 1)\right)}$$

If

$$\frac{d(\Lambda)}{d\alpha} = \frac{d(\Lambda)}{d\beta} = \frac{d(\Lambda)}{d\lambda}$$

Then

$$\hat{\alpha} = \frac{-n}{\sum_{i=1}^{n} ln\left(1 - e^{-\lambda x_i}(\lambda x_i + 1)\right)}$$

$$\hat{\beta} = \frac{-m}{\sum_{j=1}^{m} ln\left(1 - e^{-\lambda y_j}(\lambda y_j + 1)\right)}$$

$\hat{\lambda}$ can be obtained as the solution of non-linear equation:

$$g(\lambda) = 2\frac{m+n}{\lambda} - \sum_{i=1}^{n} \frac{x_i}{\left(1 - e^{-\lambda x_i}(\lambda x_i + 1)\right)} - \sum_{j=1}^{m} \frac{y_j}{\left(1 - e^{-\lambda y_j}(\lambda y_j + 1)\right)}$$
$$- \frac{n}{\sum_{i=1}^{n} ln\left(1 - e^{-\lambda x_i}(\lambda x_i + 1)\right)} \sum_{i=1}^{n} \frac{\lambda x_i^2 e^{-\lambda x_i}}{\left(1 - e^{-\lambda x_i}(\lambda x_i + 1)\right)} - \frac{m}{\sum_{j=1}^{m} ln\left(1 - e^{-\lambda y_j}(\lambda y_j + 1)\right)}$$
$$\sum_{j=1}^{m} \frac{\lambda y_j^2 e^{-\lambda y_j}}{\left(1 - e^{-\lambda y_j}(\lambda y_j + 1)\right)}$$

It can be obtained by using an iterative scheme as follows:

$$h(\lambda) = \lambda$$

where

$$h(\lambda) = 2(m+n)\left[\sum_{i=1}^{n}\frac{x_i}{\left(1-e^{-\lambda x_i}(\lambda x_i+1)\right)} + \sum_{j=1}^{m}\frac{y_j}{\left(1-e^{-\lambda y_j}\left(\lambda y_j+1\right)\right)}\right.$$
$$+\frac{n}{\sum_{i=1}^{n} ln\left(1-e^{-\lambda x_i}(\lambda x_i+1)\right)}\sum_{i=1}^{n}\frac{\lambda x_i^2 e^{-\lambda x_i}}{\left(1-e^{-\lambda x_i}(\lambda x_i+1)\right)} + \frac{m}{\sum_{j=1}^{m} ln\left(1-e^{-\lambda y_j}\left(\lambda y_j+1\right)\right)}$$
$$\left.\sum_{j=1}^{m}\frac{\lambda y_j^2 e^{-\lambda y_j}}{\left(1-e^{-\lambda y_j}\left(\lambda y_j+1\right)\right)}\right]$$

Since $\hat{\lambda}$ is a fixed point solution of the non-linear equation. Therefore, it can be obtained by using an iterative scheme as follow:

$$h(\lambda_j) = \lambda_{(j+1)}$$

where λ_k is kth iterate of $\hat{\lambda}$ the iteration procedure will be stopped when $|\lambda_k - \lambda_{k+1}| < \varepsilon$, where ε is sufficiently small. Then, the MLE of $P[Y < X]$ becomes:

$$\hat{P} = \frac{\hat{\alpha}}{\hat{\alpha}+\hat{\beta}}$$

FISHER'S INFORMATION

Asymptotic distribution of $\hat{\theta} = (\hat{\alpha}, \hat{\beta}, \hat{\lambda})$ and then we derived the asymptotic distribution of the function $P[Y < X]$.

Let the fisher's information matrix $\hat{\theta} = (\hat{\alpha}, \hat{\beta}, \hat{\lambda}) I(\theta) = (I_{i,j}(\theta))$; $i, j = 1, 2, 3.$ Therefore,

$$I(\theta) = -\begin{pmatrix} E(\frac{d^2\Lambda}{d\alpha^2}) & E(\frac{d^2\Lambda}{d\alpha\, d\beta}) & E(\frac{d^2\Lambda}{d\alpha\, d\lambda}) \\ E(\frac{d^2\Lambda}{d\beta\, d\alpha}) & E(\frac{d^2\Lambda}{d\beta^2}) & E(\frac{d^2\Lambda}{d\beta\, d\lambda}) \\ E(\frac{d^2\Lambda}{d\lambda\, d\alpha}) & E(\frac{d^2\Lambda}{d\lambda\, d\beta}) & E(\frac{d^2\Lambda}{d\lambda^2}) \end{pmatrix}$$

$$= \begin{pmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{pmatrix}$$

Where $I_{ij} = I_{ji}$

Moreover, $E\left(\frac{d^2\Lambda}{d\alpha^2}\right) = -\frac{n}{\alpha^2}$, $E\left(\frac{d^2\Lambda}{d\beta^2}\right) = -\frac{m}{\beta^2}$, $E\left(\frac{d^2\Lambda}{d\beta\, d\alpha}\right) = E\left(\frac{d^2\Lambda}{d\alpha\, d\beta}\right) = 0$

$$E\left(\frac{d^2\Lambda}{d\alpha\ d\lambda}\right)=E\left(\sum_{i=1}^{n}\frac{\lambda x_i^2 e^{-\lambda x_i}}{\left(1-e^{-\lambda x_i}(\lambda x_i+1)\right)}\right)$$

$$E\left(\frac{d^2\Lambda}{d\beta\ d\lambda}\right)=E\left(\sum_{j=1}^{m}\frac{\lambda y_j^2 e^{-\lambda y_j}}{\left(1-e^{-\lambda y_j}\left(\lambda y_j+1\right)\right)}\right)$$

$$E\left(\frac{d^2\Lambda}{d\lambda^2}\right)=E\left(\frac{2(n+m)}{\lambda^2}+(\alpha-1)\frac{d\left(q(x,\lambda)\right)}{d\lambda}+(\beta-1)\frac{d\left(q(y,\lambda)\right)}{d\lambda}\right)$$

ESTIMATION OF P IF λ IS KNOWN

Considering P when λ is known. Without loss of generality, we assume that $\lambda=1$. Therefore, in this section it is assumed that $X_1,X_2,\dots,X_n$ is a random sample from EG(α,1) and $Y_1,Y_2,\dots,Y_m$ is random sample from EG(β,1) and from the sample P is estimated. Firstly, we consider the MLE of P and its distributional properties.

Maximum Likelihood E OF P

Based on the above sample, it is clear that MLE of $\hat{P}$ will be

$$\hat{P}=\frac{\hat{\alpha}}{\hat{\alpha}+\hat{\beta}}$$

$$\hat{\alpha}=\frac{-n}{\sum_{i=1}^{n}ln\left(1-e^{-x_i}(x_i+1)\right)}$$

$$\hat{\beta}=\frac{-m}{\sum_{j=1}^{m}ln\left(1-e^{-y_j}\left(y_j+1\right)\right)}$$

Therefore,

$$\hat{P}=\frac{n\sum_{j=1}^{m}ln\left(1-e^{-y_j}\left(y_j+1\right)\right)}{n\sum_{j=1}^{m}ln\left(1-e^{-y_j}\left(y_j+1\right)\right)+m\sum_{i=1}^{n}ln\left(1-e^{-x_i}(x_i+1)\right)}$$

It is observed in (Gupta & Kundu, 2002), that:

$$-2\alpha\sum_{i=1}^{n}ln\left(1-e^{-x_i}(x_i+1)\right)\sim\chi_{2n}^2$$
$$-2\beta\sum_{j=1}^{m}ln\left(1-e^{-y_j}\left(y_j+1\right)\right)\sim\chi_{2m}^2$$

$$\hat{P} \approx \frac{V}{V+cU} \approx \frac{1}{1+\frac{\alpha}{\beta}Z}$$

$$\frac{P}{1-P} \times \frac{1-\hat{P}}{\hat{P}} \approx Z$$

Here $\approx$ indicates equivalent in distribution and $c = \frac{m\alpha}{n\beta}$. The random variables U and V are independent and follow χ^2 distribution, with 2n and 2m degrees of freedom respectively. Moreover, Z has an F distribution with 2n and 2m degrees of freedom. Therefore, the PDF of $\hat{P}$ is as follows:

$$f_{\hat{P}}(x) = k \times \frac{\left(\frac{1-x}{x}\right)^{n-1}}{\left(1+\frac{n\beta(1-x)}{m \propto x}\right)^{m+n}}, \quad 0 < x < 1$$

$$k = \frac{\Gamma(m+n)}{\Gamma(\mathrm{m})\Gamma(\mathrm{n})}\left(\frac{\beta}{\alpha}\right)^{n-1}\left(\frac{n}{m}\right)^{n}$$

The $100(1-\gamma)\%$confidence interval of P can be obtained as:

$$\left[\frac{1}{1+F_{2m,2n;1-\frac{\alpha}{2}} \times \left(\frac{1}{\hat{P}}-1\right)}, \frac{1}{1+F_{2m,2n;\frac{\alpha}{2}} \times \left(\frac{1}{\hat{P}}-1\right)}\right]$$

Where $F_{2m,2n;1-\frac{\alpha}{2}}$ and $F_{2m,2n;\frac{\alpha}{2}}$ are the lower and upper $\frac{\gamma}{2}^{th}$ percentile points of a F distribution with 2n and 2m degrees of freedom.

PERCENTILE ESTIMATOR (PCE)

If the data come from a distribution function, then we need to estimate the parameters by fitting a straight line to the points obtained by the distribution function and sample percentile point. (Murthy, Xie, and Jiang, 2004) used this method for Weibull distribution and (Gupta and Kundu, 2002) used this for generalized exponential distribution. Using this estimate for bivariate distribution, we follow these steps:

$$f(x,y,\alpha,\beta,\lambda) = f(x,\alpha,\lambda)\; f(y,\beta,\lambda)$$

So that:

$$f(x,\alpha,\beta,\lambda) = \int f(x,\alpha,\lambda)\left[\int_0^\infty f(y,\beta,\lambda)dy\right]dx$$

The estimate of parameter α can be derived from the marginal distribution of X setting the range of the distribution of Y from 0 to infinity because the two event measures are independent and random distributed. So:

$$F(x) = (1 - e^{-\lambda x}(\lambda x + 1))^{\alpha}$$

Where,

$$\int_0^\infty f(y,\beta,\lambda)dy = 1$$

$$F^{\frac{1}{\alpha}}(x) = 1 - e^{-\lambda x}(\lambda x + 1)$$

$$1 - F^{\frac{1}{\alpha}}(x) = e^{-\lambda x}(\lambda x + 1)$$

$$\ln(1 - F^{\frac{1}{\alpha}}(x)) = -\lambda x + ln(\lambda x + 1)$$

Arranging X in such $X_{1:n} < X_{2:n} < \cdots < X_{n:n}$ in order statistics obtained from the EG(α, λ).

Let P_i denote some estimate of $F(x_{i:n}, \alpha, \lambda)$ then the estimate of α and λ can be obtained by finding the derivative of:

$$\sum_{i=1}^{n}\left(\lambda x_{i:n} + ln(\lambda x_{i:n} + 1) + \ln(1 - P_i^{\frac{1}{\alpha}})\right)^2$$

With respect to α and λ to have:

$$\sum_{i=1}^{n}\left(\lambda x_{i:n} + ln(\lambda x_{i:n} + 1) + \ln(1 - P_i^{\frac{1}{\alpha}})\right)[+\ln(1 - P_i^{\frac{1}{\alpha}}]P_i^{\frac{1}{\alpha}}\frac{ln\, P_i}{\alpha^2} = 0$$

$$\sum_{i=1}^{n}\left(\lambda x_{i:n} + ln(\lambda x_{i:n} + 1) + \ln(1 - P_i^{\frac{1}{\alpha}})\right)(\lambda x_{i:n} + x_{i:n}(\lambda x_{i:n} + 1)^{-1}) = 0$$

Shawky and Bakoban (2011) considered $P_i = \frac{i}{n+1}$ as the expected value of $F(x_{i:n})$. When the shape parameter is known, then the equation will be used to obtain the percentile estimator of λ say $\tilde{\lambda}$. Consider the case when the scale parameter λ is known, then we $\lambda = 1$, then:

$$\ln F(x, \alpha) = \alpha \ln[1 - e^{-x}(x + 1)]$$

Therefore PCE of α say $\tilde{\alpha}$ can be obtained by differentiating:

$$\sum_{i=1}^{n}\{\ln p_i - \alpha \ln[1 - e^{-x_i}(x_i + 1)]\}^2$$

With respect to α. So that:

$$\tilde{\alpha} = \frac{\sum_{i=1}^{n}\ln p_i}{\sum_{i=1}^{n}\ln[1 - e^{-x_i}(x_i + 1)]}$$

Repeating the same step from the cumulative distribution function of the bivariate distribution by getting the marginal function of Y, setting $0 < x < \infty$ and

$$\tilde{\alpha} = \frac{\sum_{j=1}^{m}\ln p_j}{\sum_{j=1}^{m}\ln[1 - e^{-y_j}(y_j + 1)]}$$

Where $p_j = \frac{j}{m+1}$

NUMERICAL EXPERIMENT AND RESULTS

The data used in this research is a simulated data in Newdistn package 1.1 version in R 3.0.1 on Microsoft Windows 7. Varying the sample sizes as well as the shape parameters the results were obtained.

The sample sizes considered in this work are (n, m) (15, 15), (15, 20), (20, 20), (20, 25), (15, 25) and (25, 25). Also, the two shape parameters were (α, β) (10, 5), (5, 5), and (5, 4). Due to the restriction (that is λ = 1), the Gamma distribution used is of shape = 2 and scale = 1. This is done based on 10,000 replications

Estimation of the Parameters

The table below shows the absolute bias (bias) and mean square error (MSE) for both MLE and PCE of the estimate of the distribution parameter (shape parameter).

Table 1: Bias and the Mean Square Error (MSE) of both MLE and PCE for each Parameter α and β at Various Sample Sizes.

(n, m)	Parameters	MLE		PCE	
		Bias	MSE	Bias	MSE
(15, 15)	α=10	2.236134	9.241151	2.108347	7.358230
	β=5	1.119222	2.355891	1.057174	1.866062
	α=5	1.111975	2.363689	1.048405	1.872307
	β=5	1.107483	2.270957	1.042379	1.797903
	α=5	1.1183596	2.314677	1.0533592	1.842498
	β=4	0.8977920	1.488182	0.8415205	1.176548
(15, 20)	α=10	2.2658348	9.519624	2.1309363	7.521869
	β=5	0.9322227	1.534301	0.8963963	1.283790
	α=5	1.1239389	2.311867	1.0603257	1.838180
	β=5	0.9525801	1.637589	0.9067476	1.355308
	α=5	1.1166009	2.3259522	1.0534915	1.8549923
	β=4	0.7598382	1.0183524	0.7259391	0.8483851
(15, 25)	α=10	2.2360855	9.166983	2.1063698	7.304942
	β=5	0.8297465	1.188627	0.8013992	1.025038
	α=5	1.1183459	2.316255	1.0487645	1.838687
	β=5	0.8331466	1.190763	0.8056352	1.027665
	α=5	1.1341569	2.3396972	1.0659806	1.8552699
	β=4	0.6734376	0.7950387	0.6493400	0.6807340
(20, 20)	α=10	1.9196420	6.609348	1.8380290	5.523194
	β=5	0.9529327	1.631871	0.9092292	1.352405
	α=5	0.9501661	1.602781	0.9105428	1.340551
	β=5	0.9394890	1.557099	0.9006062	1.303118
	α=5	0.9487393	1.6055177	0.9049436	1.3325371
(20, 25)	α=10	1.9025604	6.357532	1.8215454	5.295077
	β=5	0.8413688	1.228107	0.8127488	1.050044
	α=5	0.9652006	1.650665	0.9140687	1.357707
	β=5	0.8495443	1.238929	0.8194219	1.075296
	α=5	0.9492881	1.6378983	0.9124223	1.3720299
	β=4	0.6662630	0.7728051	0.6384398	0.6588194
(25, 25)	α=10	1.6969102	5.017584	1.6324502	4.279852
	β=5	0.8442440	1.236704	0.8119407	1.061037
	α=5	0.8270969	1.190877	0.7997213	1.022007
	β=5	0.8389576	1.231559	0.8128147	1.059408
	α=5	0.8382104	1.2327004	0.8090855	1.0553639
	β=4	0.6705143	0.7740695	0.6494838	0.6696825

SUMMARY

The data is simulation from exponentiated gamma distribution (EG) using rexpg(n, "gamma", a = 1, shape,...), (Nadarajah, 2013) for X and Y. The maximum likelihood estimate of the parameters were obtained and compared with the true value in 10000 replications. Furthermore, Percentile estimate of the parameters were also obtained and compared with the true value in 10000 replications. Based on the compared results, the conclusions are made.

DISCUSSION

When having smaller sample of X that is n = 15 and m = 15, 20 and 25, it was found that percentile estimator give a better estimate and more consistent than maximum likelihood estimator. Similarly, when sample size of X increases, that is n = 20 when m = 20 and 25, it is found that percentile estimator gives better estimate and also consistent than maximum likelihood estimate. Generally, in all sample sizes considered for both X and Y, PCE give better estimate and more consistent than MLE.

Estimation of the Parameters

The table below shows the absolute bias (bias) and mean square error (MSE) for both MLE and PCE of the estimate of P.

DISCUSSION

From Table 2, for the same sample sizes (n=m), both MLE and PCE are good to estimate the event P as the Bias and MSE of the both methods are the same for all values of α and β. When n < m, for all values of α and β, the MSE of PCE are always greater than that of MLE except when α = β = 5 while the Bias of MLE are also greater than that of PCE for all values of α and β except when α = β = 5 and for the sample sizes of n = 20 and m = 25 where the Bias of MLE are less than that of PCE for all values of α and β.

This implies that when n < m, MLE is more consistent especially when (n = 15, m = 20) and (n = 15, m =25) while PCE is more accurate and the reverse is the case for n = 20 and m = 25.

Finally, when n > m, the MSE of MLE are greater than that of PCE for all values of α and β while the Bias of PCE are greater than that of MLE for all values of α and β except when α = β = 5. This implies that when α = β = 5 and n > m, PCE is more accurate and consistent method than MLE but for other values of α and β, while PCE is more consistent method, the MLE is found to be more accurate.

CONCLUSION

From the simulation study varying the sample sizes of X and Y, and also their parameters, it can be seen that percentile estimator (PCE) performs better than maximum likelihood estimator (MLE).

Also for the estimation of P at various sample sizes and parameters, it could be observed that for equal sample sizes of Y and X, any of the two methods (PCE or MLE) can be used in the estimation of P. For different sample sizes of X and Y, the choice of method depends on the values of α and β as well as the sample sizes.

Table 2: Estimated P at Various Parameters and Sample Sizes of the Event X and Y.

(n, m)	(α, β)	MLE		PCE	
		Bias	MSE	Bias	MSE
(15, 15)	10, 5	0.1167957	0.006523692	0.1167957	0.006523692
	5, 5	0.1955344	0.008141382	0.1955344	0.008141382
	5, 4	0.07967759	0.007735612	0.07967759	0.007735612
(15, 20)	10, 5	0.011311148	0.005749762	0.007717002	0.005844612
	5, 5	0.1298151	0.007020515	0.1336340	0.007019172
	5, 4	0.03099487	0.006918377	0.02700857	0.006954553
(15, 25)	10,5	0.09024011	0.005253046	0.09693402	0.005387163
	5, 5	0.10353483	0.006581826	0.09696978	0.006583766
	5, 4	0.08953235	0.006380081	0.08324359	0.006442354
(20, 15)	10, 5	0.06456067	0.005930216	0.06533882	0.005834693
	5, 5	0.06857928	0.007071319	0.06814023	0.007062773
	5, 4	0.09645631	0.006863609	0.09926026	0.006831961
(20, 20)	10, 5	0.06278784	0.004881646	0.06278784	0.004881646
	5, 5	0.06958795	0.006051847	0.06958795	0.006051847
	5, 4	0.06624281	0.005948044	0.06624281	0.005948044
(20, 25)	10,5	0.1190284	0.004398252	0.1217414	0.004439343
	5, 5	0.02405198	0.005476637	0.02678137	0.005474480
	5, 4	0.07439832	0.005438287	0.07713031	0.005458199
(25, 15)	10, 5	0.1312163	0.005438786	0.1365312	0.005297927
	5, 5	0.1988514	0.006584906	0.1922517	0.006574581
	5, 4	0.06538640	0.006387461	0.06420752	0.006323484
(25, 25)	10, 5	0.08293753	0.003983456	0.08293753	0.003983456
	5, 5	0.02044413	0.004839022	0.02044413	0.004839022
	5, 4	0.08614946	0.004859625	0.08614946	0.004859625

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