Flow and Heat Transfer Analysis of a Second Grade Fluid with Newtonian Heating in the Presence of Elastic Deformation in a Porous Medium.

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ABSTRACT

The flow and heat transfer in a second grade fluid over a stretching sheet subjected to convective boundary conditions are investigated in the presence of transverse magnetic field, internal heat generation, viscous dissipation with elastic deformation in a porous medium. Similarity transformations have been used for the reduction of partial differential equations into the ordinary differential equations. Numerically these equations are solved by fourth-order Runge-Kutta method with shooting technique. The effect of local viscoelastic parameter (second grade parameter), porosity parameter, Prandtl number, thermal radiation, magnetic number, and viscous dissipation (in the presence and in the absence of elastic deformation) with convective surface boundary condition are discussed and shown through several graphs. Computations for local Nusselt number and coefficient skin-friction have been carried out. A comparison with previously published results in special case of the problem shows an excellent agreement.

(Keywords: Runge-Kutta method, flow, heat transfer, convective boundary conditions, transverse magnetic field, heat generation, viscous dissipation, elastic deformation, porous medium)

INTRODUCTION

The field of fluid flow through a porous medium has received the attention of many researchers, because of its applications to thermal engineering, geothermal system, crude oil extraction, and energy related engineering problems such as thermal insulation of buildings, recovery of petroleum products, packed bed reactors and sensible heat storage beds, etc.

By definition, a porous medium consists of a particle medium of very small size with pores between them. The existence of particle in the medium reduces the space available for fluid flow. In the porous medium, the resistance to fluid flow is mainly due to the quality of particles present (i.e., porosity of the medium). Clearly a various drag is imminent on the surface of the particles which led to the reduction in the velocity in porous medium. The permeability, which is inverse resistance to fluid flow, depends on the porosity and the surface area per unit volume of the particles within the medium.

The porosity is an isotropic property and hence the interstitial velocity is related to super facial velocity. Clearly the resistance to fluid flow through porous media is related to the density of particles present in the medium termed as porosity.

Porous materials such as sand, rock in the underground are saturated with water or some other fluids, such fluids flow through the porous medium due to local pressure gradients and transport energy from one region to another. Fast depletion of fossil fuel reserves led to the development of alternative new energy sources.

One of the most promising new energy sources is geothermal energy which is very clean and sustainable. The geothermal reservoir is believed to have formed due to the volcanic activities or tectonic movements. As a result, magmatic intrusions may occur at shallow depths in the earth’s crust. Fluid motion in a porous medium is governed by the conservation of mass, momentum and energy equation. In simulation of flow in the porous medium, the momentum equation is given in the form of Darcy's law. Derived empirically, this law indicates a linear relationship between the fluid velocity and the pressure gradient. Further, the Reynolds number is suitably modified for porous medium and the Darcy's law holds good only for laminar flow for which modified Reynold's number must have a value less than 2.

The study of an electrically conducting fluid, which influences many natural and man-made flows, has many applications in engineering problems such as Magnetohydrodynamics (MHD) generators, plasma studies, nuclear reactors, geothermal energy extraction and the boundary layer control in the field of aerodynamics.

Heat transfer is the fundamental part of thermal engineering. It is the transfer of heat energy due to spatial temperature difference. Heat transfer occurs when the spatial temperature difference is present within a system or between systems in thermal contact with each other. Several material properties like thermal conductivity, specific heat, density, fluid viscosity, etc., modulate the heat transfer between two regions or system at differing temperatures. Taken together, these properties serves to make the solution of many heat transfer problems and involved processes. In general, three physical mechanisms of heat transfer exist viz conduction, convection, and radiation.

Conduction is the mode of heat transfer in which energy transfer occur from a region of high temperature to that of low temperature by direct molecular interaction and the drift of electrons. Convection is the mode of heat transfer in which the heat energy is transferred by the actual movement of fluid particles due to density variation. Radiation is the mode of heat transfer in which the heat energy is transferred through electromagnetic radiation. Unlike for conduction and convection which require a medium, the mode of radiation heat transfer does not require any medium.

Among the various modes of heat transfer, the mode of heat transfer through convection has received considerable attention because of its applications in wide areas of thermal engineering field. Convection is the mode of heat transfer between a solid surface and the moving fluid in contact with it. The faster the fluid motion, greater the convective heat transfer. Convective heat transfer is three type's viz., forced convection if the fluid is forced to flow over the surface, natural or free convection if the fluid motion is caused by buoyancy forces that are induced by density differences due to variation of temperature in the fluid. The third type is known as the mixed convection heat transfer which takes place when forced and natural convection act together in a system. Under this process both pressure forces and buoyant forces act together. Donald, Joseph, Bushinsky and Saylor (1989) analyzed the mixed convection heat transfer in nuclear reactors and some aspects of electronic cooling.

Salman and Mulligan (1990) analyzed the transient, buoyancy-induced flow and heat transfer adjacent to a suddenly heated vertical wall embedded in a porous medium saturated with a non-Newtonian fluid. Results obtained shows that Nusselt number decrease continuously with time and heat transfer coefficient decrease with a decrease in the power law index. Chung Liu (2004) proposed analytical solution for the flow and heat transfer in a steady laminar boundary layer flow of an electrically conducting fluid of second grade subject to a transverse uniform magnetic field past a semi-infinite stretching sheet with power law surface temperature. Also the effect of viscous dissipation, internal heat generation, work done due to deformation and joule heating in the energy equation are considered.

It was reported that the velocity component increases with visco elastic parameter and decrease with magnetic parameter. Lester, Rudman and Metcalfe (2009) applied a novel spectral method to quantity asymptotic scalar transport within Newtonian and non-Newtonian fluid over the control parameter space of a chaotic flow. The result demonstrates the ability of chaotic advection to address difficult transport problems involving non-Newtonian and highly viscous fluid. Cimpean and Pop (2012) investigated the problem of the steady fully developed mixed convection flow in an inclined porous channel filled by three types of nano fluids like Cu- water, Al_2O_3- water and T_iO_3- water. The governing equations are solved by analytical method. It has been concluded that the nano fluid greatly increase the heat transfer, even for small additions of nano particles in the base water fluid. Postelnicu (2012) investigated the heat and mass transfer in boundary layer free convection over an inclined flat plate embedded in a fluid saturated porous medium in the presence of thermophoresis. Thus the governing equations are transformed into differential equation and solved numerically by using local non similarity method.

The effect of thermophoretic coefficient and thermophoresis parameter on thermophoretic velocity deposition, Nusselt number and concentration profiles have been analyzed for both Biot and cold wall conditions. Chung Liu (2005) analyzed the flow and heat transfer of a steady laminar boundary layer flow of electrically conducting fluid of second grade in a porous medium subject to a transverse uniform magnetic field past a semi-infinite stretching sheet with power law surface temperature (or) power low surface heat flux.

Mariano, Gallego, Lugo, Camacho, Canzonieri and Pineiro (2013) analyzed the thermal conductivity, rheological behavior and the high pressure density of several non-Newtonian ethylene glycol based SnO_2 nano fluid. It has been reported that the characteristics of ethylene glycol/SnO_2 nano fluid exhibit shear thinning and rheopecty under non-Newtonian conditions and the elastic behavior is dominant.

Tai and Char (2010) numerically investigated the combined laminar free convection flow with thermal radiation and mass transfer of non-Newtonian power law fluids along a vertical plate in a porous medium and analyzed the effect of the Dufour number, Soret number, and power law index and radiation parameters. Abel, Mahantesh, Nandeppanava, Sharanagouda and Malipatil (2010) studied the boundary layer flow and heat transfer characteristics of a second grade, non-Newtonian fluid through a porous medium.

The effect of viscous dissipation, non-uniform heat source on heat transfer are considered. It has been reported that suction parameter, second order fluid parameter, Prandtl number decrease the heat transfer whereas porous parameter increase the heat transfer in the boundary layer region. Further, it can be notices that the viscous dissipation of the fluid increases the wall temperature.

Eshetu and Shankar (2014) investigated the boundary layer flow of a nano fluid through a porous medium subjected to a magnetic field, thermal radiation, viscous dissipation and chemical reaction effects. The effects of porosity, thermal radiation, magnetic field, viscous dissipation and chemical reaction to the flow field were thoroughly explained for various values of the governing parameter. Olanrewaju (2012) examined the effects of internal heat generation on Hydromagnetic non-Darcy flow and heat transfer over a stretching sheet in the presence of thermal radiation and Ohmic dissipation.

The local skin-friction and local Nusselt number with internal heat generation and radiation parameters are reported graphically for various parametric conditions to show interesting aspect of the numerical solution. Makinde and Olanrewaju (2012) studied the combined effects of internal heat generation and buoyancy force on boundary layer flow over a vertical plate with convective surface boundary condition.

The effect of Prandtl number, local Biot number, the internal heat generation parameter and the local Grashof number on the velocity and temperature profiles are illustrated and interpreted in physical terms. An overshoot of fluid velocity within the boundary layer is observed due to combined effect of buoyancy force and internal heat generation, in addition, internal heat generation causes thickening of thermal boundary layer. Olanrewaju and Hayat (2014) discussed the effects of buoyancy and transpiration on the flow and heat transfer over a moving permeable surface in a parallel stream in the presence of radiation. The effects of Prandtl number, Eckert number, the local Grashof number, and the radiation parameter on the velocity and temperature profiles are illustrated and interpreted in physical terms.

Recently, Olanrewaju and Abbas (2014) considered the corrigendum to convection heat and mass transfer in a hydromagnetic flow of a second grade fluid in the presence of thermal radiation and thermal diffusion. The researcher pointed out several errors in Olajuwon (2011) work and numerical solutions of the problem were provided with interpretations of the physical parameters to give further insight into the problem. Gbadeyan, Idowu, Ogunsola, Agboola & Olanrewaju, (2011) studied heat and mass transfer for Soret and Dufour's effect on mixed convection boundary layer flow over a stretching vertical surface in a porous medium filled with a viscoelastic fluid in the presence of magnetic field. Olanrewaju, Gbadeyan, Hayat & Hendi, (2011) studied the effects of internal heat generation, thermal radiation and buoyancy force on a boundary layer over a vertical plate with a convective surface boundary condition.

Hayat, Shehzad, Qasim & Obaidat (2011) *investigated* Flow of a second grade fluid with convective boundary conditions. Abdul Majeed, Tahira & Zarqa (2013) studied Slip Effects on the Flow of a Second Grade Fluid in a Varying Width Channel with Application to Stenosed Artery. Singh and Agarwal (2012) examined heat transfer in a second grade fluid over an exponentially stretching sheet through porous Medium with thermal radiation and elastic Deformation under the effect of magnetic field. Baris (2003) discussed the flow of a Second-Grade Visco-Elastic Fluid in a Porous Converging Channel.

The second grade fluid model is the simplest subclass of viscoelastic fluid for which one can reasonably hope to obtain the analytic solution. Even though considerable progress has been made in our understanding of the flow phenomena, more works are needed to understand the effects of the various parameters involved in the non-Newtonian models and the formulation of an accurate method of analysis for anybody shapes of engineering significance. Also, the boundary layer concept for such fluids is of special importance because of its application to many practical problems, among which we cite the possibility of reducing frictional drag on the hulls of ships and submarines. Furthermore, thermal radiation effects and MHD flow problems have assumed an increasing importance at a fundamental fabrication level.

Specifically, such flows occurs in electrical power generation, astrophysical flows, solar power technology, space vehicle re-entry and other industrial areas. Related studies regarding the thermal radiation of a gray fluid have been made in the references. Hayat and Abbas (2008) examined the heat transfer analysis on the MHD flow of a second grade fluid in a channel with porous medium. Hayat, Abbas, Sajid & Asghar, (2007) studied the influence of thermal radiation on MHD flow of a second grade fluid. Hayat, Ahmed, Sajid, & Asghar, (2007) investigated the MHD flow of a second grade fluid in a porous channel. Olajuwon (2011) discussed the Convection heat and mass transfer in a hydromagnetic flow of a second grade fluid in the presence of thermal radiation and thermal diffusion. Islam, Bano, Haroon & Siddiqui, (2011) examined the unsteady poiseuille flow of second grade fluid in a tube of elliptical cross section. Hayat, Shehzad, Muhammad and Obaidat (2011) examined the flow and heat transfer in a second grade fluid over a stretching sheet subjected to convective boundary conditions are investigated.

Similarity transformations have been used for the reduction of partial differential equations into the ordinary differential. Singh and Agarwal (2012) examined the heat transfer in a second grade fluid over an exponentially stretching sheet through porous medium with thermal radiation and elastic deformation under the effect of magnetic field. The effect of local viscoelastic parameter, porosity parameter, Prandtl number, magnetic number, viscous dissipation are discussed.

The objective of this paper is to explore the effects of the thermophysical properties coupled with elastic deformation on the fluid flow analysis under a convective surface boundary condition. The non-linear equations governing the flow are solved numerically using shooting technique with Runge-Kutta of order six. Graphical results are reported first for emerging flow parameters and then discussed.

Mathematical Formulation of the Research Problem

A two dimensional laminar flow of an incompressible, electrically conduction MHD viscoelastic liquid due to stretching sheet through porous medium of permeability k_1 is considered with heat transfer when the fluid remains stationary. The sheet is stretched with a velocity $u_w(x)=bx$ where b is a real number. The constant mass transfer velocity is denoted by v_w *with* $v_w>0$ *for injection and* $v_w<0$ *for suction*.

We choose x-axis along the stretching surface and y-axis perpendicular to the x-axis. The present flow consideration of second grade fluid is governed by the following expressions:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+\frac{\alpha_1}{\rho}\left(\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}+u\frac{\partial^3 u}{\partial x\partial y^2}+\frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2}+v\frac{\partial^3 u}{\partial y^3}\right)-\sigma_1\frac{B_0^2}{\rho}u-\frac{\upsilon}{k_1}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2-\frac{\alpha}{\rho}\frac{\partial q_r}{\partial y}+\frac{Q}{\rho c_\rho}(T-T_\infty) -\sigma\frac{\alpha_1}{\rho}\left\{\frac{\partial u}{\partial y}\frac{\partial}{\partial y}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)\right\} \tag{3}$$

Where u and v denote the velocity components in the x- and y-direction, α_1 is the second grade parameter, σ is the coefficient of elastic deformation term, μ is the viscosity coefficient, υ is the kinematic viscosity, σ_1 is the electrical conductivity, ρ is the density of the fluid, α is the thermal diffusivity, c_ρ is the specific heat at constant temperature and T is the fluid temperature. The appropriate boundary conditions are considered in the following forms:

$$u=u_w(x)=bx,\quad v=v_w,\quad -k\frac{\partial T}{\partial y}=h(T-T_f)\quad at\ y=0$$
$$u=0,\ \frac{\partial u}{\partial y}=0, T=T_\infty\qquad at\ y\to\infty \tag{4}$$

Here k is the thermal conductivity of the fluid, h- the convective heat transfer coefficient, v_w- the wall heat transfer velocity, and T_f - the convective fluid temperature below the moving sheet.
The radiative heat flux q_r is described by Roseland approximation such that:

$$q_r=-\frac{4\sigma^*}{3K}\frac{\partial T^4}{\partial y}, \tag{5}$$

where σ^* *and* K are the Stefan-Boltzmann constant and the mean absorption coefficient, respectively. Following Olanrewaju and Abbas (2014), we assume that the temperature differences within the flow are sufficiently small so that the T^4 can be expressed as a linear function after using Taylor series to expand T^4 about the free stream temperature T_∞ and neglecting higher-order terms. This result is the following approximation:

$$T^4\approx 4T_\infty^3 T-3T_\infty^4. \tag{6}$$

Using (5) and (6) in (3), we obtain:

$$\frac{\partial q_r}{\partial y}=-\frac{16\sigma^*}{3K}\frac{\partial T^4}{\partial y}. \tag{7}$$

Introducing a similarity variable *η* and a dimensionless stream function *f(η)* and temperature *θ(η)* as:

$$u = axf'(\eta), \quad v = -\sqrt{a\upsilon}\, f(\eta), \quad \theta = \frac{T - T_\infty}{T_f - T_\infty}, \quad \eta = y\sqrt{\frac{a}{\upsilon}} \tag{8}$$

where prime symbol denotes differentiation with respect to *η* and $Re_x = U_\infty x/\upsilon$ is the local Reynolds number. Eqs. (1) – (8) reduce to:

$$f''' + ff'' - f'^2 + K\left(f'f''' - f''^2 - ff^{iv}\right) - \left(M_n + P\right)f' = 0 \tag{9}$$

$$(1+R)\theta'' + \Pr f\theta' + \Pr Ecf''^2 + \Pr\lambda\theta - \Pr EcK\delta_1\left\{f'f''^2 - ff''f'''\right\} = 0 \tag{10}$$

Satisfying the conditions:

$$\begin{aligned} &f = S, \quad f' = \frac{b}{a} = \alpha_2, \quad \theta' = -\gamma[1 - \theta(0)] \ at \ \eta = 0 \\ &f' = 0, \quad f'' = 0, \quad \theta = 0 \ at \ \eta \to \infty \end{aligned} \tag{11}$$

Where $K = \dfrac{\alpha_1 a}{\rho\upsilon}$ is the second grade parameter

$M_n = \dfrac{\sigma B_0^2}{\rho a}$ is the magnetic field parameter

$P = \dfrac{\upsilon}{k_1 a}$ is the porosity material parameter

$\Pr = \dfrac{\upsilon}{\alpha}$ is the Prandtl number

$Ec = \dfrac{\mu a x^2}{\upsilon(T_f - T_\infty)}$ is the Eckert number

$\delta_1 K = \dfrac{\sigma\alpha_1 a^2}{\rho\upsilon}$ is the product of the elastic deformation and the second grade parameters

$R = \dfrac{16\sigma_1 T_\infty^3}{3m\alpha}$ is the thermal radiation parameter

$\lambda = \dfrac{Q}{\rho c_p a}$ is the internal heat generation

Numerical Procedure

The set of non-linear ordinary differential equations (9) – (10) with boundary conditions (11) have been solved numerically by using the sixth-order Runge–Kutta integration scheme with a modified version of the Newton–Raphson shooting method with *the* thermophysical parameters. The computations were done by a program which use a symbolic and computational computer language MAPLE [15].

A step size of $\Delta\eta$ = 0.001 was selected to be satisfactory for a convergence criterion of 10^{-7} in nearly all cases. The value of y_∞ was found to each iteration loop by the assignment statement $\eta_\infty = \eta_\infty + \Delta\eta$. The maximum value of η_∞, to each group of flow embedded parameters is determined when the values of unknown boundary conditions at η = 0 not change to successful loop with error less than 10^{-7}.

RESULTS AND DISCUSSION

Here, we assigned physically realistic numerical values to the embedded parameters in the system in order to gain an insight into the flow structure with respect to velocity and temperature profiles. The results are presented graphically in Figures 1-14 and conclusions are drawn for the flow field and other physical quantity of interest that have significant effects. To test the validity of our results, so comparisons of the present results with previous works are performed and excellent agreement has been obtained.

In Table 1, Table 2 and Table 3 is discussed the effects of the flow embedded parameters on the local Nusselt number, skin-friction coefficient and the wall temperature. It is understood that from Table 2, as Biot number γ and Eckert number Ec increases, the heat transfer rate decreases and the wall temperature but has no effect on the coefficient of skin-friction parameter. Similarly, when the porosity parameter P and the magnetic field parameter M_n increases, the skin-friction coefficient and the heat transfer rate (the local Nusselt number) decreases. Increasing the thermal radiation parameter brings an increase in the rate of heat transfer with the internal heat generation parameter λ.

The elastic deformation parameter δ_1 has greater influence on the local Nusselt number. As this parameter increases, the heat transfer rate at the wall decreases. In table 3, when there is no elastic deformation (δ_1= 0), as second grade parameter increases, the rate of heat transfer at the wall surface increases with the skin –friction coefficient parameter while similar effect was recorded when the second grade parameter K was increase but no effect on the skin-friction coefficient parameter at the wall. Figures 1-14 show the influence of different embedded flow parameters $K, M_n, P, R, Ec, \lambda, \delta_1, S, \alpha_2, \Pr$ *and* γ on f' *and* θ. The effects of K on the velocity profile f' are shown in Figure 1.

The fluid velocity f' increases when there is an increase in K. It thickens the velocity flow boundary layer across the channel. Figure 2, is plotted for the variations in the magnetic field parameter M_n on f' and the velocity profile f' decreases across the channel.

Figure 3 shows the variation in the porosity material P on f'. As expected. f' decreases by increasing P.

The variation of thermal radiation parameter R was plotted in Figure 4 on θ and we discovered that as this parameter increases, the thermal boundary layer thickens across the vessel. Similar experiences occurred in Figures 5 and 6 on θ as the two parameters increases which lead to an increase in the temperature distribution across the channel used.

The variations of the elastic deformation δ_1 and suction parameter S on θ were plotted in Figures 7-9. It was observed that as this so called parameter increase, the thermal boundary layer thickness decreases across the channel.

Figure 10, depicts the variations of S on f'. As expected, as suction parameter increases the velocity profile decreases which establishes that suction step down flows across the vessel.

Table 1: Comparison Results for Local Nusselt Number $-\theta'(0)$ for Parameters γ, Pr, Ec, α_2 *and* $S = 0.5, K = 0.2, P = R = M_n = \lambda = 0, \delta_1 = 1$.

γ	Pr	α_2	Ec	$-\theta'(0)$ Hayat et al (2011)	$-\theta'(0)$ Present result
0.1 0.5 1.0 2.0	0.7	0.3	0.2	0.082916 0.250173 0.334452 0.402237	0.083016 0.250178 0.334552 0.410235
0.5	0.5 0.7 1.0 2.0	0.3	0.2	0.214365 0.250132 0.288561 0.356172	0.214368 0.250142 0.289161 0.356176
0.5	1.0	0.1 0.5 1.0 2.0	0.2	0.268673 0.356192 0.305945 0.301107	0.268673 0.356192 0.305945 0.301107
0.5	0.7	0.2	0.0 0.2 0.5	0.242896 0.242184 0.241265	0.242897 0.242188 0.241271

Table 2: Numerical Values of the Skin-Friction and the Local Nusselt Number for the Flow Parameters when S = 0.5, Pr = 0.7, $\alpha_2 = 0.5$ and K = 0.2

γ	Ec	P	M_n	R	λ	δ_1	$f''(0)$	$-\theta'(0)$	$\theta(0)$
0.1 0.5 1.0	0.2	0.2	0.5	0.5	0.5	0.2	-0.608319 -0.608319 -0.608319	1.977706 0.650734 0.484862	5.9999999 2.0000000 1.4999999
0.5	0.0 0.2 0.5	0.2	0.5	0.5	0.5	0.2	-0.608319 -0.608319 -0.608319	0.663486 0.650734 0.631605	1.9999999 2.0000000 1.9999999
0.5	0.2	0.2 0.7 1.0	0.5	0.5	0.5	0.2	-0.608319 -0.695179 -0.740933	0.650734 0.597612 0.574995	2.0000000 2.0000000 1.9999999
0.5	0.2	0.2	0.5 1.0 2.0	0.5	0.5	0.2	-0.608319 -0.695179 -0.835182	0.650734 0.597612 0.536262	2.0000000 2.0000000 1.9999999
0.5	0.2	0.2	0.5	0.5 1.0 2.0	0.5	0.2	-0.608319 -0.608319 -0.608319	0.650734 1.096538 6.935615	2.0000000 1.9999999 1.9999999
0.5	0.2	0.2	0.5	0.5	0.5 1.0 1.5	0.2	-0.608319 -0.608319 -0.608319	0.650734 6.973137 -0.06569	2.0000000 1.9999999 2.0000000
0.5	0.2	0.2	0.5	0.5	0.5	0.2 0.4 0.6	-0.608319 -0.608319 -0.608319	0.650734 -0.06505 -0.06441	2.0000000 1.9999999 1.9999999

Table 3: Influence of K and δ_1 on the Skin-Friction and the Heat transfer with Fixed Flow Parameters S = 0.5, Pr = 0.72, $\alpha_2 = 0.3$, γ = 0.1, Ec = 0.2, P = 0.1, Mn = 0.1, R = 0.5, λ = 0.2.

K	δ_1	$f''(0)$	$-\theta'(0)$	$\theta(0)$
0.1	0	-0.4564729	0.79586046	4.00000000
0.2	0	-0.4342021	0.81435071	4.00000000
0.3	0	-0.4157768	0.83101059	4.00000000
0.2	1	-0.4342021	0.81562355	4.00000000
0.2	2	-0.4342021	0.81689639	3.99999999
0.2	3	-0.4342021	0.81816924	4.00000000

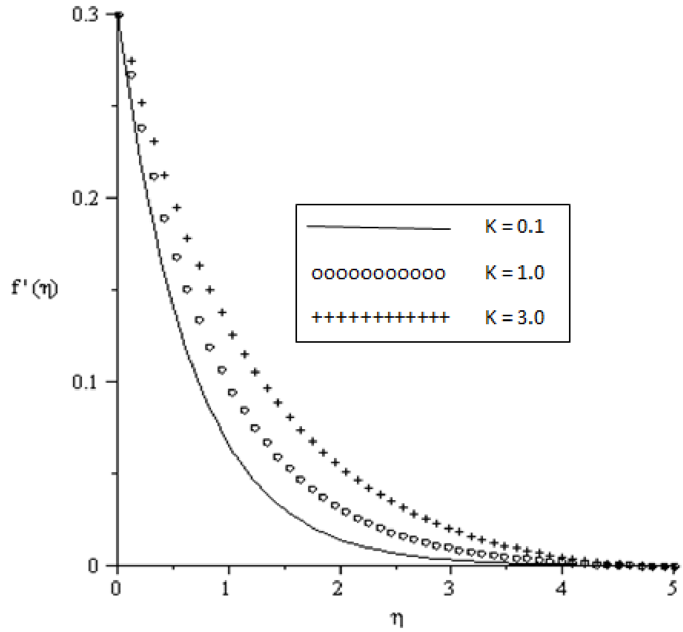

Figure 1: Influence of K on the Velocity Flow Distribution for Fixed Values of M_n =1, P=0.5, R=1, Ec = 0.2, $\lambda = 0.5$, $\delta_1 = 0.1$, S = 0.5, $\alpha_2 = 0.3$, Pr = 0.72 and $\gamma = 0.1$.

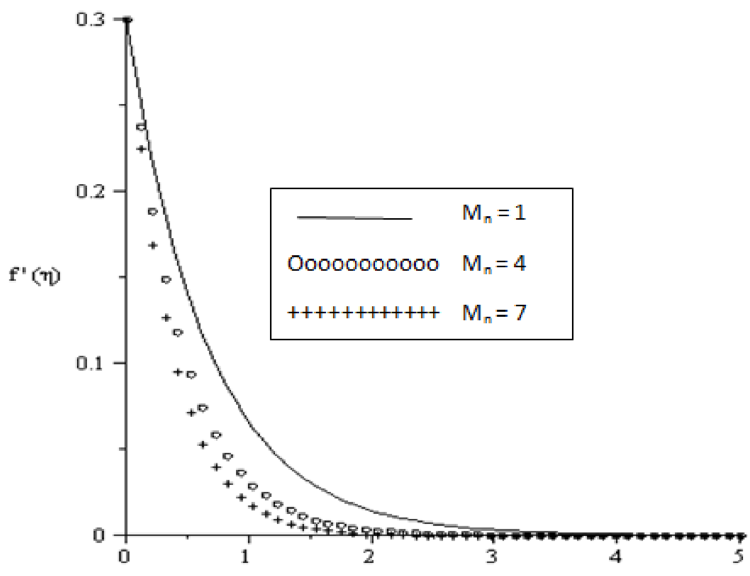

Figure 2: Influence of Mn on the Velocity Flow Distribution for Fixed Values of K =0.1, P=0.5, R=1, Ec = 0.2, $\lambda = 0.5$, $\delta_1 = 0.1$, S = 0.5, $\alpha_2 = 0.3$, Pr = 0.72 and $\gamma = 0.1$.

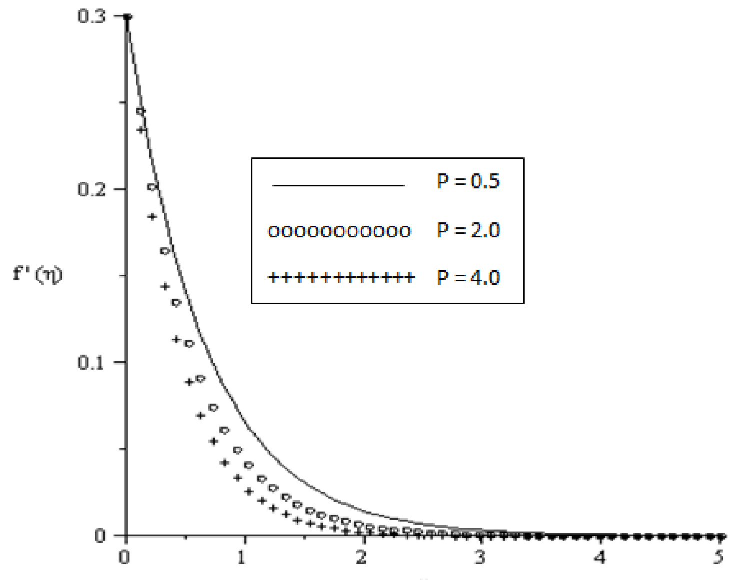

Figure 3: Influence of P on the Velocity Flow Distribution for Fixed Values of K =0.1, M_n=1, R=1, Ec = 0.2, $\lambda = 0.5$, $\delta_1 = 0.1$, S = 0.5, $\alpha_2 = 0.3$, Pr = 0.72 and $\gamma = 0.1$.

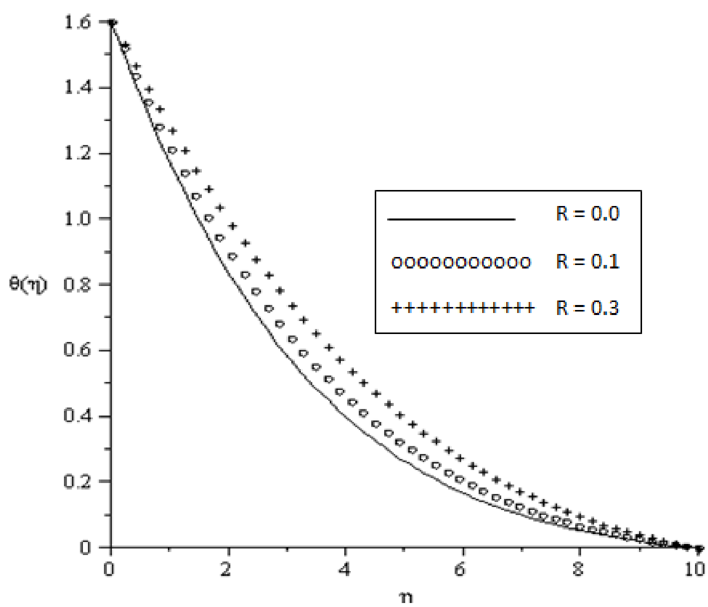

Figure 4: Influence of R on the Temperature Distribution for Fixed Values of K =0.1, M_n=1, P=0.5, Ec = 0.2, $\lambda = 0.1$, $\delta_1 = 0.1$, S = 0.5, $\alpha_2 = 0.3$, Pr = 0.72 and $\gamma = 0.5$.

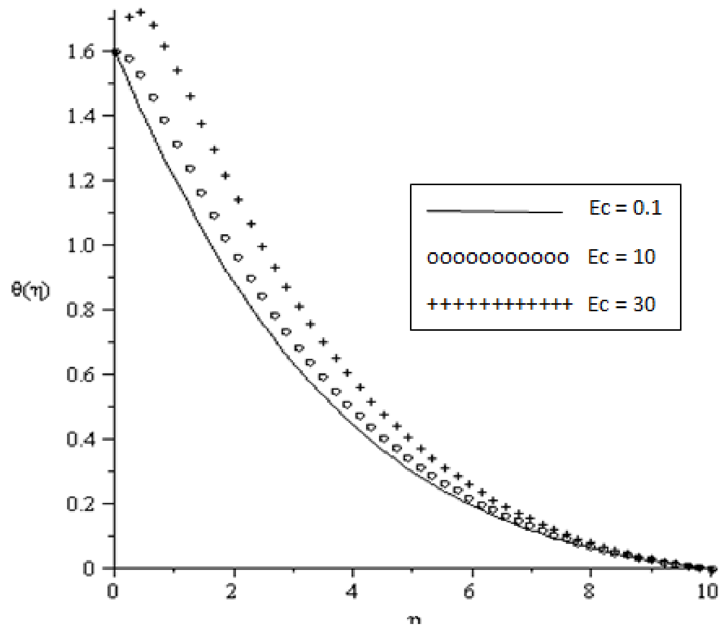

Figure 5: Influence of Ec on the Temperature Distribution for Fixed Values of K =0.1, M_n=1, P=0.5, R = 0.1, $\lambda = 0.1$, $\delta_1 = 0.1$, S = 0.5, $\alpha_2 = 0.3$, Pr = 0.72 and $\gamma = 0.5$.

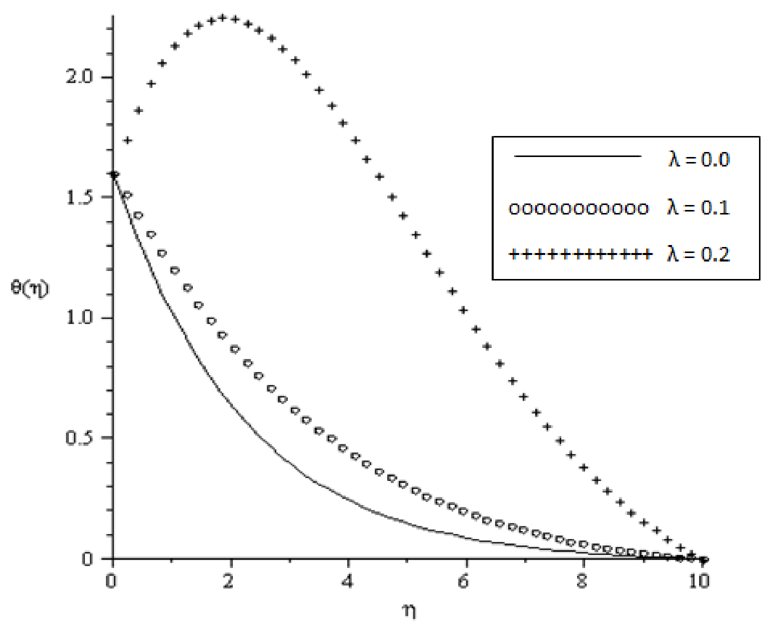

Figure 6: Influence of λ on the Temperature Distribution for Fixed Values of K =0.1, M_n=1, P=0.5, R = 0.1, $Ec = 0.2$, $\delta_1 = 0.1$, S = 0.5, $\alpha_2 = 0.3$, Pr = 0.72 and $\gamma = 0.5$.

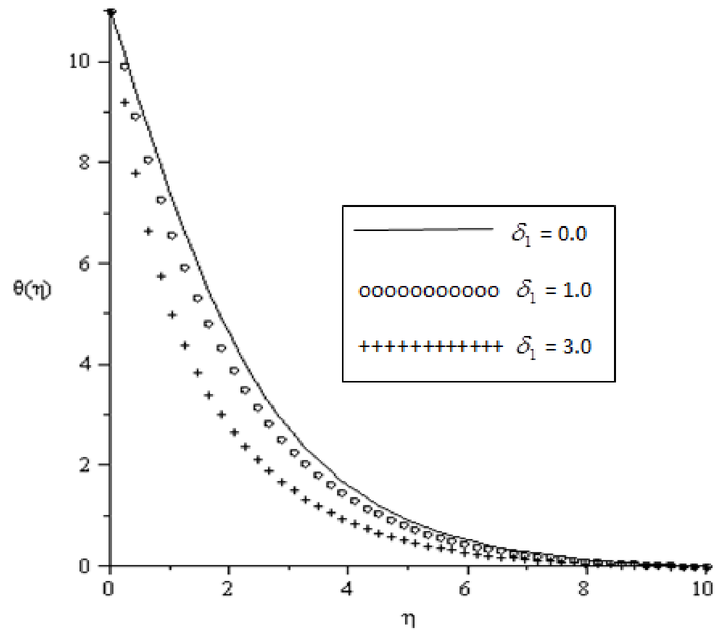

Figure 7: Influence of δ_1 on the Temperature Distribution for Fixed Values of K =2, M_n=1, P=0.5, R = 0.1, $Ec = 5$, $\lambda = 0.1$, S = 0.5, $\alpha_2 = 1$, Pr = 0.72 and $\gamma = 0.5$.

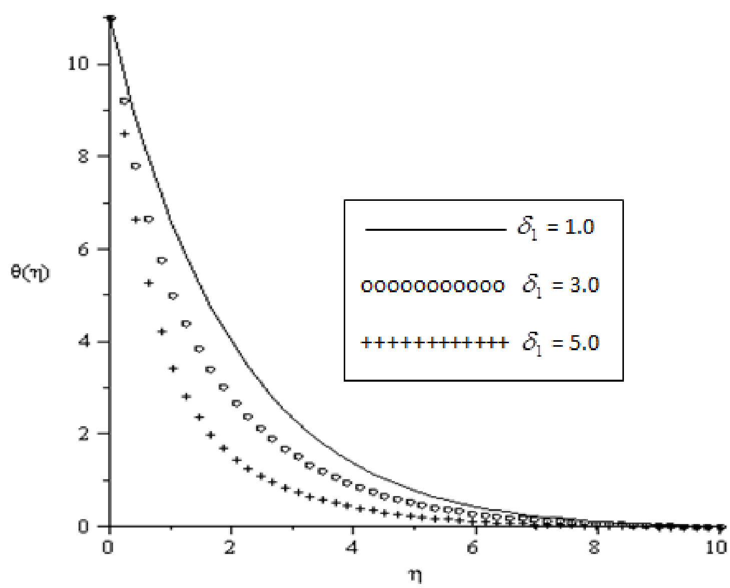

Figure 8: Influence of δ_1 on the Temperature Distribution for Fixed Values of K =2, M_n=1, P=0.5, R = 0.1, $Ec = 5$, $\lambda = 0.1$, S = 0.5, $\alpha_2 = 1$, Pr = 0.72 and $\gamma = 0.5$.

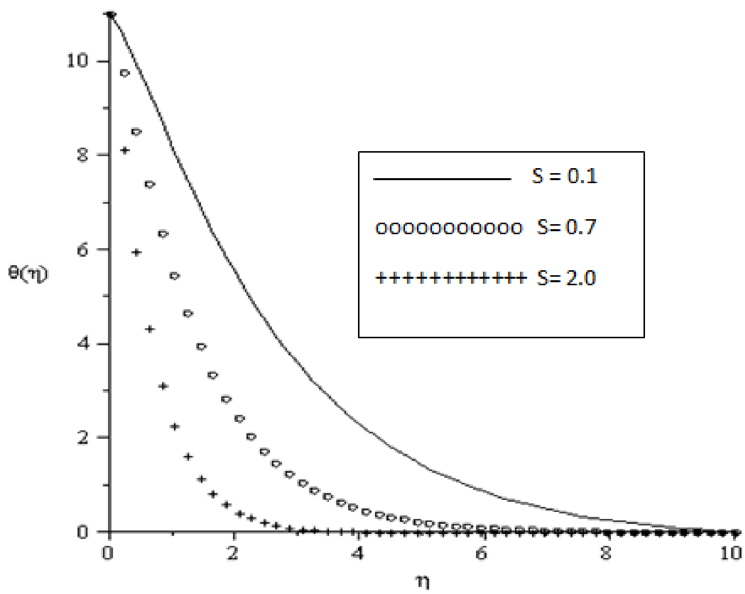

Figure 9: Influence of S on the Temperature Distribution for Fixed Values of K = 0.2, M_n=1, P=0.1, R = 0.1, $Ec = 5$, $\lambda = 0.1$, δ_1 = 1, $\alpha_2 = 1$, Pr = 0.72 and $\gamma = 0.5$.

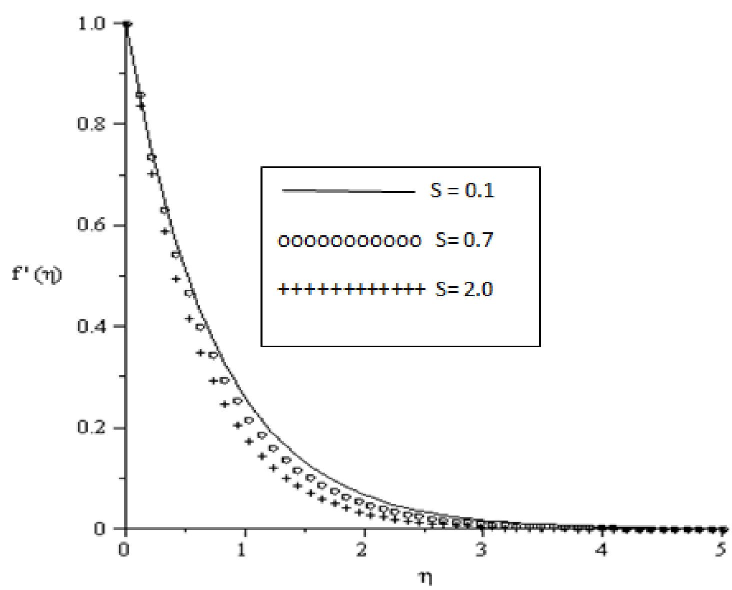

Figure 10: Influence of S on the Velocity flow Distribution for Fixed Values of K = 0.2, M_n=1, P=0.1, R = 0.1, $Ec = 5$, $\lambda = 0.1$, δ_1 = 1, $\alpha_2 = 1$, Pr = 0.72 and $\gamma = 0.5$.

Figure 11 represents the effects of α_2 on f'. As this parameter increases, the velocity profiles increases across the velocity boundary layer. Similar effects were established on the temperature distribution (Figure 12).

Figures 13 and 14 depict the variations in the Prandtl and Biot numbers on the thermal boundary layer thickness θ. As we increase these two important parameters, the thermal boundary layer thickness decreases across the channel which confirms the literature that both parameters decreases the temperature distribution.

CONCLUSION

An analytical study has been investigated on heat transfer in a second grade fluid over a stretching sheet under the effect of magnetic field with thermal radiation, internal heat generation and elastic deformation through porous medium. Non-linear partial differential equations are transformed in to ordinary differential equations by using similarity transformation.

The effect of various physical factors such as viscoelasticity, magnetic field, thermal radiation, porosity parameter, Prandtl number, Eckert number, second grade parameter, internal heat generation, and suction with convective surface boundary condition on the thermal behavior are examined and discussed in detail with the help of tables and graphs.

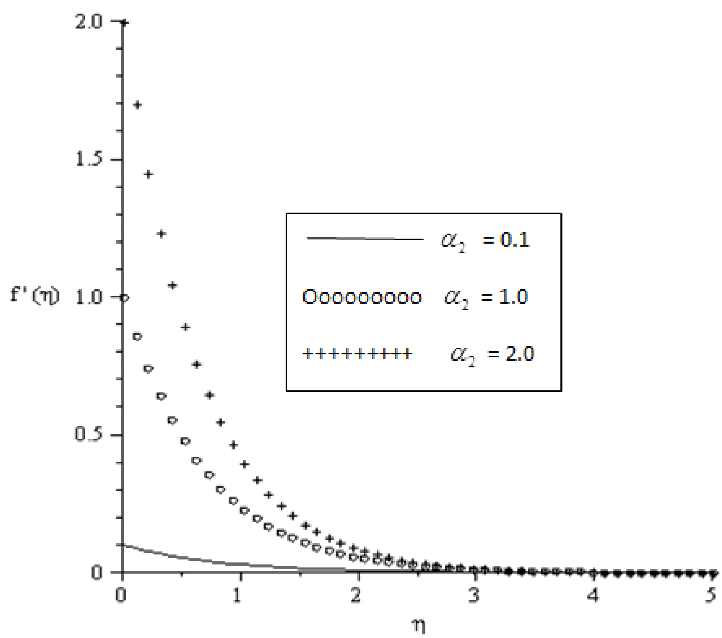

Figure 11: Influence of α_2 on the Velocity Flow Distribution for Fixed Values of K = 0.2, M_n=1, P=0.1, R = 0.1, $Ec = 5$, $\lambda = 0.1$, δ_1 = 1, S = 0.1, Pr = 0.72 and $\gamma = 0.5$.

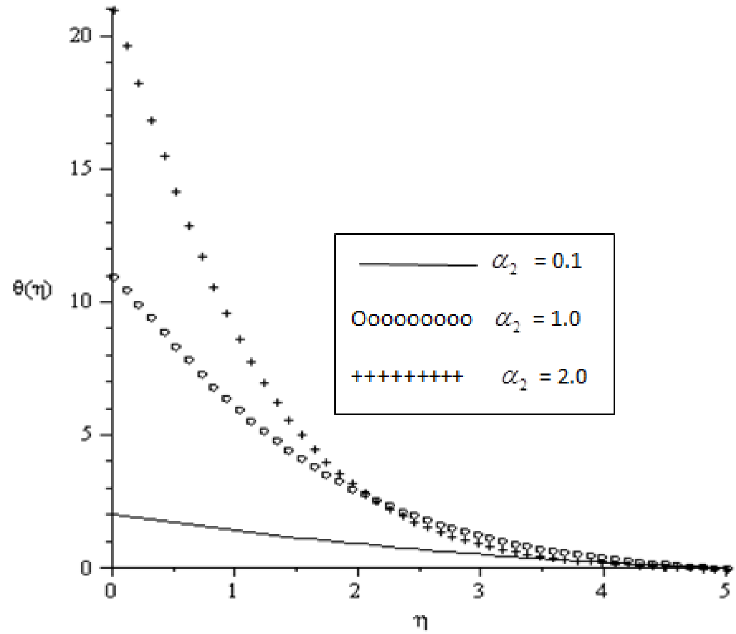

Figure 12: Influence of α_2 on the Temperature Distribution for Fixed Values of K = 0.2, M_n=1, P=0.1, R = 0.1, $Ec = 5$, $\lambda = 0.1$, δ_1 = 1, S = 0.1, Pr = 0.72 and $\gamma = 0.5$.

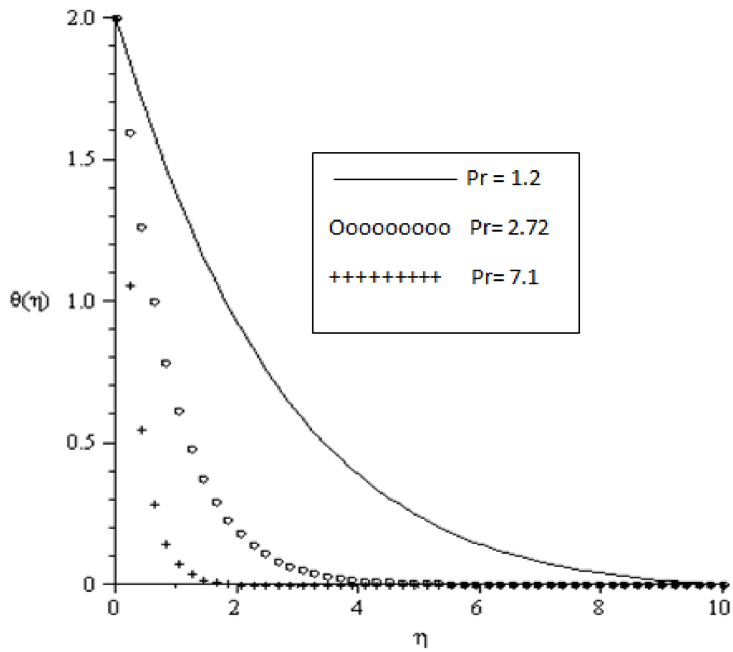

Figure 13: Influence of Pr on the Temperature Distribution for Fixed Values of K = 0.2, M_n=1, P=0.1, R = 0.1, $Ec = 5$, $\lambda = 0.1$, δ_1 = 1, $\alpha_2 = 1$, S = 0.1 and $\gamma = 0.5$.

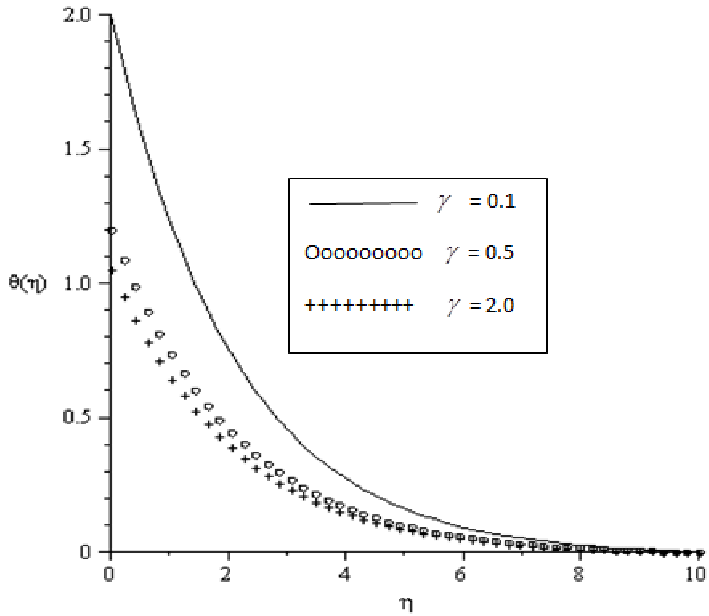

Figure 14: Influence of γ on the Temperature Distribution for Fixed Values of K = 0.2, M_n=1, P=0.1, R = 0.3, $Ec = 5$, $\lambda = 0.1$, δ_1 = 1, $\alpha_2 = 0.1$, S = 1 and P =0.72.

To check the validity of the numerical technique, comparison was made with the existing literature there was a perfect agreement. Some of the important findings drawn from the present analysis are listed as follow:

(1) The elastic deformation has greater influence on the thermal boundary layer thickness across the flow region. In fact, increasing the parameter lead to an increment in the temperature distribution across the region.

(2) The effect of increasing values of porosity parameter P, magnetic number Mn and the Eckert number Ec is to increase temperature distribution throughout the flow region.

(3) Increase the value of viscoelastic parameter (second grade fluid) K is seen to decrease the velocity profile across the flow region.

(4) The effects of α_2 on f'. As this parameter increases, the velocity profiles increases across the velocity boundary layer

(5) The effect of the Biot number is to step down the temperature distribution across the flow region.

(6) The values for the embedded flow parameters were to be well and carefully selected to give further insight and to have a stable/feasible solution within the boundary conditions.

(7) Increase the value of Prandtl number Pr is seen to decrease the thermal boundary layer thickness across the flow region.

(8) When there is no elastic deformation ($\delta_1 = 0$), the heat transfer rate at the wall is small but with the presence of elastic deformation, the heat transfer rate increases (see table 3). The higher the elastic deformation, the greater the rate of heat transfer at the wall surface.

REFERENCES

1. Gbadeyan, J.A., A.S. Idowu, A.W. Ogunsola, O.O. Agboola, and P.O. Olanrewaju. 2011. “Heat and Mass Transfer for Soret and Dufour’s Effect on Mixed Convection Boundary Layer Flow over a Stretching Vertical Surface in a Porous Medium Filled with a Viscoelastic Fluid in the Presence of Magnetic Field”. *Global Journal of Science Frontier Research*. 11(8):97-114.

2. Tasawar, H., S.A. Shehzad, Q. Muhammad, and O. Saleem. 2011. "Flow of a Second Grade Fluid with Convective Boundary Conditions". *Thermal Science*. 15(2):S253-S261.

3. Hayat, T. and Z. Abbas. 2008. "Heat Transfer Analysis on the MHD Flow of a Second Grade Fluid in a Channel with Porous Medium". *Chaos, Solitons and Fractals*. 38: 556–567.

4. Hayat, T., Z. Abbas, M. Sajid, and S. Asghar. 2007. "The Influence of Thermal Radiation on MHD Flow of a Second Grade Fluid". *International Journal of Heat and Mass Transfer*. 50:931–941.

5. Hayat, T., A. Naveed, M. Sajid, and S. Asghar. 2007. "On the MHD Fow of a Second Grade Fluid in a Porous Channel". *Computers and Mathematics with Applications*. 54:407–414.

6. Hayat, T., S. Asghar, and A.M. Siddiqui. 2000. "Some Non-Steady Flows of a Non-Newtonian Fluid". *Int. J. Eng. Sci*. 38:337–346.

7. Hossain, M.A., M.A. Alim, and D. Rees. 1999. "The Effect of Radiation on Free Convection from a Porous Vertical Plate". *Int. J. Heat Mass Transfer*. 42:181–191.

8. Hussain, M.A., K. Khanafer, and K.K. Vafai. 2001. "The Effects of Radiation on Free Convection Flow of Fluid with Variable Viscosity from a Porous Vertical Plate". *Int. J. Thermal Sci.* 40:115–124.

9. Ibrahim, F.S. 1990. "Mixed Convection-Radiation Interaction in Boundary Layer flow over Horizontal Surfaces". *Astrophys. Space Sci*. 168:263–276.

10. Islam, S., Z, Bano, T. Haroon, and A.M. Siddiqui. 2011. "Unsteady Poiseuille Flow of Second Grade Fluid in a tube of Elliptical Cross Section". *Proceedings of the Romanian Academy*. Series A.12(4):291–295.

11. Mansour, M.A. 1990. "Radiative and Free-Convection effects on the Oscillatory Flow Past a Vertical Plate". *Astrophys. Space Sci*. 166:269–275.

12. Mysels, K.J. 1949. "Flow of Thickened Fluids". US Patent No. 2: 173-492.

13. Olajuwon, B.I. 2011. "Convection Heat and Mass Transfer in a Hydromagnetic Flow of a Second Grade Fluid in the Presence of Thermal Radiation and Thermal Diffusion". *Int. J. Heat Mass Transf*. 38(3):377–382.

14. Olanrewaju, A.M. and O.D. Makinde. 2013. "On Boundary Layer Stagnation Point Flow of a Nanofluid over a Permeable Flat Surface with Newtonian Heating". *Chemical Engineering Communications*. 200:836–852.

15. Olanrewaju, P.O., J.A. Gbadeyan, A. Hayat, and T.A. Hendi. 2011. "Effects of Internal Heat Generation, Thermal Radiation and Buoyancy Force on a Boundary Layer over a Vertical Plate with a Convective Surface Boundary Condition". *S Afr J Sci.* 107:9 -10, Art. #476, 6 pages. doi:10.4102/sajs.v107i9/10.476.

16. Olanrewaju, P.O. and Z. Abbas. 2014. "Corrigendum to Convection Heat and Mass Transfer in a Hydromagnetic Flow of a Second Grade fluid in the Presence of Thermal Radiation and Thermal Diffusion". *Int. J. Heat Mass Transf*. 38(3):377–382.

17. Prandtl, L. 1904. "Verhandlugen des dritten Internationale Mathematiker". Kongresses, Heidelberg, Germany.

18. Rajagopal, K.R. 1982. "A Note on Unsteady Unidirectional Flows of a Non-Newtonian Fluid". *Int. J. Non-Linear Mech*. 17:369–373.

19. Rajagopal, K.R. 1984. "On the Creeping Flow of a Second Grade Fluid". *J. Non-Newtonian Fluid Mech*. 48:239–246.

20. Rajagopal, K.R. and P.N. Kaloni. 1989. *Continuum Mechanics and its Applications*. Hemisphere Press: Washington, DC.

21. Siddiqui, A.M., M.R. Mohyuddin, T. Hayat and S. Asghar. 2003. "Some more Inverse Solutions for Steady Flows of a Second Grade Fluid". *Arch. Mech*. 55(4):373–387.

22. Singh, V. and A. Shweta. 2012. "Heat Transfer in a Second Grade Fluid over an Exponentially Stretching Sheet through Porous Medium with Thermal Radiation and Elastic Deformation under the Effect of Magnetic Field". *Int. J. of Appl. Math and Mech*. 8(4):41-63.

23. Takhar, H.S., R. Gorla, and V.M. Soundalgekar. 1996. "Radiation Effects on MHD Free Convection Flow of a Gas Past a Semi-Infinite Vertical Plate". *Int. J. Numer. Meth. Heat Fluid Flow*. 6:77–83.

24. Tan, W.C. and T. Masuoka. 2005. "Stokes First Problem for Second Grade Fluid in a Porous Half Space". *Int. J. Non-Linear Mech*. 40:515–522.

25. Tan, W.C. and M.Y. Xu. 2002. "The Impulsive Motion of Flat Plate in Generalized Second Grade Fluid". *Mech. Res. Comm.* 29:3–9.

26. Tan, W.C. and M.Y. Xu. 2004. “Unsteady Flows of a Generalized Second Grade Fluid with the Fractional Derivative Model between Two Parallel Plates. *Acta Mech. Sin.* 20(5):471–476.

27. Toms, B.A. 1948. “Some Observations on the Flow of Linear Polymer Solutions through Straight Tubes at Large Reynolds Numbers”. *Proceeding of the First International Congress on Rheology*. 2. North Holland, Amsterdam, pp. 135–141.

28. Tulapurkara, E.G. 2005. “Hundred years of Boundary Layer”. *Sädahnä*. 30:499-5.

29. Walters, K. 1970. “Relation between Coleman-Nall, Rivlin-Ericksen, Green-Rivlin, and Oldroyd Fluids”. *ZAMP*. 21:592–600.

30. Olanrewaju, P.O. 2012. “Effects of Internal Heat Generation on Hydromagnetic Non-Darcy Flow and Heat Transfer over a Stretching Sheet in the Presence of Thermal Radiation and Ohmic Dissipation”. *World Applied Sciences Journal*. 16: 37-45.

31. Makinde, O.D. and P.O. Olanrewaju. 2012. “Combined Effects of Internal Heat Generation and Buoyancy Force on Boundary Layer Flow over a Vertical Plate with a Convective Surface Boundary Condition”. *The Canadian Journal of Chemical Engineering*. 90:1289-1294.

32. Olanrewaju, P.O. and T. Hayat. 2014. “The Effects of Buoyancy and Transpiration on the Flow and Heat Transfer over a Moving Permeable Surface in a Parallel Stream in the Presence of Radiation”. *International Journal for Computational Methods in Engineering Science and Mechanics.* 15(4):330-336.

33. Hayat, T., S.A. Shehzad, M. Qasim, and S. Obaidat. 2011. “Flow of a Second Grade Fluid with Convective Boundary Conditions”. *Thermal Science*. 15: 253-261.

34. Singh, V. and S. Agarwal. 2012. “Heat Transfer in a Second Grade Fluid over an Exponentially Stretching Sheet through Porous Medium with Thermal Radiation and Elastic Deformation under the Effect of Magnetic Field”. *Int. J. of Applied Math and Mech*. 8(4):41-63.

35. Serdar, B. 2003. “Flow of a Second-Grade Visco-Elastic Fluid in a Porous Converging Channel”. *Turkish J. Eng. Env. Sci*. 27:73 – 81.

36. Abel, M.S., M. Mahantesh, M, Nandeppanava, B. Sharanagouda, and B. Malipatil. 2010. “Heat Transfer in a Second Grade Fluid through a Porous Medium from a Permeable Stretching Sheet with Non-Uniform Heat Source/Sink”. *International Journal of Heat and Mass Transfer*. 53(9-10):1788-1795.

37. Liu, C.I. 2004. “Flow and Heat Transfer of an Electrically Conducting Fluid of Second Grade over a Stretching Sheet Subject to a Transverse Magnetic Field”. *International Journal of Heat and Mass Transfer*. 47(19-20): 4427-4437.

38. Liu, C.I. 2005. “Flow and Heat Transfer of an Electrically Conducting Fluid of Second Grade in a Porous Medium over a Stretching Sheet Subject to a Transverse Magnetic Field”. *International Journal of Non-Linear Mechanics*. 40(4):465-474.

39. Cimpean, D.S. and I. Pop. 2012. “Fully Developed Mixed Convection Flow of a Nano Fluid through an Inclined Channel Filled with a Porous Medium”. *International Journal of Heat and Mass Transfer*. 55(4):907-914.

40. Eshetu, H. and B. Shankar. 2014. “Heat and Mass Transfer through a Porous Media of MHD flow of Nano fluids with Thermal Radiation, Viscous Dissipation and Chemical Reaction Effects”. *American Chemical Science Journal*. 4(6):828-846.

41. Lester, D.R., M. Rudman, and G. Metcalfe. 2009. “Low Reynolds Number Scalar Transport Enhancement in Viscous and Non-Newtonian Fluids”. *International Journal of Heat and Mass Transfer*. 52(3-4):655-664.

42. Mariano, A., M.J.P. Gallego, L. Lugo, A. Camacho, S. Canzonieri, and M.M. Pineiro. 2013. “Thermal Conductivity, Rheological Behaviour and Density of Non-Newtonian Ethylene Glycol-Based SnO_2 Nano Fluids”. *Fluid Phase Equilibria*. 337:119-124.

43. Postelnieu, A. 2012. “Thermophoresis Particle Deposition in Natural Convection over Inclined Surface in Porous Media”. *International Journal of Heat and Mass Transfer*. 55(7-8):2087-2094.

44. Salman, H. and J.C. Mulligan. 1990. “Transient Free Convection from a Vertical Plate to a Non-Newtonian Fluid in a Porous Medium. *Journal of Non-Newtonian Fluid Mechanics*. 36:395-410.

45. Tai, B.C. and M.I. Char. 2010. “Soret and Dufour Effects on Free Convection Flow of Non-Newtonian Fluids Along a Vertical Plate Embedded in a Porous Medium with Thermal Radiation”. *International Communications in Heat and Mass Transfer*. 37(5):480-483.

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