A Modified Exponential Ratio-Type Estimator in Stratified Ranked Set Sampling.

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ABSTRACT

The combination of stratification and Ranked Set Sampling (RSS) give an advantage in obtaining an unbiased estimator for population parameters with some significant gain in efficiency. This paper presents the modified exponential estimators of population mean with known co-efficient of Variation and Co-efficient of Kurtosis of auxiliary variable in Stratified Ranked Set Sampling (SRSS) of a finite population. The bias and mean square error of the proposed estimators with large sample approximation were derived. It was shown that the proposed estimators are more efficient than the Stratified Simple Random Sampling estimators.

(Keywords: stratified ranked set sampling, exponential ratio type estimators, co-efficient of variation, co-efficient of kurtosis and efficiency)

INTRODUCTION

Ranked Set Sampling was first proposed by McIntyre (1952) to increase the accuracy of crop yield estimates without increasing the number of observations that need to be quantified. It is a method of collecting data that improves estimation by utilizing the sampler's judgement or auxiliary information about the relative sizes of the sampling units.

Dell and Clutters (1972) showed that, for comparable sample sizes, the RSS procedure results in more accurate parameter estimators than Simple Random Sampling (SRS). This means that RSS needs smaller measured observations than SRS to attain the same level of precision. Samawi (1996), introduced Stratified Ranked Set Sampling (SRSS) in order to increase the efficiency of the estimator of the population mean and several works have been done on SRSS since then. Mandowara and Mehta (2014) used the idea of SRSS to improve the precision of ratio estimators given by Kadilar and Cingi (2003). In this work, we propose estimators based on the modified exponential estimators using RSS.

The usual exponential estimator given by Bahl and Tuteja (1991) for the population mean in Stratified random sampling is:

$$t = \bar{y}_{st} \exp\left(\frac{\bar{X}_{st} - \bar{x}_{st}}{\bar{X}_{st} + \bar{x}_{st}} \right) \quad (1)$$

Where $\bar{y}_{st} = \sum_{h=1}^{L} W_h \bar{y}_h$ and $\bar{x}_{st} = \sum_{h=1}^{L} W_h \bar{x}_h$ are the unbiased estimators of population means. Here $W_h = \frac{N_h}{N}$ is the weight of h stratum where N_h is the h^{th} stratum size and N is the total population size (h = 1,2,3,…….L) and L is the total number of strata in the population.

When the population co-efficient of variation is known and motivated by Bahl and Tuteja (1991), Singh(2008) suggested a modified exponential estimator for $\bar{Y}$ in stratified random sampling as:

$$t_1 = \bar{y}_{st} \exp\left[\frac{\sum_{i=1}^{L} W_h\left(\bar{X}_h + C_{xh}\right) - \sum_{i=1}^{L} W_h\left(\bar{x}_h + C_{xh}\right)}{\sum_{i=1}^{L} W_h\left(\bar{X}_h + C_{xh}\right) + \sum_{i=1}^{L} W_h\left(\bar{x}_h + C_{xh}\right)} \right] \quad (2)$$

When the population co-efficient of kurtosis is known and motivated by Bahl and Tuteja (1991), Singh (2008) suggested a modified exponential estimator for $\bar{Y}$ in stratified random sampling as:

$$t_2 = \bar{y}_{st} \exp\left[\frac{\sum_{i=1}^{L} W_h\left(\bar{X}_h + B_{xh}\right) - \sum_{i=1}^{L} W_h\left(\bar{x}_h + B_{xh}\right)}{\sum_{i=1}^{L} W_h\left(\bar{X}_h + B_{xh}\right) + \sum_{i=1}^{L} W_h\left(\bar{x}_h + B_{xh}\right)}\right] \quad \textbf{(3)}$$

When both the population co-efficient of variation and co-efficient of kurtosis are known and motivated by Bahl and Tuteja (1991), Singh (2008) suggested the following modified exponential estimators for $\bar{Y}$ in stratified random sampling as:

$$t_3 = \bar{y}_{st} \exp\left[\frac{\sum_{i=1}^{L} W_h\left(\bar{X}_h C_{xh} + B_{xh}\right) - \sum_{i=1}^{L} W_h\left(\bar{x}_h C_{xh} + B_{xh}\right)}{\sum_{i=1}^{L} W_h\left(\bar{X}_h C_{xh} + B_{xh}\right) + \sum_{i=1}^{L} W_h\left(\bar{x}_h C_{xh} + B_{xh}\right)}\right] \quad \textbf{(4)}$$

$$t_4 = \bar{y}_{st} \exp\left[\frac{\sum_{i=1}^{L} W_h\left(\bar{X}_h B_{xh} + C_{xh}\right) - \sum_{i=1}^{L} W_h\left(\bar{x}_h B_{xh} + C_{xh}\right)}{\sum_{i=1}^{L} W_h\left(\bar{X}_h B_{xh} + C_{xh}\right) + \sum_{i=1}^{L} W_h\left(\bar{x}_h B_{xh} + C_{xh}\right)}\right] \quad \textbf{(5)}$$

The mean square errors (MSE) of the estimators to the first degree are:

$$MSE(t_1) = \sum_{h=1}^{L} \frac{W_h^2}{n_h}\left[S_{y_h}^2 + \frac{\lambda_1^2}{4} R^2 S_{x_h}^2 - \lambda_1 R S_{x_h y_h}\right] \quad \textbf{(6)}$$

$$MSE(t_2) = \sum_{h=1}^{L} \frac{W_h^2}{n_h}\left[S_{y_h}^2 + \frac{\lambda_2^2}{4} R^2 S_{x_h}^2 - \lambda_2 R S_{x_h y_h}\right] \quad \textbf{(7)}$$

$$MSE(t_3) = \sum_{h=1}^{L} \frac{W_h^2}{n_h}\left[S_{y_h}^2 + \frac{\lambda_3^2}{4} R^2 S_{x_h}^2 - \lambda_3 R S_{x_h y_h}\right] \quad \textbf{(8)}$$

$$MSE(t_4) = \sum_{h=1}^{L} \frac{W_h^2}{n_h}\left[S_{y_h}^2 + \frac{\lambda_4^2}{4} R^2 S_{x_h}^2 - \lambda_4 R S_{x_h y_h}\right] \quad \textbf{(9)}$$

Where

$$\lambda_1 = \frac{\sum_{h=1}^{L} W_h \bar{X}_h}{\sum_{h=1}^{L} W_h\left(\bar{X}_h + C_{xh}\right)},$$

$$\lambda_2 = \frac{\sum_{h=1}^{L} W_h \bar{X}_h}{\sum_{h=1}^{L} W_h\left(\bar{X}_h + B_{2h}(x)\right)},$$

$$\lambda_3 = \frac{\sum_{h=1}^{L} W_h \bar{X}_h B_{2h}(x)}{\sum_{h=1}^{L} W_h\left(\bar{X}_h B_{2h}(x) + C_{xh}\right)}$$

$$\lambda_4 = \frac{\sum_{h=1}^{L} W_h \bar{X}_h C_{xh}}{\sum_{h=1}^{L} W_h\left(\bar{X}_h C_{xh} + B_{2h}(x)\right)}$$

Samawi (1996) introduced the stratified ranked set sampling in a bid to make the population mean estimator more efficient. After some period of time, Samawi and Siam (2003) gave the performance of the combined and separate ratio estimates using the stratified ranked set sampling.

Motivated by estimators given Kadilar and Cingi (2003) that incorporation of more and more parameters on auxiliary variable help increase the efficiencies of estimators, Mandowara and Mehta (2014), propose ratio type estimator for $\bar{Y}$ using stratified ranked set sampling, when the population coefficient of variation C_{xh} and co-efficient of kurtosis B_{xh} of auxiliary variable are known from stratum to stratum *(h=1,2,…,L)* and it is given as:

$$\bar{y}_{rj} = \bar{y}_{[SRSS]}\left[\frac{\sum_{i=1}^{L} W_h\left(\bar{X}_h a + b\right)}{\sum_{i=1}^{L} W_h\left(\bar{x}_{h(r_h)} a + b\right)}\right] \qquad \textbf{(10)}$$

The Bias and MSE of the estimators $\bar{y}_{rj}$ to the first degree of approximation are respectively, given by:

$$B\left(\bar{y}_{rj}\right) = \bar{Y}\left[\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\lambda_j^2\frac{S_{x_h}^2}{\bar{X}^2} - \lambda_j\frac{S_{x_h y_h}^2}{\bar{X}\bar{Y}}\right\} - \sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{m}{n_h}\left(\lambda_j^2\sum_{i=1}^{r_h} D_{x_h(i)}^2 - \lambda_j\sum_{i=1}^{r_h} D_{x_h(i)y_h[i]}\right)\right\}\right] \qquad \textbf{(11)}$$

$$MSE\left(\bar{y}_{rj}\right) = \sum_{h=1}^{L}\frac{W_h^2}{n_h}\left[S_{y_h}^2 + \lambda_j^2 R^2 S_{x_h}^2 - 2\lambda_j R S_{x_h y_h} - \bar{Y}^2\frac{m}{n_h}\sum_{i=1}^{r_h}\left(D_{y_h[i]} - \lambda_j D_{x_h(i)}\right)^2\right] \qquad \textbf{(12)}$$

for j = 1, 2, 3, 4

where $n_h = mr_h$, $\bar{y}_{[SRSS]} = \sum_{h=1}^{L} W_h \bar{y}_{h[r_h]}$, $\bar{x}_{[SRSS]} = \sum_{h=1}^{L} W_h \bar{x}_{h(r_h)}$, $D_{y_h\{i\}}^2 = \frac{\tau_{y_h[i]}^2}{\bar{Y}^2}$, $D_{x_h\{i\}}^2 = \frac{\tau_{x_h(i)}^2}{\bar{X}^2}$ and $D_{x_h(i)y_h\{i\}} = \frac{\tau_{x_h(i)y_h[i]}}{\bar{X}\bar{Y}}$, it should also be noted that $\tau_{y_h[i]} = \mu_{y_h[i]} - \bar{Y}_h$, $\tau_{x_h[i]} = \mu_{x_h(i)} - \bar{X}_h$ and $\tau_{x_h(i)y_h[i]} = \left(\mu_{x_h(i)} - \bar{X}\right)\left(\mu_{y_h[i]} - \bar{Y}_h\right)$ where $\mu_{x_h(i)} = E[x_{h(i)}]$, $\mu_{y_h(i)} = E[y_{h(i)}]$, $\bar{X}_h$ and $\bar{Y}_h$ are the means of the h^{th} stratum for the variables *X* and *Y*, respectively.

MATERIALS AND METHODS

Motivated by estimators in Equations (1) to (5) which show the incorporation of more and more parameters on auxiliary variable to give more efficient estimators including Mandowara and Mehta (2014).This work proposes exponential ratio type estimators for population mean $\bar{Y}$ using stratified ranked set sampling. When the population coefficient of variation C_{xh} of auxiliary variable from stratum to stratum *(h=1,2,…,L)* is known, we proposed as follow:

$$t_{r1} = \bar{y}_{[SRSS]}\exp\left[\frac{\sum_{i=1}^{L} W_h\left(\bar{X}_h + C_{xh}\right) - \sum_{i=1}^{L} W_h\left(\bar{x}_{h(r_h)} + C_{xh}\right)}{\sum_{i=1}^{L} W_h\left(\bar{X}_h + C_{xh}\right) + \sum_{i=1}^{L} W_h\left(\bar{x}_{h(r_h)} + C_{xh}\right)}\right] \qquad \textbf{(13)}$$

Where $\bar{y}_{[SRSS]} = \sum_{h=1}^{L} W_h \bar{y}_{h[r_h]}$,

$\bar{x}_{[SRSS]} = \sum_{h=1}^{L} W_h \bar{x}_{h[r_h]}$ are the stratified ranked set sample means for variables respectively.

Obtaining bias and MSE of t_{r1} let $\bar{y}_{[SRSS]} = \bar{Y}\left(1 + \delta_0\right)$ and $\bar{x}_{[SRSS]} = \bar{X}\left(1 + \delta_1\right)$ so that $E\left(\delta_1\right) = E\left(\delta_0\right) = 0$. Now:

$$V(\delta) = E\left(\delta_0^2\right) = \frac{V\left(\bar{y}_{[SRSS]}\right)}{\bar{Y}^2} = \sum_{h=1}^{L} W_h^2 \tfrac{1}{mr_h}\tfrac{1}{\bar{Y}^2}\left[S_{y_h}^2 - \tfrac{m}{n_h}\sum_{i=1}^{r_h}\tau_{y_h[i]}^2\right]$$

$$= \sum_{h=1}^{L}\frac{W_h^2}{n_h}\left[\frac{S_{y_h}^2}{\bar{Y}^2} - \frac{m}{n_h}\sum_{i=1}^{r_h} D_{y_h[i]}^2\right]$$

Similarly,

$$E\left(\delta_1^2\right)=\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left[\frac{S_{y_h}^2}{\bar{X}^2}-\frac{m}{n_h}\sum_{i=1}^{r_h}D_{x_h(i)}^2\right] \text{and}$$

$$E\left(\delta_o\delta_1\right)=\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left[\frac{S_{x_hy_h}}{\bar{X}\bar{Y}}-\frac{m}{n_h}\sum_{i=1}^{r_h}D_{x_h(i)y_h[i]}^2\right]$$

Further to validate first degree of approximation, we assume that the sample is large enough to get $\left[\delta_0\right]$ and $\left[\delta_1\right]$ so small that the terms involving δ_0 and δ_1 with degree greater than two will be negligible.

The Bias and MSE of the estimator t_{r1} to the first degree of approximation are respectively, given by:

$$\left(t_{r1}-\bar{Y}\right)=\bar{Y}\left(\delta_0-\tfrac{1}{2}\lambda_1\delta_0\delta_1+\tfrac{3}{8}\lambda_1^2\delta_1^2\right)$$

$$\left(t_{r1}-\bar{Y}\right)^2=\bar{Y}^2\left(\delta_0^2-\tfrac{1}{4}\lambda_1^2\delta_1^2+\lambda_1\delta_0\delta_1\right)$$

$$B\left(t_{r1}\right)=E\left(t_{r1}-\bar{Y}\right)=\bar{Y}\left(E\left(\delta_0\right)-\tfrac{1}{2}\lambda_1E\left(\delta_0\delta_1\right)+\tfrac{3}{8}\lambda_1^2E\left(\delta_1^2\right)\right)$$

$$B\left(t_{r1}\right)=\bar{Y}\left[\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{3\lambda_1^2}{8}\frac{S_{x_h}^2}{\bar{X}^2}-\frac{\lambda_1}{2}\frac{S_{x_hy_h}^2}{\bar{X}\bar{Y}}\right\}\right.$$
$$\left.-\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{m}{n_h}\left(\frac{3\lambda_1^2}{8}\sum_{i=1}^{r_h}D_{x_h(i)}^2-\frac{\lambda_1}{2}\sum_{i=1}^{r_h}D_{x_h(i)y_h[i]}^2\right)\right\}\right] \quad \textbf{(14)}$$

$$MSE\left(t_{r1}\right)=E\left(t_{r1}-\bar{Y}\right)^2=\bar{Y}^2\left(E\left(\delta_0^2\right)+\tfrac{1}{4}\lambda_1^2E\left(\delta_1^2\right)+\lambda_1E\left(\delta_0\delta_1\right)\right)$$

Since $E\left(\delta_1\right)=E\left(\delta_0\right)=0$

$$MSE\left(t_{r1}\right)=\bar{Y}\left[\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{S_{y_h}^2}{\bar{Y}^2}-\frac{m}{n_h}\sum_{i=1}^{r_h}D_{y_h[i]}^2\right\}-\frac{\lambda_1^2}{4}\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{m}{n_h}\left(\frac{S_{x_h}^2}{\bar{X}^2}-\frac{m}{n_h}\sum_{i=1}^{r_h}D_{x_h(i)}^2\right)\right\}\right.$$
$$\left.-\lambda_1\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left(\frac{S_{x_hy_h}}{\bar{X}\bar{Y}}-\frac{m}{n_h}\sum_{i=1}^{r_h}D_{x_h(i)y_h[i]}\right)\right]$$

$$MSE\left(t_{r1}\right)=\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left[S_{y_h}^2+\frac{\lambda_1^2}{4}R^2S_{x_h}^2-\lambda_1RS_{x_hy_h}-\bar{Y}^2\frac{m}{n_h}\sum_{i=1}^{r_h}\left(D_{y_h[i]}-\frac{\lambda_1}{2}D_{x_h(i)}\right)^2\right] \quad \textbf{(15)}$$

Adapting the estimators in Equation (3) given by Singh et al. (2008), a new exponential ratio type estimator in stratified ranked set sampling is being proposed as follows:

$$t_{r2}=\bar{y}_{[SRSS]}\exp\left[\frac{\sum_{i=1}^{L}W_h\left(\bar{X}_h+B_{2h}(x)\right)-\sum_{i=1}^{L}W_h\left(\bar{x}_{h(r_h)}+B_{2h}(x)\right)}{\sum_{i=1}^{L}W_h\left(\bar{X}_h+B_{2h}(x)\right)+\sum_{i=1}^{L}W_h\left(\bar{x}_{h(r_h)}+B_{2h}(x)\right)}\right] \quad \textbf{(16)}$$

Obtaining bias and MSE of t_{r3} let $\bar{y}_{[SRSS]}=\bar{Y}\left(1+\delta_0\right)$ and $\bar{x}_{[SRSS]}=\bar{X}\left(1+\delta_1\right)$ so that $E\left(\delta_1\right)=E\left(\delta_0\right)=0$

The Bias and MSE of the estimator t_{r2} to the first degree of approximation are respectively, given by:

$$\left(t_{r2}-\bar{Y}\right)=\bar{Y}\left(\delta_0-\tfrac{1}{2}\lambda_2\delta_0\delta_1+\tfrac{3}{8}\lambda_2^2\delta_1^2\right)$$
$$\left(t_{r2}-\bar{Y}\right)^2=\bar{Y}^2\left(\delta_0^2-\tfrac{1}{4}\lambda_2^2\delta_1^2+\lambda_2\delta_0\delta_1\right)$$
$$B\left(t_{r2}\right)=E\left(t_{r1}-\bar{Y}\right)=\bar{Y}\left(E\left(\delta_0\right)-\tfrac{1}{2}\lambda_2E\left(\delta_0\delta_1\right)+\tfrac{3}{8}\lambda_2^2E\left(\delta_1^2\right)\right)$$

$$B(t_{r2})=\bar{Y}\left[\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{3\lambda_2^2}{8}\frac{S_{x_h}^2}{\bar{X}^2}-\frac{\lambda_2}{2}\frac{S_{x_hy_h}^2}{\bar{X}\bar{Y}}\right\}-\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{m}{n_h}\left(\frac{3\lambda_2^2}{8}\sum_{i=1}^{r_h}D_{x_h(i)}^2-\frac{\lambda_2}{2}\sum_{i=1}^{r_h}D_{x_h(i)y_h[i]}^2\right)\right\}\right] \quad \textbf{(17)}$$

$$MSE(t_{r2})=E(t_{r2}-\bar{Y})^2=\bar{Y}^2\left(E(\delta_0^2)+\tfrac{1}{4}\lambda_2^2E(\delta_1^2)+\lambda_2E(\delta_0\delta_1)\right)$$

Since $E(\delta_1)=E(\delta_0)=0$

$$MSE(t_{r2})=\bar{Y}\left[\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{S_{y_h}^2}{\bar{Y}^2}-\frac{m}{n_h}\sum_{i=1}^{r_h}D_{y_h[i]}^2\right\}-\frac{\lambda_2^2}{4}\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{m}{n_h}\left(\frac{S_{x_h}^2}{\bar{X}^2}-\frac{m}{n_h}\sum_{i=1}^{r_h}D_{x_h(i)}^2\right)\right\}\right.$$
$$\left.-\lambda_2\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left(\frac{S_{x_hy_h}}{\bar{X}\bar{Y}}-\frac{m}{n_h}\sum_{i=1}^{r_h}D_{x_h(i)y_h[i]}\right)\right]$$

$$MSE(t_{r2})=\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left[S_{y_h}^2+\frac{\lambda_2^2}{4}R^2S_{x_h}^2-\lambda_2RS_{x_hy_h}-\bar{Y}^2\frac{m}{n_h}\sum_{i=1}^{r_h}\left(D_{y_h[i]}-\frac{\lambda_2}{2}D_{x_h(i)}\right)^2\right] \quad \textbf{(18)}$$

Adapting the estimators in (1.4) given by Singh et al. (2008), another new exponential ratio type estimator in stratified ranked set sampling is being proposed as follows:

$$t_{r3}=\bar{y}_{[SRSS]}\exp\left[\frac{\sum_{i=1}^{L}W_h\left(\bar{X}_hB_{2h}(x)+C_{xh}\right)-\sum_{i=1}^{L}W_h\left(\bar{x}_{h(r_h)}B_{2h}(x)+C_{xh}\right)}{\sum_{i=1}^{L}W_h\left(\bar{X}_hB_{2h}(x)+C_{xh}\right)+\sum_{i=1}^{L}W_h\left(\bar{x}_{h(r_h)}B_{2h}(x)+C_{xh}\right)}\right] \quad \textbf{(19)}$$

Obtaining bias and MSE of t_{r3} let $\bar{y}_{[SRSS]}=\bar{Y}(1+\delta_0)$ and $\bar{x}_{[SRSS]}=\bar{X}(1+\delta_1)$ so that $E(\delta_1)=E(\delta_0)=0$
The Bias and MSE of the estimator t_{r3} to the first degree of approximation are respectively, given by:

$$(t_{r3}-\bar{Y})=\bar{Y}\left(\delta_0-\tfrac{1}{2}\lambda_3\delta_0\delta_1+\tfrac{3}{8}\lambda_3^2\delta_1^2\right)$$
$$(t_{r3}-\bar{Y})^2=\bar{Y}^2\left(\delta_0^2-\tfrac{1}{4}\lambda_3^2\delta_1^2+\lambda_3\delta_0\delta_1\right)$$

$$B(t_{r3})=E(t_{r3}-\bar{Y})=\bar{Y}\left(E(\delta_0)-\tfrac{1}{2}\lambda_3E(\delta_0\delta_1)+\tfrac{3}{8}\lambda_3^2E(\delta_1^2)\right)$$

$$B(t_{r3})=\bar{Y}\left[\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{3\lambda_3^2}{8}\frac{S_{x_h}^2}{\bar{X}^2}-\frac{\lambda_3}{2}\frac{S_{x_hy_h}^2}{\bar{X}\bar{Y}}\right\}-\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{m}{n_h}\left(\frac{3\lambda_3^2}{8}\sum_{i=1}^{r_h}D_{x_h(i)}^2-\frac{\lambda_3}{2}\sum_{i=1}^{r_h}D_{x_h(i)y_h[i]}^2\right)\right\}\right] \quad \textbf{(20)}$$

$$MSE(t_{r3})=E(t_{r3}-\bar{Y})^2=\bar{Y}^2\left(E(\delta_0^2)+\tfrac{1}{4}\lambda_3^2E(\delta_1^2)+\lambda_3E(\delta_0\delta_1)\right)$$

Since $E(\delta_1)=E(\delta_0)=0$

$$MSE(t_{r3}) = \bar{Y}\left[\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{S_{y_h}^2}{\bar{Y}^2} - \frac{m}{n_h}\sum_{i=1}^{r_h}D_{y_h[i]}^2\right\} - \frac{\lambda_3^2}{4}\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{m}{n_h}\left(\frac{S_{x_h}^2}{\bar{X}^2} - \frac{m}{n_h}\sum_{i=1}^{r_h}D_{x_h(i)}^2\right)\right\}\right.$$

$$\left. - \lambda_3\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left(\frac{S_{x_h y_h}}{\bar{X}\bar{Y}} - \frac{m}{n_h}\sum_{i=1}^{r_h}D_{x_h(i)y_h[i]}\right)\right]$$

$$MSE(t_{r3}) = \sum_{h=1}^{L}\frac{W_h^2}{n_h}\left[S_{y_h}^2 + \frac{\lambda_3^2}{4}R^2 S_{x_h}^2 - \lambda_3 R S_{x_h y_h} - \bar{Y}^2\frac{m}{n_h}\sum_{i=1}^{r_h}\left(D_{y_h[i]} - \frac{\lambda_3}{2}D_{x_h(i)}\right)^2\right] \quad \textbf{(21)}$$

Adapting the estimators in (5) given by Singh et al. (2008), another new exponential ratio type estimator in stratified ranked set sampling is being proposed as follows:

$$t_{r4} = \bar{y}_{[SRSS]}\exp\left[\frac{\sum_{i=1}^{L}W_h\left(\bar{X}_h C_{xh} + B_{2h}(x)\right) - \sum_{i=1}^{L}W_h\left(\bar{x}_{h(r_h)}C_{xh} + B_{2h}(x)\right)}{\sum_{i=1}^{L}W_h\left(\bar{X}_h C_{xh} + B_{2h}(x)\right) + \sum_{i=1}^{L}W_h\left(\bar{x}_{h(r_h)}C_{xh} + B_{2h}(x)\right)}\right] \quad \textbf{(22)}$$

Obtaining bias and MSE of t_{r4} let $\bar{y}_{[SRSS]} = \bar{Y}(1+\delta_0)$ and $\bar{x}_{[SRSS]} = \bar{X}(1+\delta_1)$ so that $E(\delta_1) = E(\delta_0) = 0$
The Bias and MSE of the estimator t_{r4} to the first degree of approximation are respectively, given by:

$$(t_{r4} - \bar{Y}) = \bar{Y}\left(\delta_0 - \tfrac{1}{2}\lambda_4\delta_0\delta_1 + \tfrac{3}{8}\lambda_4^2\delta_1^2\right)$$
$$(t_{r4} - \bar{Y})^2 = \bar{Y}^2\left(\delta_0^2 - \tfrac{1}{4}\lambda_4^2\delta_1^2 + \lambda_4\delta_0\delta_1\right)$$
$$B(t_{r4}) = E(t_{r4} - \bar{Y}) = \bar{Y}\left(E(\delta_0) - \tfrac{1}{2}\lambda_4 E(\delta_0\delta_1) + \tfrac{3}{8}\lambda_4^2 E(\delta_1^2)\right)$$
$$B(t_{r4}) = \bar{Y}\left[\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{3\lambda_4^2}{8}\frac{S_{x_h}^2}{\bar{X}^2} - \frac{\lambda_4}{2}\frac{S_{x_h y_h}^2}{\bar{X}\bar{Y}}\right\} - \sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{m}{n_h}\left(\frac{3\lambda_4^2}{8}\sum_{i=1}^{r_h}D_{x_h(i)}^2 - \frac{\lambda_4}{2}\sum_{i=1}^{r_h}D_{x_h(i)y_h[i]}^2\right)\right\}\right] \quad \textbf{(23)}$$

$$MSE(t_{r4}) = E(t_{r4} - \bar{Y})^2 = \bar{Y}^2\left(E(\delta_0^2) + \tfrac{1}{4}\lambda_4^2 E(\delta_1^2) + \lambda_4 E(\delta_0\delta_1)\right)$$

Since $E(\delta_1) = E(\delta_0) = 0$

$$MSE(t_{r4}) = \bar{Y}\left[\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{S_{y_h}^2}{\bar{Y}^2} - \frac{m}{n_h}\sum_{i=1}^{r_h}D_{y_h[i]}^2\right\} - \frac{\lambda_4^2}{4}\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{m}{n_h}\left(\frac{S_{x_h}^2}{\bar{X}^2} - \frac{m}{n_h}\sum_{i=1}^{r_h}D_{x_h(i)}^2\right)\right\}\right.$$

$$\left. - \lambda_4\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left(\frac{S_{x_h y_h}}{\bar{X}\bar{Y}} - \frac{m}{n_h}\sum_{i=1}^{r_h}D_{x_h(i)y_h[i]}\right)\right]$$

$$MSE(t_{r4}) = \sum_{h=1}^{L}\frac{W_h^2}{n_h}\left[S_{y_h}^2 + \frac{\lambda_4^2}{4}R^2 S_{x_h}^2 - \lambda_4 R S_{x_h y_h} - \bar{Y}^2\frac{m}{n_h}\sum_{i=1}^{r_h}\left(D_{y_h[i]} - \frac{\lambda_4}{2}D_{x_h(i)}\right)^2\right] \quad \textbf{(24)}$$

EFFICIENCY COMPARISON

On comparing Equations 2, 3, 4 and 5 with 13, 16, 19, and 22, we have:

1. $MSE(t_1) - MSE(t_{r1}) = B_1 \geq 0$

Where $B_1 = \bar{Y}^2 \frac{m}{n_h} \sum_{i=1}^{r_h} \left(D_{y_h[i]} - \frac{\lambda_1}{2} D_{x_h(i)} \right)^2$

$\Rightarrow MSE(t_1) \geq MSE(t_{r1})$

2. $MSE(t_2) - MSE(t_{r2}) = B_1 \geq 0$

Where $B_2 = \bar{Y}^2 \frac{m}{n_h} \sum_{i=1}^{r_h} \left(D_{y_h[i]} - \frac{\lambda_2}{2} D_{x_h(i)} \right)^2$

$\Rightarrow MSE(t_2) \geq MSE(t_{r2})$

$MSE(t_3) - MSE(t_{r3}) = B_3 \geq 0$

Where $B_3 = \bar{Y}^2 \frac{m}{n_h} \sum_{i=1}^{r_h} \left(D_{y_h[i]} - \frac{\lambda_3}{2} D_{x_h(i)} \right)^2$

$\Rightarrow MSE(t_3) \geq MSE(t_{r3})$

1. $MSE(t_4) - MSE(t_{r4}) = B_4 \geq 0$

Where $B_4 = \bar{Y}^2 \frac{m}{n_h} \sum_{i=1}^{r_h} \left(D_{y_h[i]} - \frac{\lambda_4}{2} D_{x_h(i)} \right)^2$

$\Rightarrow MSE(t_4) \geq MSE(t_{r4})$

CONCLUSION

We have examined the properties of exponential ratio type estimators with stratified ranked set sampling when the co-efficient of variation and co-efficient of kurtosis of the auxiliary variable are known. The bias and MSE of the resulting estimators were estimated. These MSE were compared with that of modified exponential estimators with stratified simple random sampling. The exponential estimators base on SRSS are more efficient that those of SSRS.

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