On the Stability of Endemic Equilibrium State of HIV/AIDS Model with Irresponsible Infective Immigrants.

Ayoola Emmanuel Lekan*[1]; A.A. Momoh[1]; A. Tahir[1]; and U.M. Modibbo[2].

[1]Department of Mathematics, Modibbo Adama University of Technology, Yola, Adamawa State, Nigeria.
[2]Department of Statistics and Operations Research, Modibbo Adama University of Technology, Yola. Adamawa State. Nigeria.

E-mail: abdulfatai@mautech.edu.ng*

ABSTRACT

In this paper, a non–linear mathematical model is proposed to study the effects of irresponsible infected immigrants on the spread of HIV/AIDS in a heterogeneous population with a constant recruitment of susceptible receptors. The equilibrium points, stability analysis, and numerical solution on the model are presented. The Routh–Hurwitz stability condition was employed to examine the stability of the disease–free equilibrium and Next Generation matrix was adopted to generate the Basic Reproduction Number. Also, the endemic equilibrium is stable as it satisfies the Bellman and Cooke's condition for stability. It is realised that at the disease–free equilibrium, the model is stable when the basic reproduction number $R_0 < 1$ and unstable otherwise.

The condition for stability of the model near the endemic equilibrium state is presented. Numerical solution reveals that the presence of infective immigrants into the population significantly affects the spread of the disease. The analysis further shows that strict immigration policies such as screening and reduction in the number of immigrants into a given population, and behavioural change of all classes of individuals should be considered in efforts aimed at controlling the spread of the disease.

(Mathematics Subject Classification: Primary 92D40, 92D25; Secondary: 34D20)

(Keywords: AIDS epidemic, infective immigrants, numerical solution, heterogeneous, stability)

INTRODUCTION

HIV/AIDS remains one of the world's most significant public health challenges, particularly in low- and middle-income countries. It has claimed more than 39 million lives so far. In 2013, at least 1.5 million people died from HIV related causes globally. There were approximately 35 million people living with HIV at the end of 2013 with about 2.1 million people becoming newly infected with HIV in 2013 globally. Sub –Saharan Africa is the most affected region with about 24.7 million people living with HIV in 2013. Also sub-Saharan Africa accounts for almost 70% of the global total of new HIV infections [11].

The Human Immunodeficiency Virus (HIV) belongs to the family of Retroviruses. In 1984, researchers discovered the primary causative viral agent; the Human Immunodeficiency Virus type 1 (HIV-1). In 1986, a second type of HIV, called HIV-2 was discovered in West Africa; where it may have been present decades earlier. Both HIV-1 and HIV-2 have the same modes of transmission and are associated with similar opportunistic infections. In person infected with HIV-2, immunodeficiency seems to develop more slowly compared with person infected with HIV-1, and those with HIV-2 are less infectious early in the course of infection [3].

Human Immunodeficiency Virus (HIV) is a virus while Acquired Immunodeficiency Syndrome (AIDS) is a condition brought about by the virus HIV. You can have HIV without having AIDS, and many people live for many years with HIV without ever developing to AIDS. But if you have AIDS, you have to have HIV.

HIV does not survive well outside the body. Therefore, it cannot be transmitted through casual, everyday contact; Mosquitoes and other insects do not transmit HIV. HIV is primarily spread through unprotected vaginal or anal intercourse with someone who is HIV positive, by sharing contaminated needles, syringes and other injecting equipment, and less commonly through transmissions of infected blood.

Globally, there were 232 million international immigrants in 2013, of these, nearly 59 percent lived in the developed regions while the developing regions hosted 41 percent of the world`s total. The world international migrants accounted for a relatively small share of total population, comprising about 3.2 percent of the world population in 2013; compared to 2.9 in 1990 [9].

WHO has affirmed that HIV screening of international travelers would be ineffective, impractical and wasteful. Rather than screening international travelers, resources must be applied to prevent HIV transmission among each population, based on information and education, and with the support of health and social services [10].

Ying-yen and Cooke [12] studied a model in change of behavior can help in the control of the spread of the disease. The attitude towards sex and other modes of transmission of HIV can play a major role in spreading the disease.

Simwa and Pokhariyal [7] in there paper dealt with a deterministic model for HIV epidemic with three stages of disease progression among infected patients. It is assumed that the patient once infected experiences disease progression up to full blown AIDS.

Bashiru [1] considered a population consisting of adults (15 and more years) and stated that it is this group that is sexually mature and active, and therefore, capable of reproduction.

McCoy [4] used a mathematical model to predict the effect of routine HIV screening that would have on survival, disease transmission and cost effectiveness. The model also estimated how many fewer people would be infected as a result of earlier defection and counselling to prevent transmission of infection to others.

Shim [6] conducted a research study entitled, "A note on Epidemic Models with Infective Immigrants". The purpose was to take a close look at the endemic behavior of SIS (Susceptible-Infective-Susceptible) and SIR (Susceptible-Infective-Recovery) models with infective immigrants and vaccination under standard incidence.

Tripathi and Omar [8] developed an epidemiological model to analyze the effect of screening of unaware infective on the spread of HIV infection with constant immigration of susceptible. In this paper, we established stability of the endemic equilibrium state of a non-linear mathematical model proposed by Mohammed et al. [5] to study the effects of infective immigrants on the spread of HIV/AIDS in a population.

THE MODEL

Considering a population of size $N(t)$ at time t with constant inflow of susceptible at a rate; Q_0. The population is subdivided into four classes: susceptible $S(t)$, infective; irresponsible infective $I_1(t)$, responsible infective $I_2(t)$ and full blown AIDS patients $A(t)$ with natural mortality rate d in all classes. The following assumptions are made in the development of the model:

1. The population under study is heterogeneous and varying with time.
2. The population under study is subdivided into four groups.
3. The HIV can only be transmitted through sexual intercourse or through infection from infected needle and blood.
4. The full – blown AIDS class is sexually inactive.
5. The rate at which irresponsible infective infect people with the disease is higher than that of responsible infective.

Model Diagram

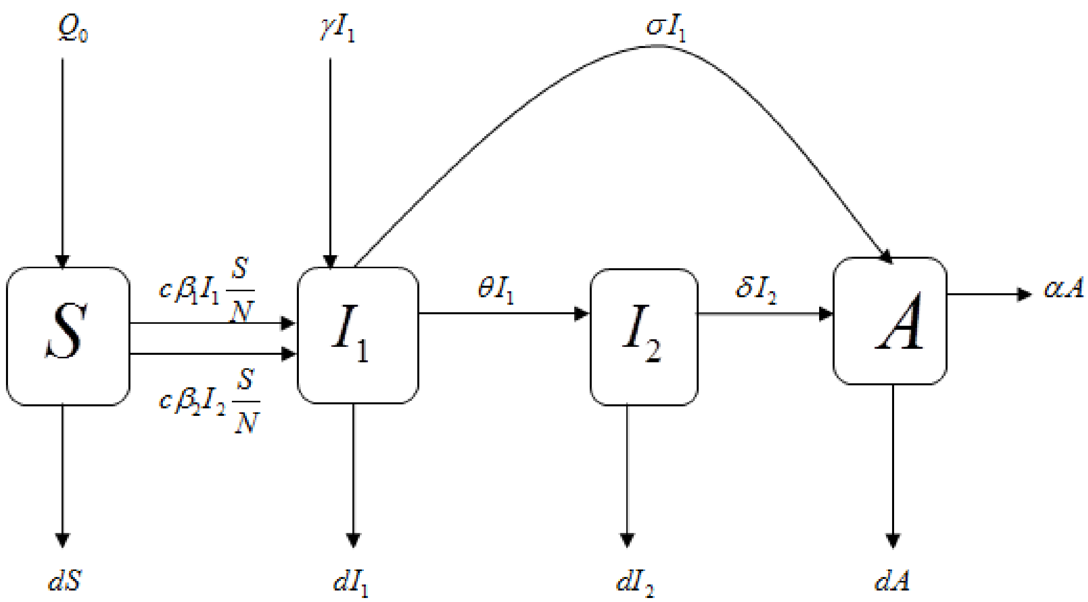

Figure 1: The Modified Model Flow Chart

Model Equations

$$\frac{dS}{dt} = Q_0 - \frac{c\beta_1 I_1 S}{N} - \frac{c\beta_2 I_2 S}{N} - dS \quad \textbf{(1)}$$

$$\frac{dI_1}{dt} = \frac{c\beta_1 I_1 S}{N} + \frac{c\beta_2 I_2 S}{N} + \gamma I_1 - (\theta + \sigma + d) I_1 \quad \textbf{(2)}$$

$$\frac{dI_2}{dt} = \theta I_1 - (\delta + d) I_2 \quad \textbf{(3)}$$

$$\frac{dA}{dt} = \sigma I_1 + \delta I_2 - \alpha A - dA \quad \textbf{(4)}$$

With initial conditions:

$$S(0) = S_0,\ I_1(0) = I_{10},\ I_2(0) = I_{20},\ A(0) = A_0,\ \beta_1 > \beta_2$$

Definition of Variables and Parameters of the Modified Model

Table 1: Definition of variables and parameters

Variables and Parameters	Definition
$N(t)$	Total population size at time t.
$S(t)$	The size of the susceptible population at time t.
$I_1(t)$	The size of the irresponsible infective population at time t.
$I_2(t)$	The size of the responsible infective population at time t.
$A(t)$	The size of the full blown AIDS population at time t.
c	The number of sexual partners an infective individual has.
β_1	The contact rate of irresponsible infective.
β_2	The contact rate of responsible infective
d	The natural death rate.
σ	The conversion rate of irresponsible infective to full-blown AIDS
θ	The conversion rate of irresponsible infective to responsible infective.
δ	The conversion rate of responsible infective to full-blown AIDS.
α	The AIDS - induced mortality rate.
γ	Rate of recruitment of infective immigrants into the population
Q_0	The rate of recruitment of susceptible into the population.

STABILITY ANALYSIS OF THE MODEL

The system exhibits two types of equilibriums; disease – free and endemic equilibrium states.

Stability Disease – free Equilibrium State

At the disease – free equilibrium, there are no infective and full – blown AIDS patients.
Hence; $I_1 = I_2 = A = 0$ and $N = \frac{Q_0}{d}$

$$\textit{Therefore, the disease-free equilibrium is}$$
$$E_0 = \left(\frac{Q_0}{d}, 0, 0, 0 \right)$$

$$\textit{Hence;}$$
$$J\left(\frac{Q_0}{d}, 0, 0, 0 \right) = \begin{pmatrix} -d & 0 & 0 & 0 \\ 0 & c\beta_1 + \gamma - (\theta + \sigma + d) & c\beta_2 & 0 \\ 0 & \theta & -(\delta + d) & 0 \\ 0 & \sigma & \delta & -(\alpha + d) \end{pmatrix} \quad \textbf{(5)}$$

$$\textit{Therefore, the characteristics equation corresponding to } E_0 = \left(\frac{Q_0}{d}, 0, 0, 0 \right)$$
$$\textit{is given by:}$$
$$H(\lambda) = (d + \lambda)(\alpha + d + \lambda)(\lambda^2 - \rho\lambda + \omega) \quad \textbf{(6)}$$
$$\textit{Where;}$$
$$\rho = (\delta + d)\left(-c\beta_1 - \gamma + \theta + \sigma + d\right) \textit{ and } \omega = (\delta + d)\left(-c\beta_1 - \gamma + \theta + \sigma + d\right) - c\beta_2\theta$$

Routh – Hurtwitz Stability Criterion

Thus, by Routh – Hurtwitz Stability criterion; which is a method for determining whether a linear system is stable or not, by examining the conditions of the roots of the characteristics equation of the system. To determine whether this system is stable or not, the two necessary conditions of all the roots, having negative real parts are:

a. All the polynomial coefficients must have the same sign.

b. All the polynomial coefficients must be non - zero.

$$\textit{It can be seen that}: \rho > 0 \textit{ and } \omega > 0$$
$$\textit{Thus}, \text{ using } \omega > 0$$
$$(\delta + d)(\theta + \sigma + d) > (\delta + d)\left(\gamma + c\beta_1\right) + c\beta_2\theta \quad \textbf{(7)}$$

The Condition (7) is sufficient to make E_0 locally asymptotically stable.

Basic Reproduction Number Using Next Generation Matrix

The linear stability of E_0 can be established by its basic reproduction number. It is determined by using next generation method on Equations (1), (2), (3) and (4) in the form of matrices F and V

F *be the rate of appearance of new* infection *in compartment*
V *be the transfer of individuals out of compartment by another means*
x_0 *be the disease free equilibrium*

Hence;

$R_0 = F \times V^{-1}$

$$F = \begin{pmatrix} c\beta_1 + \gamma & c\beta_2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad V = \begin{pmatrix} (\theta+\sigma+d) & 0 & 0 \\ \theta & -(\delta+d) & 0 \\ \sigma & \delta & -(\alpha+d) \end{pmatrix}$$

To obtain V^{-1}

$$V^{-1} = \frac{V'}{|V|} \qquad i.e \qquad \frac{\text{Adjoint } of \text{ } matrix\ V}{\text{Determinant } of \text{ } matrix\ V}$$

$$|V| = (\theta+\sigma+d)\left[(\delta+d)(\alpha+d)\right]$$

Hence, the basic reproduction number of the system is giving by:

$$R_0 = \frac{c\beta_1 + \gamma(\delta+d) + c\beta_2\theta}{(\theta+\sigma+d)(\delta+d)} \tag{8}$$

It is clear that for $R_0 < 1$ which corresponds to the Condition (7), the disease – free equilibrium is locally asymptotically stable so that infection fades out from the population and thus the endemic equilibrium does not exist. However, for $R_0 > 1$, E_0 is unstable and then the infection is maintained in the population.

Stability of the Endemic Equilibrium State

The endemic equilibrium point is given by $E^* = (N^*, I_1^*, I_2^*, A^*)$

At equilibrium;

$$\frac{dN}{dt}=\frac{dI_1}{dt}=\frac{dI_2}{dt}=\frac{dA}{dt}=0 \quad (8)$$

Hence;

$$Q_0-dN+\gamma I_1-\alpha A=0 \quad (9)$$

$$\frac{c\left(\beta_1 I_1+\beta_2 I_2\right)\left(N-I_1-I_2-A\right)}{N}+\gamma I_1-\left(\theta+\sigma+d\right)I_1=0 \quad (10)$$

$$\theta I_1-(\delta+d)I_2=0 \quad (11)$$

$$\sigma I_1+\delta I_2-(\alpha+d)A=0 \quad (12)$$

Therefore;

$$N^*=\frac{c\left(\beta_1(\delta+d)+\beta_2\theta\right)\left(\theta+\sigma+\delta\right)\left(\alpha+d+\delta\right)}{(\delta+d)(\alpha+d)\mu}I_1^* \quad (13)$$

$$I_1^*=\frac{Q_0(\delta+d)(\alpha+d)\mu}{cd\left(\beta_1(\delta+d)+\beta_2\theta\right)\left(\theta+\sigma+\delta\right)\left(\alpha+d+\delta\right)+\alpha\left(\theta+\sigma+\delta\right)\mu-(\delta+d)(\alpha+d)\mu\gamma} \quad (14)$$

$$I_2^*=\frac{\theta}{\delta+d}I_1^* \quad (15)$$

$$A^*=\frac{\delta\left(\theta+\sigma+d\right)}{(\delta+d)(\alpha+d)}I_1^*$$

Where; $\mu=c\beta_1(\delta+d)+c\beta_2\theta\left(\gamma-\theta-\sigma-d\right)(\delta+d)$ (16)

We note here that E^* *is positive only when* $\mu>0$ *or* $R_O>1$ *and* $\alpha\delta\left(\theta+\sigma+d\right)$-$(\delta+d)(\alpha+d)>0$

Therefore, the characteristics equation corresponding to $E^*=(N^*,I_1^*,I_2^*,A^*)$ is given by;

$$
\begin{aligned}
H(\lambda) = {} & \lambda^4 + \lambda^3(\alpha + 2d - \delta c\beta_1 - \delta\gamma + \theta\delta + \delta\sigma + d\delta - cd\beta_1 - d\gamma + d\theta + d\sigma + d^2) \\
& + \lambda^2(d\alpha + d^2 - \alpha\delta c\beta_1 - \alpha\delta\gamma + \alpha\theta\delta + \alpha\delta\sigma + \alpha d\delta - \alpha cd\beta_1 - \alpha d\gamma + \alpha d\theta + \alpha d\sigma \\
& + \alpha d^2 - 2d\delta c\beta_1 - 2d\delta\gamma + 2\theta\delta d + 2\delta\sigma d + 2d^2\delta - 2cd^2\beta_1 - 2d^2\gamma + 2d^2\theta \\
& + 2d^2\sigma + 2d^3 - \delta\beta_1 - \delta\gamma + \delta\theta + \delta^2 + \delta d - d\beta_1 - d\gamma + d\theta + d\delta + d^2 - c\beta_2\theta) \\
& + \lambda(-d\alpha\delta c\beta_1 - d\alpha\delta\gamma + d\alpha\theta\delta + d\alpha\delta\sigma + d^2\alpha\delta - d^2\alpha c\beta_1 - d^2\gamma\alpha + d^2\theta\alpha \\
& + d^2\delta\alpha + d^3\alpha - d^3\delta c\beta_1 - d^2\delta\gamma + d^2\theta\delta + d^2\delta\sigma + d^3\delta - d^3c\beta_1 - d^3\gamma + d^3\theta \\
& + d^3\sigma + d^4 - \alpha\delta\beta_1 - \alpha\delta\gamma + \delta\theta\alpha + \alpha\delta^2 + \alpha\delta d - \alpha d\beta_1 - \alpha d\gamma + \alpha d\theta \\
& + \alpha d\delta + \alpha d^2 - \alpha c\beta_2\theta - 2d\delta\beta_1 - 2d\delta\gamma + 2d\delta\theta + 2d\delta^2 + 2d^2\delta - 2d^2\beta_1 - 2d^2\gamma \\
& + 2d^2\theta + 2d^2\delta + 2d^3 - 2c\beta_2\theta) - d\alpha\delta\beta_1 - d\alpha\delta\gamma + d\alpha\delta\theta + d\alpha\delta^2 + d^2\alpha\delta \\
& - d\alpha\beta_1 - d^2\alpha\gamma + d^2\alpha\theta + d^2\alpha\delta + d^3\alpha - d\alpha c\beta_2\theta - d^2\delta\beta_1 - d^2\delta\gamma + d^2\delta\theta \\
& + d^2\delta^2 + d^3\delta - d^3\beta_1 - d^3\gamma + d^3\theta + d^3\delta + d^4 - d^2c\beta_2\theta
\end{aligned} \quad \textbf{(17)}
$$

As applied in Bellman and Cooke (1963), the result of Bellman and Cooke theorem is such that if $\Delta(iy), y \in I$ is separated into its real and imaginary parts $\Delta(iy) = F(y) + iG(y)$ and all zeros of $\Delta(z)$ have negative real parts. Then, the zeros of $F(y)$ *and* $G(y)$ are real, simple and alternate, and

By applying the result of Bellman and Cooke above; by setting $\lambda = iw$

We get:

$$F(0)G'(0) - F'(0)G(0) > 0 \quad \textbf{(18)}$$

$H(iw) = F(w) + iG(w)$

Where $F(w)$ and $G(w)$ are the real and imaginary parts of $H(iw)$

Hence;

Substituting $\lambda = iw$ into the $H(\lambda)$

$$
\begin{aligned}
H(iw) = {} & w^4 + iw^3(\alpha + 2d - \delta c\beta_1 - \delta\gamma + \theta\delta + \delta\sigma + d\delta - cd\beta_1 - d\gamma + d\theta + d\sigma + d^2) \\
& + w^2(d\alpha + d^2 - \alpha\delta c\beta_1 - \alpha\delta\gamma + \alpha\theta\delta + \alpha\delta\sigma + \alpha d\delta - \alpha cd\beta_1 - \alpha d\gamma + \alpha d\theta + \alpha d\sigma \\
& + \alpha d^2 - 2d\delta c\beta_1 - 2d\delta\gamma + 2\theta\delta d + 2\delta\sigma d + 2d^2\delta - 2cd^2\beta_1 - 2d^2\gamma + 2d^2\theta \\
& + 2d^2\sigma + 2d^3 - \delta\beta_1 - \delta\gamma + \delta\theta + \delta^2 + \delta d - d\beta_1 - d\gamma + d\theta + d\delta + d^2 - c\beta_2\theta) \\
& + iw(-d\alpha\delta c\beta_1 - d\alpha\delta\gamma + d\alpha\theta\delta + d\alpha\delta\sigma + d^2\alpha\delta - d^2\alpha c\beta_1 - d^2\gamma\alpha + d^2\theta\alpha \\
& + d^2\delta\alpha + d^3\alpha - d^3\delta c\beta_1 - d^2\delta\gamma + d^2\theta\delta + d^2\delta\sigma + d^3\delta - d^3 c\beta_1 - d^3\gamma + d^3\theta \\
& + d^3\sigma + d^4 - \alpha\delta\beta_1 - \alpha\delta\gamma + \delta\theta\alpha + \alpha\delta^2 + \alpha\delta d - \alpha d\beta_1 - \alpha d\gamma + \alpha d\theta \\
& + \alpha d\delta + \alpha d^2 - \alpha c\beta_2\theta - 2d\delta\beta_1 - 2d\delta\gamma + 2d\delta\theta + 2d\delta^2 + 2d^2\delta - 2d^2\beta_1 - 2d^2\gamma \\
& + 2d^2\theta + 2d^2\delta + 2d^3 - 2c\beta_2\theta) - d\alpha\delta\beta_1 - d\alpha\delta\gamma + d\alpha\delta\theta + d\alpha\delta^2 + d^2\alpha\delta \\
& - d\alpha\beta_1 - d^2\alpha\gamma + d^2\alpha\theta + d^2\alpha\delta + d^3\alpha - d\alpha c\beta_2\theta - d^2\delta\beta_1 - d^2\delta\gamma + d^2\delta\theta \\
& + d^2\delta^2 + d^3\delta - d^3\beta_1 - d^3\gamma + d^3\theta + d^3\delta + d^4 - d^2 c\beta_2\theta
\end{aligned} \qquad \textbf{(19)}
$$

Evaluating for $F(w)$ and $G(w)$ we obtain

$$
\begin{aligned}
F(w) = {} & w^4 + w^2(d\alpha + d^2 - \alpha\delta c\beta_1 - \alpha\delta\gamma + \alpha\theta\delta + \alpha\delta\sigma + \alpha d\delta - \alpha cd\beta_1 - \alpha d\gamma + \alpha d\theta + \alpha d\sigma \\
& + \alpha d^2 - 2d\delta c\beta_1 - 2d\delta\gamma + 2\theta\delta d + 2\delta\sigma d + 2d^2\delta - 2cd^2\beta_1 - 2d^2\gamma + 2d^2\theta \\
& + 2d^2\sigma + 2d^3 - \delta\beta_1 - \delta\gamma + \delta\theta + \delta^2 + \delta d - d\beta_1 - d\gamma + d\theta + d\delta + d^2 - c\beta_2\theta) \\
& - d\alpha\delta\beta_1 - d\alpha\delta\gamma + d\alpha\delta\theta + d\alpha\delta^2 + d^2\alpha\delta - d\alpha\beta_1 - d^2\alpha\gamma + d^2\alpha\theta + d^2\alpha\delta + d^3\alpha - d\alpha c\beta_2\theta \\
& - d^2\delta\beta_1 - d^2\delta\gamma + d^2\delta\theta + d^2\delta^2 + d^3\delta - d^3\beta_1 - d^3\gamma + d^3\theta + d^3\delta + d^4 - d^2 c\beta_2\theta
\end{aligned} \qquad \textbf{(20)}
$$

$$
\begin{aligned}
G(w) = {} & iw^3(\alpha + 2d - \delta c\beta_1 - \delta\gamma + \theta\delta + \delta\sigma + d\delta - cd\beta_1 - d\gamma + d\theta + d\sigma + d^2) \\
& + iw(-d\alpha\delta c\beta_1 - d\alpha\delta\gamma + d\alpha\theta\delta + d\alpha\delta\sigma + d^2\alpha\delta - d^2\alpha c\beta_1 - d^2\gamma\alpha + d^2\theta\alpha \\
& + d^2\delta\alpha + d^3\alpha - d^3\delta c\beta_1 - d^2\delta\gamma + d^2\theta\delta + d^2\delta\sigma + d^3\delta - d^3 c\beta_1 - d^3\gamma + d^3\theta \\
& + d^3\sigma + d^4 - \alpha\delta\beta_1 - \alpha\delta\gamma + \delta\theta\alpha + \alpha\delta^2 + \alpha\delta d - \alpha d\beta_1 - \alpha d\gamma + \alpha d\theta \\
& + \alpha d\delta + \alpha d^2 - \alpha c\beta_2\theta - 2d\delta\beta_1 - 2d\delta\gamma + 2d\delta\theta + 2d\delta^2 + 2d^2\delta - 2d^2\beta_1 - 2d^2\gamma \\
& + 2d^2\theta + 2d^2\delta + 2d^3 - 2c\beta_2\theta)
\end{aligned} \qquad \textbf{(21)}
$$

$$
\begin{aligned}
F'(w) = \frac{\partial F(w)}{\partial w} = {} & 4w^3 + 2w(d\alpha + d^2 - \alpha\delta c\beta_1 - \alpha\delta\gamma + \alpha\theta\delta + \alpha\delta\sigma + \alpha d\delta - \alpha cd\beta_1 - \alpha d\gamma + \alpha d\theta \\
& + \alpha d\sigma + \alpha d^2 - 2d\delta c\beta_1 - 2d\delta\gamma + 2\theta\delta d + 2\delta\sigma d + 2d^2\delta - 2cd^2\beta_1 - 2d^2\gamma \\
& + 2d^2\theta + 2d^2\sigma + 2d^3 - \delta\beta_1 - \delta\gamma + \delta\theta + \delta^2 + \delta d - d\beta_1 - d\gamma + d\theta + d\delta + d^2 - c\beta_2\theta) \\
& - d\alpha\delta\beta_1 - d\alpha\delta\gamma + d\alpha\delta\theta + d\alpha\delta^2 + d^2\alpha\delta - d\alpha\beta_1 - d^2\alpha\gamma + d^2\alpha\theta + d^2\alpha\delta + d^3\alpha - d\alpha c\beta_2\theta \\
& - d^2\delta\beta_1 - d^2\delta\gamma + d^2\delta\theta + d^2\delta^2 + d^3\delta - d^3\beta_1 - d^3\gamma + d^3\theta + d^3\delta + d^4 - d^2 c\beta_2\theta
\end{aligned} \qquad \textbf{(22)}
$$

$$G'(w) = \frac{\partial G(w)}{\partial w} = G(w) = 3w^2(\alpha + 2d - \delta c\beta_1 - \delta\gamma + \theta\delta + \delta\sigma + d\delta - cd\beta_1 - d\gamma + d\theta + d\sigma + d^2)$$
$$- d\alpha\delta c\beta_1 - d\alpha\delta\gamma + d\alpha\theta\delta + d\alpha\delta\sigma + d^2\alpha\delta - d^2\alpha c\beta_1 - d^2\gamma\alpha + d^2\theta\alpha$$
$$+ d^2\delta\alpha + d^3\alpha - d^3\delta c\beta_1 - d^2\delta\gamma + d^2\theta\delta + d^2\delta\sigma + d^3\delta - d^3 c\beta_1 - d^3\gamma + d^3\theta$$
$$+ d^3\sigma + d^4 - \alpha\delta\beta_1 - \alpha\delta\gamma + \delta\theta\alpha + \alpha\delta^2 + \alpha\delta d - \alpha d\beta_1 - \alpha d\gamma + \alpha d\theta$$
$$+ \alpha d\delta + \alpha d^2 - \alpha c\beta_2\theta - 2d\delta\beta_1 - 2d\delta\gamma + 2d\delta\theta + 2d\delta^2 + 2d^2\delta - 2d^2\beta_1$$
$$- 2d^2\gamma + 2d^2\theta + 2d^2\delta + 2d^3 - 2c\beta_2\theta \quad \textbf{(23)}$$

Setting $w = 0$

$$F(0) = -d\alpha\delta\beta_1 - d\alpha\delta\gamma + d\alpha\delta\theta + d\alpha\delta^2 + d^2\alpha\delta - d\alpha\beta_1 - d^2\alpha\gamma + d^2\alpha\theta + d^2\alpha\delta + d^3\alpha$$
$$- d\alpha c\beta_2\theta - d^2\delta\beta_1 - d^2\delta\gamma + d^2\delta\theta + d^2\delta^2 + d^3\delta - d^3\beta_1 - d^3\gamma + d^3\theta + d^3\delta + d^4 - d^2 c\beta_2\theta \quad \textbf{(24)}$$

$$G(0) = 0 \quad \textbf{(25)}$$

$$F'(0) = 0 \quad \textbf{(26)}$$

$$G'(0) = -d\alpha\delta c\beta_1 - d\alpha\delta\gamma + d\alpha\theta\delta + d\alpha\delta\sigma + d^2\alpha\delta - d^2\alpha c\beta_1 - d^2\gamma\alpha + d^2\theta\alpha + d^2\delta\alpha + d^3\alpha$$
$$- d^3\delta c\beta_1 - d^2\delta\gamma + d^2\theta\delta + d^2\delta\sigma + d^3\delta - d^3 c\beta_1 - d^3\gamma + d^3\theta + d^3\sigma + d^4 - \alpha\delta\beta_1 - \alpha\delta\gamma$$
$$+ \delta\theta\alpha + \alpha\delta^2 + \alpha\delta d - \alpha d\beta_1 - \alpha d\gamma + \alpha d\theta + \alpha d\delta + \alpha d^2 - \alpha c\beta_2\theta - 2d\delta\beta_1 - 2d\delta\gamma$$
$$+ 2d\delta\theta + 2d\delta^2 + 2d^2\delta - 2d^2\beta_1 - 2d^2\gamma + 2d^2\theta + 2d^2\delta + 2d^3 - 2c\beta_2\theta \quad \textbf{(27)}$$

We have from the Bellman and Cooke`s theorem that; the condition $R_e\lambda > 0$ is given by inequality:

$$F(0)G'(0) - F'(0)G(0) > 0 \quad \textbf{(28)}$$

The Equation (28) is the stability condition for the endemic equilibrium state.
From (25) and (26), we have $G(0) = 0$ and $F'(0) = 0$

Hence,(28) becomes;

$$F(0)G'(0) > 0 \quad \textbf{(29)}$$

Let;

$$J = F(0)G'(0) \quad \textbf{(30)}$$

The endemic equilibrium state will be stable when $J > 0$

Table 2: Variation of Infective Immigrants.

β_1	β_2	α	d	Q_0	θ	c	δ	σ	γ	$F(0)$	$G'(0)$	J	Remark
0.08	0.015	0.5	0.02	0.4	0.955	10	0.25	0.25	0.1	0.00065984	-0.2047724	-0.0001351182345	Unstable
0.08	0.015	0.5	0.02	0.4	0.955	10	0.25	0.25	0.2	0.00037904	-0.2196338	-0.0008324999555	Unstable
0.08	0.015	0.5	0.02	0.4	0.955	10	0.25	0.25	0.3	0.00049824	-0.2344946	-0.0001168345895	Unstable
0.08	0.015	0.5	0.02	0.4	0.955	10	0.25	0.25	0.4	-0.00018256	-0.2493554	0.0004552232182	Stable
0.08	0.015	0.5	0.02	0.4	0.955	10	0.25	0.25	0.5	-0.00081086	-0.2634162	0.0002135936599	Stable
0.08	0.015	0.5	0.02	0.4	0.955	10	0.25	0.25	0.6	-0.00074416	-0.266577	0.0001983759403	Stable
0.08	0.015	0.5	0.02	0.4	0.955	10	0.25	0.25	0.7	-0.00102496	-0.2939378	0.0003012744875	Stable
0.08	0.015	0.5	0.02	0.4	0.955	10	0.25	0.25	0.8	-0.00130576	-0.3087986	0.0004032168599	Stable
0.08	0.015	0.5	0.02	0.4	0.955	10	0.25	0.25	0.9	-0.001586656	-0.3236594	0.0005135050577	Stable
0.08	0.015	0.5	0.02	0.4	0.955	10	0.25	0.25	1.0	-0.00186736	-0.3385202	0.0006321390807	Stable

Based on Bellman and Cooke's theorem, the endemic equilibrium state will be stable when $J > 0$. In the case of table 2; as the rate of recruitment of infective immigrants into the population (γ) increases, J diverges (i.e. greater than zero) and will eventually become $J > 0$; we conclude that the endemic equilibrium state is stable. Therefore, in order to minimize the spread of the disease and prevent the total population from being wiped away; effective immigration policies such as screening should be put in place to ensure that the infected individuals who might act irresponsibly may be denied entry or are exposed so as to minimize their potential of infecting others.

CONCLUSION

It is shown that the basic reproductive number $R_0 < 1$ corresponds to a disease–free equilibrium indicating that the disease is under control. The disease however becomes endemic when $R_0 > 1$ and thus the disease remains in the population.

From the numerical solution in Table 2, we have: $J > 0$. Since this value is greater than zero. We conclude that the endemic equilibrium is stable; the stability of the endemic equilibrium implies that strict immigration policies such as screening and reduction in the number of immigrants into a given population could help control the spread of the disease. Also certain model parameters such as contact rate and number of sexual partners of infected persons are likely to increase amongst irresponsible infective and therefore could increase the spread of the disease.

Understanding the magnitude and future trends of the HIV/AIDS epidemics is a necessary pre-requisite for proper planning and mobilization of resources for its prevention and control. Hence, the findings of this project provide useful inputs to policy formulation and execution in the fight against the spread of HIV/AIDS.

In order to control the disease, the transmission of persons from the susceptible to infective population should be reduced to the barest minimum. We recommend that Behavioral survey of immigrants should be conducted so as to generate data that can explain the various factors driving the epidemic in the various health zones in the country and inform intervention nationwide. Civil society and other identifiable groups should be involved in engaging society on the disease with particular emphasis on abstinence as the safest option.

REFERENCES

1. Bashiru, K.A. 2005. "Stochastic Modeling of Specific Mode of Transmission of HIV/AIDS Epidemic with Reference to Nigeria". A Project for M.Tech. in Mathematical Sciences Department, Federal University of Technology: Akure Ondo State, Nigeria.

2. Bellman, R. and K.C. Cooke. 1963. *Differential Difference Equations*. Academic Press: London, UK.

3. Bhattacharjee, S. 2007. "Why does HIV Manifest into AIDS in Humans". *Everyman`s Science*. XL(6):401.

4. McCoy. 2005. "Routine HIV Screening Can Prolong Life at a Reasonable Cost". African Institute for Mathematical Sciences (AIMS).

5. Mohammed, I.D. and B. Seidu. 2012. "Modelling the Effect of Irresponsible Infective Immigrants on the Transmission Dynamics of HIV/AIDS". *Advances in Applied Mathematical Biosciences*. 3: 31-40.

6. Shim, E. 2006. "A Note on Epidemic Models with Infective Immigrants". SAMSI. Retrieved from: www.samsi.info/.../tr2006-04.pdf

7. Simwa, R. and G.P. Pokhariyal. 2003. "A Dynamical Model for Stage-Specific HIV Incidences with Application to Sub-Saharan Africa". Department of Mathematics, University of Nairobi: Nairobi, Kenya.

8. Tripathi, A. and S. Omar. 2006. "Modelling the Spread of AIDS Epidemic with Vertical Transmission". *Applied Mathematics and Computation*. 178:262-272.

9. United Nation Population Division. 2013. *International Migration Report*. Retrieved from: www.un.org/en/development/desa/population.

10. World Health Organization. 1998. "Statement on Screening of International Travelers for Infection with HIV". Retrieved from: citation.allacademic.com/meta/p_mla…

11. World Health Organization. 2013. "HIV, Fact Sheets". Retrieved from: www.who.int/mediacentre/factsheets/fs360/en/

12. Ying-Yen, H. and K. Cooke. 2000. "Behavior Change and Treatment of Core Groups: It Effect on the Spread of HIV/AIDS". *IMA. J. Math. Appl. Med. Biol*. 17:213-241.

SUGGESTED CITATION

Lekan, A.E., A.A. Momoh, A. Tahir, and U.M. Modibbo. 2015. "On the Stability of Endemic Equilibrium State of HIV/AIDS Model with Irresponsible Infective Immigrants". *Pacific Journal of Science and Technology*. 16(2):225-233.