

# Analysis of Diffraction Losses Using the Geometric Theory of Diffraction (GTD) over Varying Heights.

N.T. Makanjuola<sup>1</sup>; L.A. Akinyemi<sup>1,3</sup>; O.O. Shoewu<sup>1</sup>; and F.O. Ojeyemi<sup>2</sup>.

<sup>1</sup>Department of Electronic and Computer Engineering, Lagos State University, Epe Campus, Epe, Lagos, Nigeria.

<sup>2</sup>ICT Unit, Administrative Staff College of Nigeria, Lagos, Nigeria.

<sup>3</sup>Department of Electrical Engineering, Faculty of Engineering and the Built Environment, University of Cape Town, South Africa.

E-mail: [tunjimakanjuola@yahoo.com](mailto:tunjimakanjuola@yahoo.com)  
[akinyemi@crg.ee.uct.ac.za](mailto:akinyemi@crg.ee.uct.ac.za)  
[letua034@yahoo.com](mailto:letua034@yahoo.com)  
[engrshoewu@yahoo.com](mailto:engrshoewu@yahoo.com)  
[ojeyemifestus@yahoo.com](mailto:ojeyemifestus@yahoo.com)

## ABSTRACT

This research work presents an analysis of radio wave propagation model based on the Geometric Theory of Diffraction (GTD) technique, for computing the diffraction losses associated with the technique. The GTD model analysis was done at a frequency of 2GHz with the respective distances ( $d$ ) from 1000m while the height ( $h$ ) above the line of sight was varied between 2m above the ground to 20m taking a step of 2m interval. The above process was repeated with respective distances from obstacle ( $d_1, d_2$ ) as 2000m, 3000m, 4000m, and 5000m. The height ( $h$ ) above the line of sight was also varied between 2m and 20m with a step of 2m interval also. The excess path length ( $\Delta d$ ) was first calculated, the *Fresnel-Kirchhoff diffraction* ( $v$ ) was computed and finally the Lee's approximation for diffraction losses was used to arrive at the diffraction losses in each case.

Inferring and observation from the graphs, diffraction losses followed the same pattern. At  $d_1 = d_2 = 1000m$ , there was diffraction gain up to a point before diffraction losses set in. The point at which diffraction losses started in each case increased as ( $d_1, d_2$ ) was increased to 2000m, 3000m, and 4000m. However, the point where diffraction losses started became relatively constant as ( $d_1, d_2$ ) increased from 4000m to 5000m. It was ascertained from this study that diffraction losses is only increased appreciably and steady when the height of the obstacle above the line of sight is increased. An increase in the distance between the transmitter and receiver to

the obstacle does not affect the diffraction losses seriously provided the line of sight is fixed and unchanged.

(Keywords: radiowaves, wave, diffraction, losses, Fresnelzones, frequency, propagation)

## INTRODUCTION

The accurate and precise modeling of electromagnetic wave propagation is necessary and important for predicting the performance of a radio system. A thorough and proper estimation of the effects of reflection, refraction and diffraction is also necessary to successfully model troposphere radiowave propagation over terrain because obstacles between the radio terminals will seriously affect the propagation conditions.

Hence there are three major theories for solving diffraction problems, namely the Huygens-Fresnel diffraction theory, boundary integral representations using the Helmholtz-Kirchoff integral theorem, and the Geometrical Theory of Diffraction.<sup>[1]</sup>

According to Huygens' construction, every point on a wavefront can be regarded as a source of a secondary disturbance (a spherical wavelet). The wavefront is at any instant defined by the envelope of these wavelets. Fresnel augmented and supplemented this principle by adding interference between the wavelets to treat diffraction phenomena. He also defined a subdivision of space between the source and the

receiver into concentric ellipsoids with frequency dependent radii: the Fresnel ellipsoids. By modeling, diffraction effects as a loss in signal intensity, Bertoni, Tsingos and Gascuel used Fresnel ellipsoids to determine the relevant obstacle at any given frequency.

The Helmholtz-Kirchoff integral theorem is a formalization of the Huygens-Fresnel principle. It expresses the scattered field at any point in space as a function of the field on the surface of the diffracting objects. Mathematically, it can be expressed as a surface integral and solved by numerical methods such as Boundary Element Method (BEM), which discretize surfaces into patches into the ray theory of light.

The Geometric Theory of Diffraction (GTD) incorporates diffraction effects into the ray theory of light.<sup>[1]</sup> The wedges of the model serve as secondary sources and generate new diffracted rays.<sup>[2]</sup> Each diffracted ray is attenuated by a diffraction coefficient in the same way a reflection ray is attenuated by a reflection coefficient. As for reflected rays, diffracted rays follow Fermat's principle: if the propagation medium is homogeneous, the rays follow the shortest path from the source to the receiver, stabbing the diffracting edges. Thus, incident and diffracted rays make equal angle with the edge direction.

In this study, an analysis of diffraction losses with the GTD with the obstacle varying by the refractive index, the Geometric Theory of Diffraction (GTD) technique models the obstacle by diffracting wedges.

## LITERATURE REVIEW AND THEORETICAL BACKGROUND

Many models currently exists that use the combination of spherical earth diffraction, multiple knife-edge diffraction, wedge diffraction, and geometric optics to arrive at a solution for the field of a given transmitter to receiver geometry and a specified terrain path. One model, called SEKE (Spherical Earth Knife Edge), was developed at Lincoln Laboratory in 1967. This model is based on the assumption that the propagation losses over any path (in the frequency range from VHF to X-band) can be approximated by one of the multipath, multiple knife-edge diffraction or spherical earth diffraction losses alone, or a weighted average of these three basic losses.<sup>[3]</sup>

Another model is based on the geometrical theory of diffraction (GTD) and works by determining what ray paths exist for a given height to receiver geometry and terrain profile, form a family of 16 ray types. The total field at the target is then found by adding the ray amplitudes from each possible ray.<sup>[4]</sup> While these models may adequately account for reflection and diffraction, they lack a proper accounting for range-dependent atmospheric environments. SEKE allows for a variable earth radius factor, but this assumes a constant gradient and horizontal homogeneity. The GTD model will allow inhomogeneous environments but problem still persists and arises as a result of region of caustics. For many years now, other methods such as the parabolic equation (PE) method has been used to model radiowave propagation in the troposphere for over-ocean paths.<sup>[5]</sup>

Ivanchov and Savitska (2012) worked on an inverse problem for a parabolic equation in a free-boundary domain degenerating at the initial time moment. Striking results were obtained with unknown time –dependent major coefficient approach. The biggest advantage using the PE method is that it gives a full-wave solution for the field in the presence of range-dependent environments. Two methods may be used to solve the PE. One uses finite-difference method and the other uses the slit-step Fourier algorithm.<sup>[6]</sup>

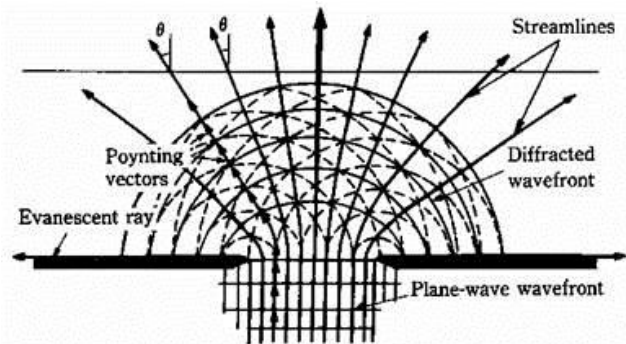
Also, Hryntsiv (2012) who researched on the inverse problem with free boundary for a weakly degenerate parabolic equation and an appreciable result was obtained.

Xu (2006) also worked on the complex variables and Elliptic equation. At the end of the work, a great result was achieved in terms of the performance of the approach.<sup>[7]</sup> There have been many published journals in recent years on the development of the split-step parabolic equation method as applied to electromagnetic wave propagation. The parabolic equation method and the geometric theory of diffraction which was originally developed by Fock in 1946, but it wasn't until 1973 that a practical solution for complicated refractive environments was developed by Hardin and Tappert, called the split-step Fourier method.<sup>[8]</sup> This method was originally applied to model acoustic propagation, but the radar community has since used the split-step algorithm to model propagation in the troposphere.<sup>[9]</sup>

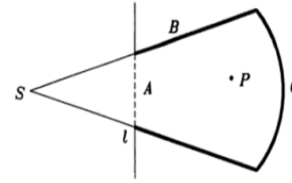
The importance of the geometric theory of diffraction is that, not only does it provide an exact solution to field equations (within the operator approximation) for a homogeneous atmosphere, but also can predict (with relatively small errors) field strengths for vertically and laterally inhomogeneous environments in a relatively short time. The more conventional mode-conversion methods can also be used, but the time involved in obtaining numerical results is generally too long to be of any practical consideration.<sup>[10]</sup>

Zhao and Li (1990) who also worked in the area of free boundary problem for quasi-linear degenerate parabolic equations with degeneracy. Hence, the uniqueness and regularity of generalized solution were obtained in terms of results. This now justifies the efforts put forward in comparative analysis and performance of diffraction losses using the geometric theory of diffraction technique.

Over the sea the troposphere usually exhibits horizontal homogeneity over long distances. It has been found that assumption of a horizontally stratified troposphere leads to valid operational propagation of assessment rate of 86%. However, the environment can change drastically at air/mass boundaries associated with wave cyclones and land/ocean interface.<sup>[12]</sup>



**Figure 1:** Wave Front in Different Directions due to Obstacle<sup>[13]</sup>



**Figure 2:** Schematic Layout Used to Introduce the Geometric Theory of Diffraction.<sup>[13]</sup>

## GEOMETRICAL THEORY OF DIFFRACTION

The Geometrical theory of diffraction (GTD) was devised to eliminate many of the problems associated with Geometrical Optics (GO). The strongest diffracted fields arise from edges, but ones of lesser strength originate from point discontinuities (tips and corners).

The behavior of the diffracted field is based on the following postulates of GTD:

- Wave fronts are locally plane and waves are TEM.
- Diffracted rays emerge radially from an edge
- Rays travel in straight lines in a homogeneous medium.
- Polarization is constant along a ray in an isotropic medium.
- The diffracted field strength is inversely proportional to the cross sectional area of the flux tube.
- The diffracted field is linearly related to the incident field at the diffraction point by a diffraction coefficient.

Diffraction is actually the phenomenon that radio signal can propagate around curved surface or sharp-edged obstacles and this is based on Huygens's principle which states that each point on a wave front acts as a secondary point source.

Diffraction is caused by propagation of secondary wavelets into the shadowed area.

When objects exist in free space that block or attenuate some of the wave field, the radiation enables EM waves to "bend" around objects. In

order to calculate the field at a point in (or near) the “shadow” of an object, Huygens’ Principle can be used to find accurate numerical results. Excess path length:

$$\begin{aligned} \Delta d &= \sqrt{d_1^2 + h^2} + \sqrt{d_2^2 + h^2} - (d_1 + d_2) \\ &= d_1 \sqrt{1 + \left(\frac{h}{d_1}\right)^2} + d_2 \sqrt{1 + \left(\frac{h}{d_2}\right)^2} - (d_1 + d_2) \\ &\approx \frac{h^2}{2d_1} + \frac{h^2}{2d_2} = \frac{h^2}{2} \left( \frac{d_1 + d_2}{d_1 d_2} \right) \end{aligned}$$

Assuming  $h \ll d_1, d_2$   
Phase Change:

$$\begin{aligned} \Delta \phi &= \frac{2\pi \Delta d}{\lambda} \\ &= \frac{2\pi}{\lambda} \frac{h^2}{2} \frac{d_1 + d_2}{d_1 d_2} \\ &= \frac{\pi}{2} h^2 \frac{2(d_1 + d_2)}{\lambda d_1 d_2} \\ &= \frac{\pi}{2} V^2 \end{aligned}$$

Where  $v$  is the dimensions Fresnel-Kirchhoff diffraction parameter:

$$\begin{aligned} V &= h \sqrt{\frac{2(d_1 + d_2)}{\lambda d_1 d_2}} \\ &= \sqrt{\frac{4\Delta d}{\lambda}} \end{aligned}$$

Fresnel Zones: Successive regions where secondary waves have an  $\lambda/2$  excess path length. These successive zones provides constructive and destructive interference alternately. Let  $r_n$  be the  $n$ -th Fresnel zone circle radius:

$$\begin{aligned} \Delta d &= \frac{\lambda n}{2} = \frac{r_n^2}{2} \frac{d_1 + d_2}{d_1 d_2} \\ r_n &= \sqrt{\frac{n \lambda d_1 d_2}{d_1 + d_2}} \end{aligned}$$

Radius is maximized when obstacle is in midway. Diffraction is affected by frequency and obstacle location. Higher frequency, smaller  $r_n$ , less diffraction. If the Fresnel zone is unobstructed, diffraction effect can be neglected i.e. free space propagation if the first Fresnel zone is clear.

When obstacle is present, received E-field is a vector sum of all the fields from the secondary Huygen sources above the knife edge:

$$F(v) = \frac{E_r}{E_0} = \frac{(1+j)}{2} \int_0^\infty \exp\left(-\frac{j\pi t^2}{2}\right) dt$$

$F(v)$  is the Fresnel integral

$E_r$  is the received E-field

$E_0$  is the E-field in the free space propagation

Diffraction gain in dB

$$G_d(v) = 20 \log |F(v)|$$

Approximated solution by Lee’85

$$G_d(v) = 0 \quad v \leq -1$$

$$G_d(v) = 20 \log(0.5 - 0.62v) \quad -1 \leq v \leq 0$$

$$G_d(v) = 20 \log(0.5 \exp(-0.95v)) \quad 0 \leq v \leq 1$$

$$G_d(v) = 20 \log(0.4 - \sqrt{0.1184 - (0.38 - 0.1v)^2}) \quad 1 \leq v \leq 2.4$$

$$G_d(v) = 20 \log(0.225^{1/v}) \quad v > 2.4$$

Multiple knife-edge diffraction: The multiple knife edges can be appropriated as a single equivalent obstacle but that is too optimistic. The wave diffraction can also be calculated in series which is too complicated.

## METHODOLOGY AND ANALYSIS

### GTD Computations

In the GTD model, a frequency of 2GHz was used. The respective distances from the Receiver ( $d_1$ ) and Transmitter ( $d_2$ ) to the obstacle was assumed to be the same and fixed at 100m, 200m, 300m, 400m and 500m while the height of the obstacle above the line sight ( $h$ ) was varied between 2m and 20m using a range step of 2m interval.

$$F = 2\text{GHz}$$

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{2 \times 10^9} = 0.15\text{m}$$

$$d_1 = d_2 = 1000\text{m}$$

$$h = 2\text{m} - 20\text{m step } 2\text{m}$$

$$\text{At } h = 2\text{m}$$

$$\Delta d = \frac{h^2}{2} \frac{d_1 + d_2}{d_1 d_2}$$

$$\Delta d = \frac{2^2 \cdot 1000 + 1000}{2 \cdot 1000000}$$

$$\Delta d = 0.004$$

$$v = \sqrt{\frac{4\Delta d}{\lambda}} = \sqrt{\frac{4 \times 0.004}{0.15}} = 0.3266$$

$$G_d(v) = 20 \log(0.5 \exp(-0.95v))$$

$$G_d(v) = 20 \log(0.5 \exp(-0.95v \times 0.3266))$$

$$G_d(v) = 20 \log 0.5 \exp(-0.3103)$$

$$G_d(v) = -3.3256 \text{ dB}$$

$$\text{At } h = 4 \text{ m}$$

$$\Delta d = \frac{h^2 d_1 + d_2}{2 d_1 d_2}$$

$$\Delta d = \frac{4^2 \cdot 1000 + 1000}{2 \cdot 1000000}$$

$$\Delta d = 0.016$$

## RESULT AND DISCUSSIONS

### Computations

Diffraction losses in the GTD model are analyzed after computation. The GTD model was done at a frequency of 2GHz with the respective distances from the obstacle ( $d_1$ ,  $d_2$ ) as 1000m while the height  $h$  was varied between 2m and 20m taking a range step of 2m interval. The excess path length ( $\Delta d$ ) was first calculated thereafter the *Fresnel-Kirchoff diffraction parameter* ( $v$ ) was computed and finally the Lee's approximation for diffraction losses was used to arrive at the diffraction losses. The diffraction losses was also computed with the respective distances from the obstacle ( $d_1$ ,  $d_2$ ) as 2000m, 3000m, 4000m and 5000m while the height  $h$  varied between the same interval with the same range step.

### Results

After the above computations were carefully done, the following results were obtained.

**Table 1:** Results at 2GHz for  $d_1$  and  $d_2 = 1000\text{m}$ .

Height (h) m	Diffraction Losses (GTD) dB
2	-3.33
4	-0.63
6	2.06
8	-15.76
10	-17.54
12	-19.23
14	-20.82
16	-21.30
18	-22.32
20	-23.24

**Table 2:** Results at 2GHz for  $d_1$  and  $d_2 = 2000\text{m}$ .

Height (h) m	Diffraction Losses (GTD) dB
2	-4.11
4	-2.21
6	-0.30
8	1.60
10	-14.90
12	-16.20
14	-17.45
16	-18.66
18	-19.82
20	-20.93

**Table 3:** Results at 2GHz for  $d_1$  and  $d_2 = 3000\text{m}$ .

Height (h) m	Diffraction Losses (GTD) dB
2	-4.46
4	-2.91
6	-1.35
8	0.20
10	1.76
12	-14.76
14	-15.84
16	-16.87
18	-17.88
20	-18.86

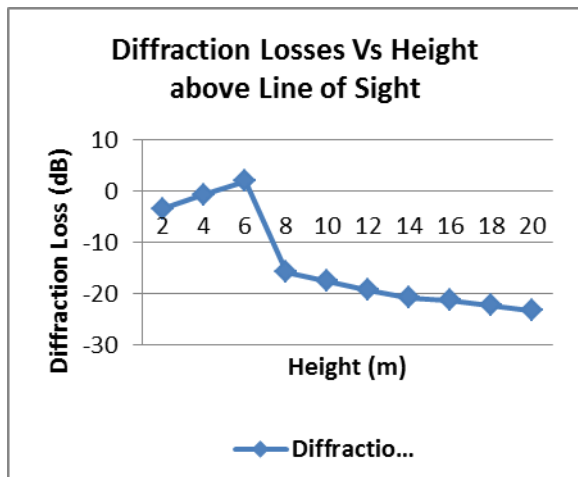
**Table 4:** Results at 2GHz for  $d_1$  and  $d_2 = 4000\text{m}$ .

Height (h) m	Diffraction Losses (GTD) dB
2	-4.67
4	-3.33
6	-1.98
8	-0.63
10	0.72
12	2.06
14	-14.83
16	-15.76
18	-16.66
20	-17.54

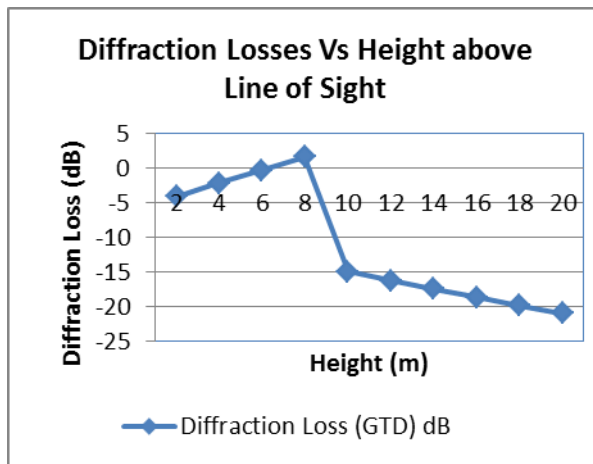
**Table 5:** Results at 2GHz for  $d_1$  and  $d_2 = 5000\text{m}$ .

Height (h) m	Diffraction Losses (GTD) dB
2	-4.82
4	-3.61
6	-2.40
8	-1.20
10	0.01
12	1.21
14	-14.12
16	-14.98
18	-15.81
20	-16.61

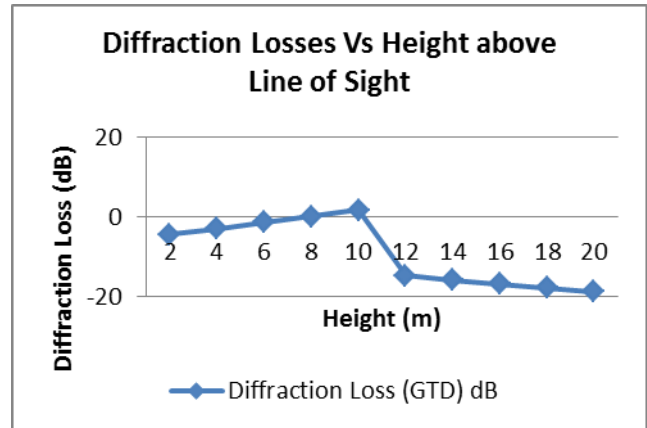
**COMPARISON OF DIFFRACTION LOSSES VS HEIGHT ABOVE LINE OF SIGHT FOR THE VARIOUS DISTANCES  $d_1$  AND  $d_2$**



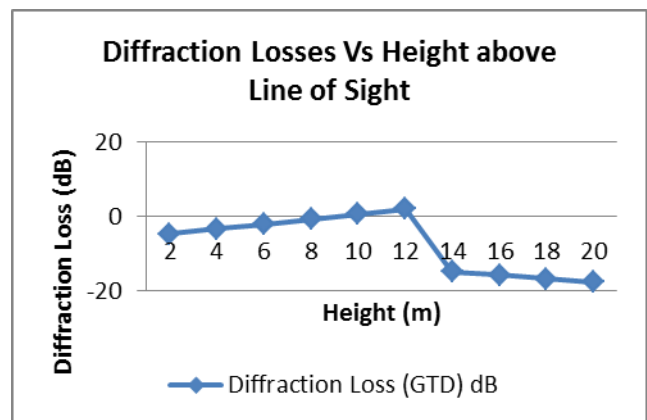
**Figure 1:** Diffraction Losses versus Obstacle Height above Line of Sight for 2GHz Frequency and at Respective Distance of Transmitter ( $d_1$ ) and Receiver ( $d_2$ ) from the Obstacle = 1000m.



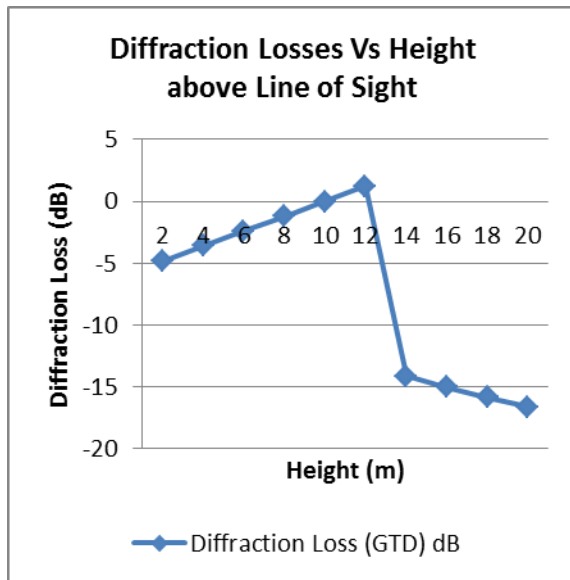
**Figure 2:** Diffraction Losses versus Obstacle Height above Line of Sight for 2GHz Frequency and at the Respective Distance of Transmitter ( $d_1$ ) and Receiver ( $d_2$ ) from the Obstacle = 2000m.



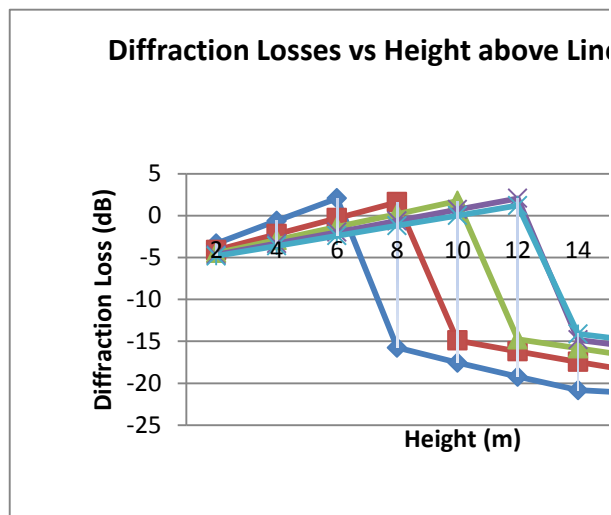
**Figure 3:** Diffraction Losses versus Obstacle Height above Line of Sight for 2GHz Rrequency and at the Respective Distance of Transmitter ( $d_1$ ) and Receiver ( $d_2$ ) from the Obstacle = 3000m.



**Figure 4:** Diffraction Losses versus Obstacle Height above Line of Sight for 2GHz Frequency and at the Respective Distance of Transmitter ( $d_1$ ) and Receiver ( $d_2$ ) from the Obstacle = 4000m.



**Figure 5:** Diffraction Losses versus Obstacle Height above Line of Sight for 2GHz Frequency and at Respective Distance of Transmitter ( $d_1$ ) and Receiver ( $d_2$ ) from the Obstacle.



**Figure 6:** Diffraction Losses versus Obstacle Height above Line of Sight for 2GHz Frequency and at the Respective Distance of Transmitter ( $d_1$ ) and ( $d_2$ ) from the Obstacle =1000m, 2000m, 3000m, 4000m, and 5000

## CONCLUSIONS

Analysis of diffraction losses have been carried out in this study using the Geometric Theory of Diffraction (GTD) technique. From the analysis

using the Geometric Theory of Diffraction, it was noted that diffraction losses followed the same trend judging from the graphs at  $d_1=d_2=1000m$ . The diffraction losses was negative at the initial height of 2m and as the height was increased in steps of 2m the diffraction losses reached the maximum positive value at 6m thereafter it went into the negative region again. This means that there was diffraction gain up to height of 6m then diffraction losses set in. The same graph pattern was noticed for  $d_1=d_2=2000m, 3000m, 4000m$  and 5000m from the obstacle except for the slight variation in diffraction gain/losses.

It has been found out from this thesis that diffraction losses only increased appreciably when the height of the obstacle above the line of sight is increased. An increase in the distances between the transmitter and receiver to the obstacle does not affect diffraction losses seriously. Hence the height of obstacle above the line of sight should be made minimal by increasing the height of the transmitter and receiver so that diffraction losses will be reduced. Nevertheless, the higher the height of the transmitter and receiver, also the closer the distance between the transmitter and receiver the lesser the diffraction losses due to obstacles.

## RECOMMENDATIONS

After all the test and survey carried out, it was observed that different factors also result into diffraction losses which lead to poor quality of service as a result of path losses in the communications gadgets mainly between a transmitter and a receiver which has been discussed extensively in this research work and the possible ways to reduce this effect is through avoiding obstacle between the transmitting and receiving antennas and the associate distance.

However, it would be much clearer if a more advanced model can be used with the aid of simulation using computer aided techniques. It is on this note that I am recommending that readers of this piece of work to take a step further to improve on it by using more advanced methods for better results. Also, our communication companies must consider city centers as areas where more methods of path losses prevention should be applied considering the economic effect of diffraction losses when setting up any repeater station in those states. A regular drive and survey test is equally recommended to be

carried out in low land areas especially places below the sea level because the presence of trees and hills affects the signal transmission and reception.

## REFERENCES

1. Holm, P. and A. Waern. 2003. "Wave Propagation over a Forest Edge Parabolic Equation Modeling vs. GTD Modeling". IEEE International Symposium on Electromagnetic Compatibility.
2. Ogunsola, A. 2010. "Lecture Notes on Radio Wave Propagation for M.Sc. Students". UNILAG.
3. McNamara, D.A., C.W. I. Pistorius, and J.A.G. Malherbe. 1990. *Introduction to the Uniform Geometrical Theory of Diffraction*. Atech House: Norwood, MA.
4. Ayasli, S. 1986. "SEKE: A Computer Model for Low Altitude Radar Propagation over Irregular Terrain". *IEEE Trans. On Antenna Propagat.* AP-34(8):1013-1023.
5. Tappert, F.D. 1977. "The Parabolic Approximation Method". In: *Wave Propagation and Underwater Acoustics*. 70 (Lectures Notes in Physics). J.B. Keller and J.S. Papadakis (eds). Springer-Verlag: New York, NY. 224-287.
6. Dockery G.D. and G.C. Konstanzer. 2007. "Recent Advances in Prediction of Tropospheric Propagation using the Parabolic Equation". *Johns Hopkins APL Tech Dig.* 8:404-412.
7. Ayyappan, K. and P. Dananjayan. 2006. "Propagation Model for Highway in Mobile Communication System". *Ubiquitous Computing and Communication Journal.* 3(4):61-66.
8. Mohammed, S. Al-Basheir, M.S. Raed, and M.S. Sami. 2006. "Measurements and Analysis for Signal Attenuation through Date Palm Trees at 2.1GHz Frequency". *Sudan Engineering Society Journal.* 52(45).
9. Phaiboon, S., P. Phokharutkul, and S. Somkurnpanich. 2004. "Mobile Propagation Path Losses Models for Suburban Areas using Fuzzy Logic Approximation". *IEEE Trans. On Communication.* 0.7803 – 8560 – 8/04, pp 133 – 136.
10. Shalangwa, D.A. and S.K. Singh. 2010. "Measurement and Modeling of Path Losses for GSM 900 in Subs Urban Environment over Irregular Terrain". *International Journal of Computer Science and Nationals Security.* 10(8):268 – 274.
11. Website: <http://www.gov.Br./index.asp:accessin> January/2006.
12. Adebayo, T.L. and F.O. Edeko. 2006. "Characterization of Propagation Path Losses at 1.8GHz: A Case Study of Benin City, Nigeria". *Research Journal of Applied Sciences: Medwell Online.*
13. Someda, C.G. 2006. *Electromagnetic Waves. Second edition.* CRC Press: Boca Raton, FL. 514-515.

## SUGGESTED CITATION

Makanjuola, N.T., L.A. Akinyemi, O.O. Shoewu, and F.O. Ojeyemi. 2015. "Analysis of Diffraction Losses Using the Geometric Theory of Diffraction (GTD) over Varying Heights". *Pacific Journal of Science and Technology.* 16(1):139-146.

 [Pacific Journal of Science and Technology](http://www.akamaiuniversity.us/PJST.htm)