Sophistication versus Performance: The Case of Time Series Modeling.

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ABSTRACT

This research compared the forecasting efficiency of three univariate time series models on monthly minimum temperature data. Data (in ^{0}C) on monthly minimum temperature of Bida, Niger State, Nigeria, spanning January 2000 to August 2013 were sourced from the National Cereals Research Institute (NCRI), Baddegi, Bida. The research used data for January 2000 to December 2012 for modeling while the rest data were used for gauging model performance. The Decomposition (Multiplicative and Additive), Holt-Winters' (Multiplicative and Additive) and SARIMA models were fitted to the temperature data and forecasting efficiency was measured by Root Mean Squared Error (RMSE). SARIMA (1, 1, 1) X $(1, 0, 1)_{12}$, possessed RMSE of 1.66, while SARIMA (1, 0, 0) X $(1, 0, 1)_{12}$ and SARIMA (1, 0, 1) X $(1, 0, 1)_{12}$ both recorded 1.84. Holt-Winters' Multiplicative and Additive both had RMSE of 2.21; Decomposition Multiplicative and Additive models also recorded same RMSE of 1.06, being the least of all. The Decomposition Multiplicative and Additive models (despite their simplicity) were found to produce the best out-of-sample forecasts, while the Holt-Winters' Multiplicative and Additive produced the least accurate, based on RMSE. Model sophistication is hence, not a measure of performance.

(Keywords: temperature, forecasting, Holt-Winters, decomposition, additive, multiplicative, SARIMA)

INTRODUCTION

In recent times, research efforts on applied and theory of time series modeling and forecasting have been on the increase, probably owing to the growing need to have a good idea of the possible value of variables in future periods, majorly for planning purposes, coupled with greater access to computing facilities. For instance, the need for a share investor to know the likely price of a share in future is critical for investment decisions. One area, in which forecasting is quite invaluable and whose forecasts are well known and appreciated by the public is meteorology. It has become the practice to have rainfall and temperature pattern forecasts as part of the news, all over the world. Such forecasts provide valuable information to the public.

Various forecasting approaches have been adopted in time series modeling. These approaches range from simple methods like Exponential Smoothing to more sophisticated ones like Seasonal Autoregressive Integrated Moving Average (SARIMA) models, due to Box and Jenkins (1976), Threshold Autoregressive (TAR) model due to Tong (1983), Self Excited Threshold Autoregressive (SETAR) model, and Smooth Transitional Autoregressive (STAR) model due to Teräsvirta (1994). Some of these models are quite straightforward while others are very sophisticated and highly computationally intensive. There is no guarantee that methods that are more sophisticated would perform better than the less sophisticated ones. For a particular data set, some would outperform others but such display of supremacy should not be generalized, as same results may not result when applied on other data sets.

Reid, in Kendall and Ord (1990) models 113 macroeconomic series in United Kingdom and United States of America, using Brown's, modified Brown's, Winters' and Harrison's methods. The study observes that certain methods are suitable for some types of data; Kendal and Ord also report extensive comparative studies by Markridakis and Hibon; the studies conclude that simple methods like exponential smoothing perform as well as more, sophisticated ones. Bolarinwa (2005) compares three univariate forecasting methods: Decomposition, SARIMA and Holt-Winters on meteorological data. The study concludes that

Decomposition method produces best 1-step ahead forecasts, while Box-Jenkins approach produces best 7-step ahead forecasts; Olowe (2009) models Nigeria Naira/Dollar exchange rate volatility; Etuk (2012) is a modeling of Nigerian Naira-US Dollar exchange rates using SARIMA approach while Etuk (2013) fits SARIMA model to Naira/Euro exchange rates. Bolarinwa (2010), performs SARIMA modeling of maximum temperature of Bida; Oyetunji and Akinsete (1987); and Foster and Rahmstorf (2011) are two of other works on time series modeling of temperature data.

The focus of this article is to model monthly minimum temperature of Bida, using the Decomposition, Holt-Winters and the SARIMA approaches, with a view to comparing their forecasting efficiency. The article is organized as follows: Section 2 presents the methods; Section 3 presents the results and discussions while the last Section concludes the article.

METHODS

Data

Data (in ^{0}C) on monthly minimum temperature of Bida, Niger State, Nigeria, spanning January 2000 to August 2013 were sourced from the National Cereals Research Institute (NCRI), Baddegi, Bida. The research used data for January 2000 to December 2012 for modeling while the rest data were used for gauging model performance.

The Model

Models under consideration are Decomposition (Additive and Multiplicative), Holt-Winters' (Additive and Multiplicative) and SARIMA.

Decomposition Method

Under this method, the time series variable (Y) is thought as composing of four components, namely: Trend (T), Seasonal (S), Cyclical (C) and Irregular (I). If the components are assumed to interact in additive manner, additive model results as follows:

$$Y = T + S + C + I \qquad \textbf{(1)}$$

On the other hand, if the components are assumed to interact in multiplicative manner, multiplicative model results as:

$$Y = T.S.C.I \qquad \textbf{(2)}$$

The trend is usually, estimated by the *least squares* method and the seasonal component, by the *seasonal index.*

SARIMA

The Box-Jenkins SARIMA (p, d, q) X (P, D, Q)$_s$ model has the functional form:

$$\phi_p(L)\Phi_P(L^s)(1-L)^d(1-L^s)^D y_t = \theta_q(L)\Theta_Q(L^s)\varepsilon_t \qquad \textbf{(3)}$$

where

L is a backward shift operator defined $L^j X_t = X_{t-j}$;

$\phi_1, \phi_2, \ldots, \phi_p$ are the non-seasonal autoregressive parameters;

$\Phi_1, \Phi_2, \ldots, \Phi_P$ are the seasonal autoregressive parameters;

$\theta_1, \theta_2, \ldots, \theta_q$ are the non-seasonal moving average parameters;

$\Theta_1, \Theta_2, \ldots, \Theta_Q$ are the seasonal moving average parameters;

d and D are the orders of non-seasonal and seasonal differencing respectively;

s is the seasonality;

$\{\varepsilon_t\}$ is a white noise process.

Box and Jenkins (1976) suggest a model building process based on the following procedure: Postulate general class of models; identify model; estimate model parameters, perform diagnostic checks; use model for forecasting or control if found adequate, else, re-identify model.

Model Identification

Box and Jenkins (1976) suggest that autocorrelation function (ACF) and partial autocorrelation function (PACF) be employed in determining the order of the model. For AR process of order p, AR(p), the PACF cuts off after lag p, while the ACF decays exponentially to zero; for MA process of order q, MA(q), the ACF cuts off after lag q while the PACF decays to zero; for ARMA process, both the ACF and the PACF tail off. Identification in this circumstance is more difficult. Courtesy of improved access to computing facilities, one can try a number of models and select out of those found adequate by diagnostic procedure, on basis of selected measure of accuracy.

Parameter Estimation

After model identification, parameters are to be estimated. Box and Jenkins (1976) suggest a non-linear least squares approach to estimating the model parameters.

Diagnostic Checking

After model fitting, the adequacy of the model has to be ascertained by inspection of the residuals. The Chi-Square test due to Ljung and Box (1978) is adopted in this research. The basis of all the tests is that the residuals should behave like a white noise process for model to be adequate.
The test of choice proceeds as follows:

$$H_0 : \rho_1 = \rho_2 = \ldots.\rho_k = 0$$
$$H_1 : Not\ all\ \rho_j = 0$$

Test Statistic:

$$Q = Q(k) = n(n+2)\sum_{j=1}^{K}\frac{r_j^2}{(n-j)} \rightarrow \chi^2_{K-p-q-P-Q} \quad \textbf{(4)}$$

Decision Criterion: Reject H_0 if:

$$Q(k) > \chi^2_{(1-\alpha), K-p-q-P-Q}$$

where n is number of observations left after differencing defined: n = N-d-D; and N is the original length of the series.

The statistic, (4) is evaluated at several choices of K (number of lags) and acceptance of H_0 suggests model adequacy.

Forecasting with SARIMA Model

Forecasting, using the model is the next line of action if model is found to be adequate by the diagnostic procedure. To illustrate the principle of SARIMA forecasting, let us use SARIMA (1, 1, 1) X (1, 0, 1)$_{12}$. If we denote by $\hat{y}_t(l)$, the forecast for lead time *l*.

Noting that the functional form of SARIMA (1, 1, 1) X (1, 0, 1)$_{12}$ is:

$$\phi_1(L)\Phi_1(L^{12})(1-L)^1(1-L^{12})^0 y_t = \theta_1(L)\Theta_1(L^{12})\varepsilon_t$$

$$(1-\phi L)(1-\Phi L^{12})(1-L)y_t = (1-\theta L)(1-\Theta L^{12})\varepsilon_t \quad \textbf{(4a)}$$

After some manipulations, (4a) becomes:

$$y_t = \Phi y_{t-12} + \phi y_{t-1} - \phi\Phi y_{t-13} + y_{t-1} - \Phi y_{t-13} - \phi y_{t-2} + \phi\Phi y_{t-14} + \varepsilon_t - \Theta\varepsilon_{t-12} - \theta\varepsilon_{t-1} + \theta\Theta\varepsilon_{t-13} \quad \textbf{(4b)}$$

The 1-step-ahead forecast from the origin, t is:

$$\hat{y}_t(1) = \Phi y_{t-11} + \phi y_t - \phi\Phi y_{t-12} + y_t - \Phi y_{t-12} - \phi y_{t-1} + \phi\Phi y_{t-13} - \Theta\varepsilon_{t-11} - \theta\varepsilon_t + \theta\Theta\varepsilon_{t-12} \quad \textbf{(5)}$$

For lead times *l* = 1, 2…13, the noise terms: $\varepsilon_1, \varepsilon_2 \ldots, \varepsilon_{13}$, in addition to the autoregressive components, enter into the model. However, for lead times, *l* >13, only the autoregressive components remain in the model:

$$\hat{y}_t(l) = \Phi y_{t+l-12} + \phi y_{t+l-1} - \phi\Phi y_{t+l-13} + y_{t+l-1} - \Phi y_{t+l-13} - \phi y_{t+l-2} + \phi\Phi y_{t+l-14} \quad (2.6)$$

Holt-Winters

This method has both the multiplicative and additive versions. For the multiplicative version, the forecast function is

$$\hat{Y}_t(l) = \{a_0(t) + l a_1(t)\} c\{t + l - s\}$$
$$l = 1, 2, \ldots \quad \textbf{(7)}$$

where

C (t) is the multiplicative seasonal effect and s, the seasonality.

The updating formulae are:

$$a_0(t) = \alpha_1 \frac{y_t}{c(t-s)} + (1-\alpha_1)\{a_0(t-1) + a_1(t-1)\}$$

(8a)

$$a_1(t) = \alpha_2\{a_0(t) - a_0(t-1)\} + (1-\alpha_2) + a_2(t-1)$$

(8b)

$$c(t) = \alpha_3 \frac{y_t}{a_0(t)} + (1-\alpha_3) c(t-s)$$

(8c)

where

$0 < \alpha_1, \alpha_2, \alpha_3 < 1$ and

$\alpha_1, \alpha_2, and\ \alpha_3$ represent, respectively, the level, trend, and seasons parameters.

For the additive model, the forecast function is:

$$\hat{Y}_t(l) = \{a_0(t) + l a_1(t)\} + c\{t + l - s\}$$
$$l = 1, 2, \ldots \quad \textbf{(9)}$$

and the updating formulae are:

$$a_0(t) = \alpha_1\{y_t - c(t-s)\} + (1-\alpha_1)\{a_0(t-1) + a_1(t-1)\}$$

(10a)

$$c(t) = \alpha_3(y_t - a_0(t) + (1-\alpha_3) c(t-s)$$

(10b)

$a_1(t)$ is same as that of the multiplicative version. i.e. (2.8b).

Measure of Forecast Accuracy

The measure of accuracy adopted in this research is the forecast Root Mean Squared Error (RMSE). For each fitted model, it was computed based on forecasts for January to August 2013 and forecasts were rounded to their nearest integers for the purpose of computations. RMSE is defined as:

$$RMSE = \sqrt{\frac{1}{W}\sum_{t=1}^{W}(\hat{y}_t - y_t)^2}$$

(11)

Where

y_t is the actual observation;

$\hat{y}_t$ is the forecast.;

W is the number of forecasts.

1 to 24-steps ahead forecasts were obtained for each fitted model. Final model selection was based on the RMSE.

RESULTS AND DISCUSSIONS

Seasonal indices obtained from the Multiplicative Decomposition approach (Table 1) revealed that typically, minimum temperatures for January, February, November and December were lower than monthly average (of monthly minimum temperature), while for the other months, they were higher; forecasts for January 2013 to December 2014 are presented in Table 2. Seasonal indices produced by the Additive model (Table 3) corroborate the suggestion of the multiplicative model; the forecasts are also, presented in Table 4.

The smoothing constants utilized by the Holt-Winters' are presented in Table 5. The choice of zero for the trend was, informed by lack of visible trend in the data, as suggested by the plot of original data (Figure 1). Tables 6 and 8 present the forecasts and available actual values for the multiplicative and additive versions, respectively.

Tables 9, 12, and 15 present the parameter estimates of the fitted SARIMA models: SARIMA (1, 0, 0) X (1, 0, 1)12, SARIMA (1, 0, 0) X (1, 0, 1)12, SARIMA (1, 0, 0) X (1, 0, 1)$_{12}$. The results of the diagnostic tests (Tables 10, 13, and 16) for the models suggested adequacy (p-value>.05) at all lags involved; the forecasts, their 95% confidence limits and available actual values are presented in Tables 11, 14, and 17. It is worthy of mention that forecasts were rounded to the nearest whole numbers in the course of calculating the forecast RMSE (based on period: January to August 2013).

Of the SARIMA models (Table 18), SARIMA (1, 1, 1) X (1, 0, 1)$_{12}$ produced the least RMSE of 1.66, while SARIMA (1, 0, 0) X (1, 0, 1)$_{12}$ and SARIMA (1, 0, 1) X (1, 0, 1)$_{12}$ recorded 1.84. Holt-Winters' Multiplicative and Additive performed equally, having RMSE of 2.21; Decomposition Multiplicative and Additive models also recorded same value of 1.06. Generally, Decomposition has produced best out-of-sample forecasts, followed by Holt-Winters', then SARIMA (1, 1, 1) X (1, 0, 1)$_{12}$ and lastly, SARIMA (1, 0, 0) X (1, 0, 1)$_{12}$ and SARIMA (1, 0, 1) X (1, 0, 1)$_{12}$.

CONCLUSION

The monthly minimum temperature of Bida has been modeled, using Box-Jenkins, Holt-Winters' and the Decomposition methods. The Decomposition Multiplicative and Additive models (despite their simplicity) were found to produce the best out-of-sample forecasts, while the Holt-Winters' Multiplicative and Additive produced the least accurate, based on RMSE. Model sophistication is hence, not a measure of performance.

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APPENDIX I: Results of Decomposition Method

Trend Line Equation: Yt = 21.3199 + 6.46E-03*t

Table 1: Seasonal Indices for Decomposition (Multiplicative Model).

Period	Index
1	0.743621
2	0.919600
3	1.10277
4	1.17335
5	1.11766
6	1.07208
7	1.06684
8	1.05260
9	1.04157
10	1.05881
11	0.899271
12	0.751822

Table 2: Forecasts for Decomposition (Multiplicative Model).

Row	Period	Forecast	Actual
1	157	16.6080	19
2	158	20.5443	23
3	159	24.6436	26
4	160	26.2284	26
5	161	24.9907	25
6	162	23.9785	24
7	163	23.8682	23
8	164	23.5563	25
9	165	23.3162	
10	166	23.7090	
11	167	20.1424	
12	168	16.8446	
13	169	16.6657	
14	170	20.6156	
15	171	24.7291	
16	172	26.3193	
17	173	25.0773	
18	174	24.0616	
19	175	23.9509	
20	176	23.6379	
21	177	23.3969	
22	178	23.7911	
23	179	20.2121	
24	180	16.9029	

Trend Line Equation: Yt = 21.3199 + 6.46E-03*t

Table 3: Seasonal Indices for Decomposition (Additive model).

Period	Index
1	-5.52257
2	-1.73090
3	2.26910
4	3.72743
5	2.51910
6	1.53993
7	1.43576
8	1.14410
9	0.894097
10	1.24826
11	-2.18924
12	-5.33507

Table 4: Forecasts for Decomposition (Additive Model)

Row	Period	Forecast	Actual
1	157	16.8114	19
2	158	20.6096	23
3	159	24.6160	26
4	160	26.0808	26
5	161	24.8789	25
6	162	23.9062	24
7	163	23.8085	23
8	164	23.5233	25
9	165	23.2798	
10	166	23.6404	
11	167	20.2094	
12	168	17.0700	
13	169	16.8889	
14	170	20.6871	
15	171	24.6935	
16	172	26.1583	
17	173	24.9564	
18	174	23.9837	
19	175	23.8860	
20	176	23.6008	
21	177	23.3573	
22	178	23.7179	
23	179	20.2869	
24	180	17.1475	

APPENDIX II: Results of Holt-Winters' Modeling

Table 5: Smoothing Constants for Holt-Winters' Multiplicative Model.

Alpha (level): 0.1
Gamma (trend): 0.0
Delta (seasonal): 0.8

Table 6: Forecasts for Holt-Winter's multiplicative model

Row	Period	Forecast	Lower	Upper	Actual
1	157	15.7860	12.9957	18.5763	19
2	158	20.1697	16.4262	23.9133	23
3	159	21.7672	16.9991	26.5352	26
4	160	27.4020	21.5757	33.2283	26
5	161	25.1775	18.2747	32.0803	25
6	162	24.1092	16.1191	32.0994	24
7	163	23.0790	13.9945	32.1635	23
8	164	22.8757	12.6921	33.0593	25
9	165	22.8073	11.5212	34.0933	
10	166	22.7430	10.3521	35.1339	
11	167	20.3304	6.8328	33.8280	
12	168	15.9794	1.3737	30.5852	
13	169	15.6281	-0.0869	31.3431	
14	170	19.9679	3.1426	36.7931	
15	171	21.5491	3.6130	39.4853	
16	172	27.1273	8.0797	46.1750	
17	173	24.9249	4.7652	45.0846	
18	174	23.8671	2.5950	45.1393	
19	175	22.8470	0.4621	45.2320	
20	176	22.6456	-0.8525	46.1437	
21	177	22.5776	-2.0339	47.1892	
22	178	22.5138	-3.2113	48.2390	
23	179	20.1254	-6.7136	46.9644	
24	180	15.8182	-12.1349	43.7712	

Table 7: Smoothing Constants for Holt-Winter's Additive Model.

Alpha (level): 0.1
Gamma (trend): 0.0
Delta (seasonal): 0.8

Table 8: Forecasts for Holt-Winter's Additive Model.

Row	Period	FORE1	LOWE1	UPPE1	Actual
1	157	15.7706	13.0087	18.5324	19
2	158	20.2054	16.5001	23.9108	23
3	159	21.8618	17.1423	26.5812	26
4	160	27.2909	21.5240	33.0577	26
5	161	25.1307	18.2983	31.9631	25
6	162	24.0927	16.1840	32.0014	24
7	163	23.0835	14.0916	32.0753	23
8	164	22.8784	12.7987	32.9581	25
9	165	22.8135	11.6426	33.9844	
10	166	22.7514	10.4869	35.0159	
11	167	20.3359	6.9760	33.6959	
12	168	15.9301	1.4733	30.3868	
13	169	15.5608	0.0060	31.1155	
14	170	19.9956	3.3420	36.6493	
15	171	21.6520	3.8988	39.4052	
16	172	27.0811	8.2277	45.9345	
17	173	24.9209	4.9668	44.8750	
18	174	23.8829	2.8277	44.9381	
19	175	22.8737	0.7170	45.0304	
20	176	22.6686	-0.5899	45.9271	
21	177	22.6037	-1.7568	46.9642	
22	178	22.5416	-2.9212	48.0044	
23	179	20.1261	-6.4391	46.6914	
24	180	15.7203	-11.9477	43.3882	

APPENDIX III: Results of SARIMA Modeling

Table 9: Final Estimates of Parameters for SARIMA (1, 0, 0) X $(1, 0, 1)_{12.}$

Type	Coef	SE Coef	T	P
AR 1	0.3827	0.0757	5.06	0.000
SAR 12	0.9986	0.0069	145.17	0.000
SMA 12	0.8776	0.0565	15.53	0.000
Constant	0.02296	0.02556	0.90	0.370
Mean	26.55	29.55		

Table 10: Results of Ljung-Box test SARIMA (1, 0, 0) X $(1, 0, 1)_{12.}$

Lag	12	24	36	48
Chi-Square	5.6	22.4	38.5	45.0
DF	8	20	32	44
P-Value	0.693	0.322	0.200	0.428

Table 11: Forecasts from period 156 for SARIMA (1, 0, 0) X $(1, 0, 1)_{12}$.

Period	Forecast	95 Percent Limits Lower	Upper	Actual
157	16.2352	13.7471	18.7232	19
158	20.2557	17.5918	22.9197	23
159	23.6831	20.9944	26.3719	26
160	25.6114	22.9190	28.3038	26
161	24.4605	21.7676	27.1534	25
162	23.6267	20.9337	26.3197	24
163	23.3087	20.6157	26.0016	23
164	22.9523	20.2593	25.6453	25
165	23.0666	20.3736	25.7596	
166	23.0683	20.3753	25.7613	
167	20.0745	17.3815	22.7675	
168	16.7071	14.0141	19.4001	
169	16.5145	13.8048	19.2243	
170	20.3659	17.6537	23.0782	
171	23.7260	21.0134	26.4385	
172	25.6276	22.9149	28.3402	
173	24.4691	21.7564	27.1818	
174	23.6329	20.9203	26.3456	
175	23.3140	20.6014	26.0267	
176	22.9577	20.2450	25.6703	
177	23.0716	20.3589	25.7842	
178	23.0732	20.3606	25.7859	
179	20.0836	17.3710	22.7963	
180	16.7209	14.0082	19.4335	

Table 12: Final Estimates of Parameters for SARIMA (1, 0, 1) X $(1, 0, 1)_{12}$

Type	Coef	SE Coef	T	P
AR 1	0.7092	0.1342	5.28	0.000
SAR 12	0.9987	0.0060	166.52	0.000
MA 1	0.4014	0.1740	2.31	0.022
SMA 12	0.8791	0.0557	15.79	0.000
Constant	0.01013	0.01311	0.77	0.441
Mean	27.15	35.14		

Table 13: Ljung-Box Chi-Square statistic for SARIMA (1, 0, 1) X $(1, 0, 1)_{12}$

Lag	12	24	36	48
Chi-Square	1.5	16.8	29.6	36.4
DF	7	19	31	43
P-Value	0.981	0.605	0.540	0.751

Table 14: Forecasts from period 156 for SARIMA (1, 0, 1) X $(1, 0, 1)_{12}$

		95 Percent Limits		
Period	Forecast	Lower	Upper	Actual
157	16.4305	13.9565	18.9046	19
158	20.2898	17.7012	22.8783	23
159	23.6730	21.0287	26.3172	26
160	25.5960	22.9242	28.2679	26
161	24.4464	21.7608	27.1320	25
162	23.6138	20.9213	26.3063	24
163	23.2999	20.6039	25.9958	23
164	22.9483	20.2506	25.6460	25
165	23.0637	20.3651	25.7622	
166	23.0651	20.3661	25.7641	
167	20.0656	17.3663	22.7648	
168	16.6742	13.9748	19.3735	
169	16.5348	13.8174	19.2523	
170	20.3628	17.6434	23.0822	
171	23.7230	21.0026	26.4434	
172	25.6303	22.9094	28.3512	
173	24.4728	21.7516	27.1939	
174	23.6345	20.9133	26.3558	
175	23.3163	20.5950	26.0377	
176	22.9619	20.2405	25.6832	
177	23.0747	20.3533	25.7961	
178	23.0745	20.3531	25.7959	
179	20.0776	17.3562	22.7990	
180	16.6897	13.9683	19.4111	

Table 15: Final Estimates of Parameters for SARIMA (1, 1, 1) X $(1, 0, 1)_{12}$

Type	**Coef**	**SE Coef**	**T**	**P**
AR 1	0.3290	0.0846	3.89	0.000
SAR 12	0.9956	0.0096	103.68	0.000
MA 1	0.9581	0.0362	26.48	0.000
SMA 12	0.8131	0.0743	10.95	0.000
Constant	-0.000693	0.002893	-0.24	0.811

Table 16: Ljung-Box Chi-Square statistic for SARIMA (1, 1, 1) X $(1, 0, 1)_{12}$

Lag	12	24	36	48
Chi-Square	1.5	21.7	38.3	47.2
DF	7	19	31	43
P-Value	0.982	0.299	0.173	0.304

Table 17: Forecasts from period 156 for SARIMA (1, 1, 1) X $(1, 0, 1)_{12}$

		95 Percent Limits		
Period	**Forecast**	**Lower**	**Upper**	**Actual**
157	16.1587	13.6403	18.6772	19
158	20.5037	17.8177	23.1897	23
159	23.6872	20.9697	26.4047	26
160	25.6219	22.8937	28.3501	26
161	24.3587	21.6243	27.0932	25
162	23.5943	20.8548	26.3338	24
163	23.2207	20.4765	25.9648	23
164	22.8016	20.0529	25.5503	25
165	22.9411	20.1879	25.6943	
166	22.9370	20.1793	25.6947	
167	19.9571	17.1949	22.7193	
168	16.7904	14.0238	19.5571	
169	16.4130	13.5785	19.2475	
170	20.5620	17.7086	23.4154	
171	23.6726	20.8097	26.5354	
172	25.5787	22.7088	28.4486	
173	24.3138	21.4376	27.1901	
174	23.5497	20.6673	26.4320	
175	23.1760	20.2876	26.0643	
176	22.7574	19.8631	25.6518	
177	22.8953	19.9950	25.7956	
178	22.8901	19.9839	25.7964	
179	19.9223	17.0101	22.8345	
180	16.7686	13.8505	19.6866	

Table 18: Forecast RMSE based on January to August 2013 forecasts

Model	Model I	Model II	Model III	Model IV	Model V	Model VI	Model VII
RMSE	1.06	1.06	2.21	2.21	1.84	1.84	1.66

Key:
Model I: Decomposition (Multiplicative Model)
Model II: Decomposition (Additive Model)
Model III: Holt Winters' Multiplicative Model
Model IV: Holt Winters' Additive Model
Model V: SARIMA (1, 0, 0) X $(1, 0, 1)_{12}$
Model VI: SARIMA (1, 0, 1) X $(1, 0, 1)_{12}$
Model VII: SARIMA (1, 1, 1) X $(1, 0, 1)_{12}$

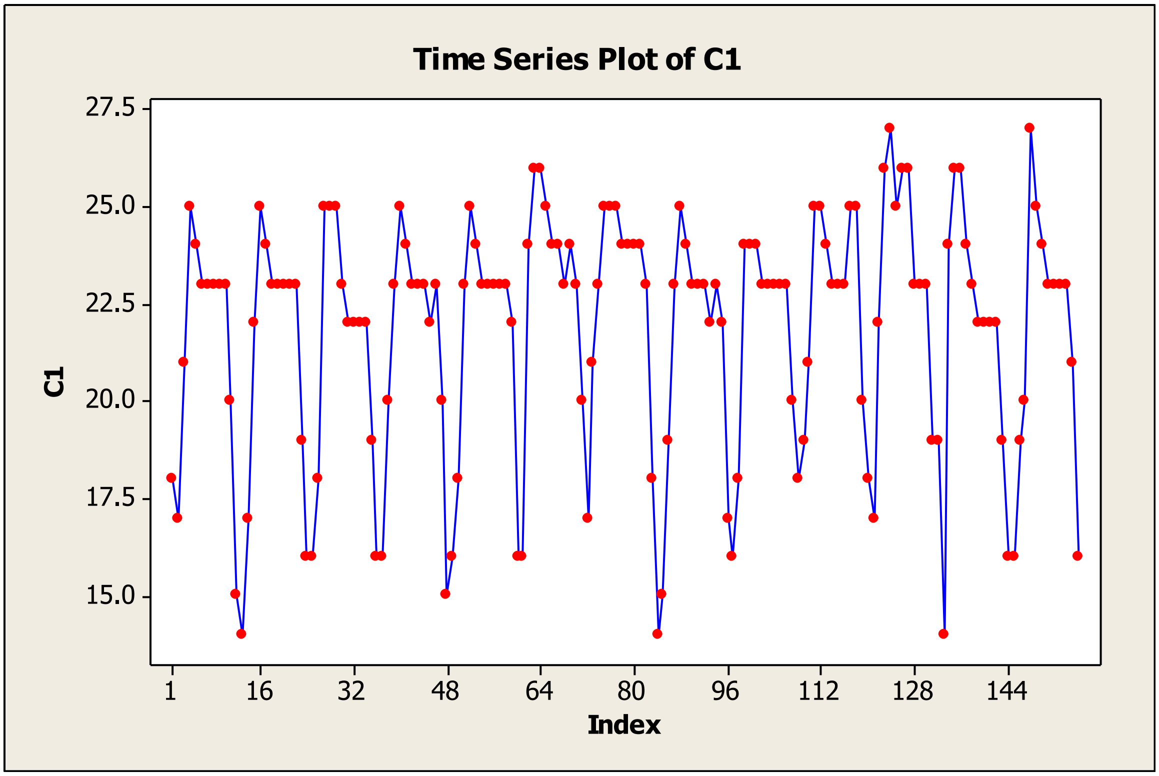

Figure 1: Plot of Original Data.

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