A Deterministic Compartmental Model of Tuberculosis Control Strategy Adopted by the National Tuberculosis and Leprosy Control Programme in Nigeria.

Abdullah Idris Enagi, Ph.D.

Department of Mathematics and Statistics, Federal University of Technology, Minna, Nigeria.

E-mail: aienagi@yahoo.com

ABSTRACT

In 1989 National Tuberculosis and Leprosy Control Programme was established to provide framework for the control of Tuberculosis and Leprosy in Nigeria. This was followed by the official launching in 1991 and adoption of Directly Observed Treatment Strategy for the control of Tuberculosis in Nigeria in 1993. In 2006 Tuberculosis was declared an emergency in Nigeria and the Stop Tuberculosis strategy for the control of Tuberculosis in Nigeria was adopted to reduce the prevalence of Tuberculosis to a level at which the disease will no longer constitute public health problems in the country.

In this work we present a deterministic model interpretation of method of control adopted by National Tuberculosis and Leprosy Control Program. We established the Disease free and the endemic equilibrium states and carried out the stability analysis of the disease free equilibrium state. We also carried out numerical simulations of the model using maple mathematical software to have an insight into the dynamics of the model. We found out that the disease free equilibrium state is stable. The numerical simulations showed that it will be very difficult to completely eradicate Tuberculosis from Nigeria using this method.

(Keywords: equilibrium state, disease free, stability and numerical simulations)

INTRODUCTION

Despite the fact that Tuberculosis (TB) is currently well-controlled in most countries, recent data indicate that the overall global incidence of TB is rising as a result of resurgence of disease in Africa and parts of Eastern Europe and Asia (Dye, 2006). In these regions, the emergence of drug-resistant TB and the convergence of the HIV (human immunodeficiency virus) and TB epidemics have created substantial new challenges for disease control.

Nigeria, a country with population of about 140,000,000 people, has an estimated incidence of 311 cases per 100,000 population. The estimated incidence for SM+ cases is 137 per 100,000 population and estimated prevalence of MDR-TB among new TB cases is 1.9% TB burden is further compounded by high National HIV prevalence of 4.4%. (National Tuberculosis and Leprosy Control Programme, Abuja, 2008).

In 1989 National Tuberculosis and Leprosy Control Program was established to provide framework for the control of TB and Leprosy in Nigeria. This was followed by the official launching in 1991 and adoption of DOTS strategy for the control of TB in Nigeria in1993. In 2006 TB was declared an emergency in Nigeria and the Stop TB strategy for the control of TB in Nigeria was adopted to reduce the prevalence of Tuberculosis to a level at which the disease will no longer constitute public health problems in the country.

The goal of the National TB programme is to reduce, significantly, the burden of TB by 2015 in line with the Millennium Development Goals (MDGs) and the STOP TB Partnership targets. (National Tuberculosis and Leprosy Control Programme, Abuja, 2008).

Mathematical models can be defined as the process of creating a mathematical representation of some phenomena in order to gain a better understanding of then. It is therefore, an abstraction of reality in to the world of mathematics. Any phenomena which have the ability to grow or decay over time can be represented by a mathematical model and then solved analytically where feasible or in several cases tools of advanced calculus and Functional Analysis are employed to study and interpret the

dynamics. Sowumi (1997) described this as experimenting on paper which is safer than using human or animal lives. Also, numerical or computer simulations of such models can be carried out. The analysis of such models will then give an insight into the dynamics of the real life situation. Mathematical knowledge such as the existence of equilibrium states and their stability analysis are of great interest in the mathematical models of population dynamics.

MATERIALS AND METHODS

Model Formulation

Consider the Susceptible-Infected (SI) model. Let S(t) and I(t) be the number of Susceptibles (i.e., those who can get the disease) and Infected Persons (i.e., those who have already gotten the disease), respectively. The number of the infected persons grows at a rate proportional to the product of Susceptibles and Infected Persons and the number of Susceptible persons decreases at the same rate so that we get the system of differential equations:

$$\frac{dS}{dt} = -\beta SI, \qquad \textbf{(1)}$$

$$\frac{dI}{dt} = \beta SI \qquad \textbf{(2)}$$

Modifying the above model given by (1) and (2) to allow recovery of infected person lead us to Susceptible-Infected-Susceptible (SIS) model. Here a susceptible person can become infected at a rate proportional to ***SI*** and an infected person can recover and become susceptible again at a rate γI , so that:

$$\frac{dS}{dt} = -\beta SI + \gamma I, \qquad \textbf{(3)}$$

$$\frac{dI}{dt} = \beta SI - \gamma I. \qquad \textbf{(4)}$$

We modify this model to allow for increase of the susceptible compartment through births and immigrations at the rate ρ and also split the Infected class into Latent and Infectious class. The latent class consists of individuals that have been infected by Mycobacterium Tuberculosis but have intact cell mediated immune system which attracts the bacteria towards the alveolar macrophages which ingested the bacteria thereby preventing them from replicating and spreading to other parts of the body. Members of this class show no symptom of Tuberculosis and cannot infect others but can progress to Infectious class whenever their immune system got broken down.

The infectious compartment consists of those individuals whose immune system got compromised by any immunosuppressive condition. The bacteria overcome the immune system replicates and spreads through the blood streams and the lymphatic system to other parts of the body. Members of this class show symptoms of tuberculosis and can spread it to others. The last compartment is the recovered class individuals that were treated and declared cure of tuberculosis.

The Susceptible population changes due to the coming in of new Susceptible into the population where we assumed that people come into the location of interest at a constant rate ρ (Through birth or immigration). The Susceptible population also diminishes due to natural death at rate μ and infection with an incident rate of infection β .

In the same way the population dynamic of the Latent class grows with the instantaneous incidence rate of infection β . The population of this class diminishes by natural death rate μ , and occasional breakdown to the infectious class at the rate τ .

The dynamics of the infectious class population is dependent on the Latent class degenerating to infectious class at the rate τ . This class also reduces by natural death rate μ , successful cure of infectious TB patients at the rate γ and death caused as a result of chronic TB infection at the rate δ .

In our study, we assumed that there is homogeneous mixing of the population where all people are equally likely to be infected by infectious individuals in case of contact. We assumed equal natural death rate μ for each compartment. We also assumed that successfully

treated individuals moved to recovered or removed class meaning there will be no reinfection (i.e., permanent immunity). Note that in this model we have no latent TB treatment and this is the model interpretation of the TB control programme we have in Nigeria today.

Model Diagram

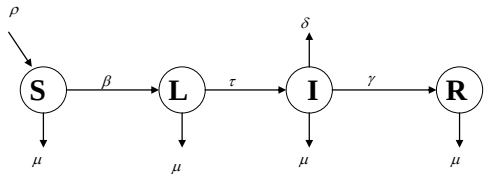

Figure 1: Showing Schematic Presentation of the Model.

Model Equations

$$\frac{dS}{dt} = \rho - \beta SI - \mu S \quad \textbf{(5)}$$

$$\frac{dL}{dt} = \beta SI - (\tau + \mu)L \quad \textbf{(6)}$$

$$\frac{dI}{dt} = \tau L - (\gamma + \mu + \delta)I \quad \textbf{(7)}$$

$$\frac{dR}{dt} = \gamma I - \mu R \quad \textbf{(8)}$$

Equilibrium Solutions

We now solve the model equations to obtain the equilibrium states as in Enagi (2011).

At the equilibrium state $\frac{dS}{dt} = \frac{dL}{dt} = \frac{dI}{dt} = \frac{dR}{dt} = 0$.

Let S(t)=w, L(t) = x, I(t) = y and R(t) = z.

Then the system of equations becomes,

$$\rho - \beta wy - \mu w = 0 \quad \textbf{(9)}$$

$$\beta wy - (\tau + \mu)x = 0 \quad \textbf{(10)}$$

$$\tau x - (\gamma + \mu + \delta)y = 0 \quad \textbf{(11)}$$

$$\gamma y - \mu z = 0 \quad \textbf{(12)}$$

We now solve this system of equations simultaneously to obtain:

(w,x,y,z) = $(\frac{\rho}{\mu},0,0,0)$ as the disease free equilibrium state and,

$$w = \frac{(\tau + \mu)(\gamma + \mu + \delta)}{\tau\beta},$$

$$x = \frac{[\tau\beta\rho - \mu(\tau + \mu)(\gamma + \mu + \delta)]}{\beta(\mu + \tau)\tau},$$

$$y = \frac{[\tau\beta\rho - \mu(\tau + \mu)(\gamma + \mu + \delta)]}{\beta(\gamma + \mu + \delta)(\mu + \tau)},$$

$$z = \frac{[\tau\beta\rho - \mu(\tau + \mu)(\gamma + \mu + \delta)]\gamma}{\beta\mu(\gamma + \mu + \delta)(\mu + \tau)},$$

as the endemic equilibrium state.

Stability Analysis

Having established the equilibrium states. We now investigate the stability of the disease free equilibrium states. To obtain this, we examine the behaviour of the model population near the equilibrium state (Yusuf, 2008).

The Characteristic Equation

Recall that the system of equations in model one at equilibrium state is:

$$\rho - \beta wy - \mu w = 0$$

$$\beta wy - (\tau + \mu)x = 0$$

$$\tau x - (\gamma + \mu + \delta)y = 0$$

$$\gamma y - \mu z = 0$$

We obtain the Jacobian matrix of this system of equations as presented by Benyah (2008).

$$J_1 = \begin{pmatrix} -(\mu+\beta y) & 0 & -\beta w & 0 \\ \rho y & -(\tau+\mu) & 0 & 0 \\ 0 & \tau & -(\gamma+\mu+\delta) & 0 \\ 0 & 0 & \gamma & -\mu \end{pmatrix}$$

The characteristic equation is obtained from the Jacobian determinant with the eigenvalues λ and (w,x,y,z) = ($\frac{\rho}{\mu}$,0,0,0) as:

$$(\mu+\lambda)^2(\tau+\mu+\lambda)(\gamma+\mu+\delta+\lambda)=0$$
$$\Rightarrow$$
$$\lambda_1=-\mu,\ \lambda_2=-\mu,\ \lambda_3=-(\tau+\mu)$$
and $\lambda_4=-(\gamma+\mu+\delta)$.

Since all the eigenvalues are negative, the disease free equilibrium state is stable.

Numerical Simulations of the Models (Using Maple software)

The numerical simulations are meant to study the profile of the population in respect of the distinct compartments in the model and to consider the effect of varying contraction and recovery rates on the population.

From the available literature we adopted the following values for the parameters in the model.
Birth rate ρ = 0.045 (National Population Commission, Abuja, 2008).

Natural death rate μ= 0.014 (National Population Commission, Abuja, 2008).

Movement rate from Latent class to infectious class τ = 0.03 (Sanchez and Blower 1997,WHO 2006a, WHO 2006b).

Initial recovery rate of I(t) γ =0.23 (National Tuberculosis and Leprosy Control Programme, Abuja, 2008).

Tuberculosis induced death rate δ = 0.001 (National Tuberculosis and Leprosy Control Programme, Abuja, 2008).

Tuberculosis contraction rate β (varied hypothetically).

RESULTS AND DISCUSSION

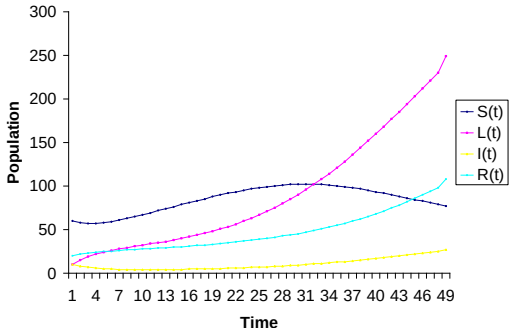

Figure 2: Graphical Profile of each Compartment for β=0.01 and γ=0.23.

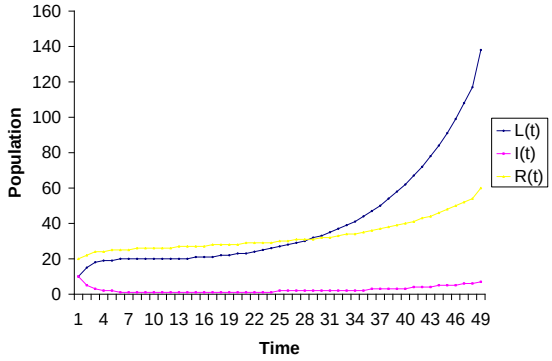

Figure 3: Graphical Profile of each Compartment for β=0.01 and γ=0.23.

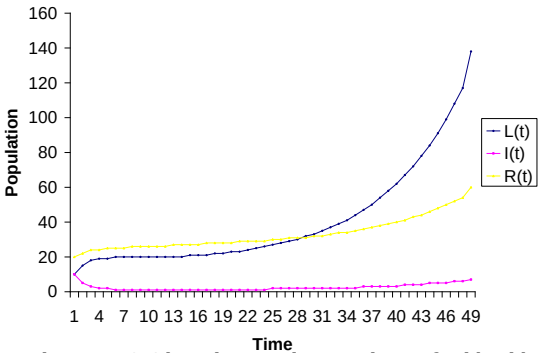

Figure 4: Closer View of L(t), I(t), and R(t) for β=0.01 and γ=0.23.

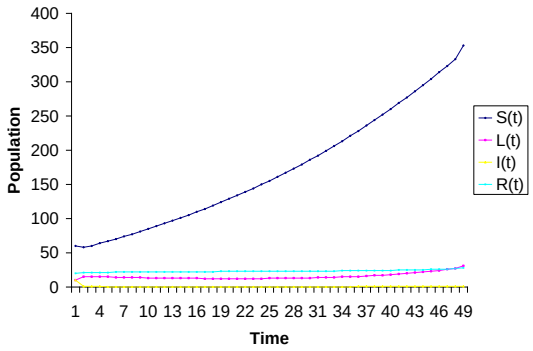

Figure 5: Graphical Profile of each Compartment for β=0.01 and γ=0.9.

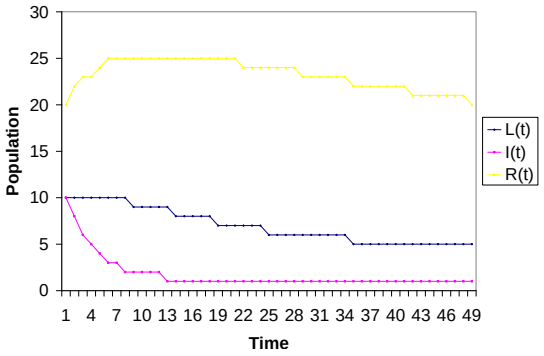

Figure 8: Closer View of L(t), I(t), and R(t) for β=0.001 and γ=0.23.

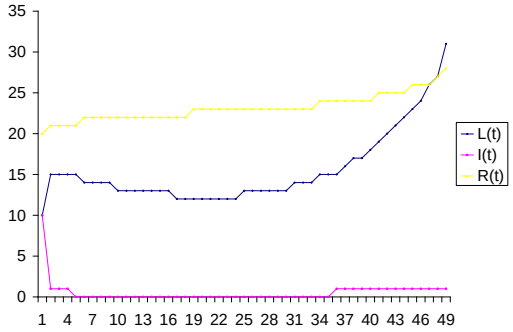

Figure 6: Closer View of L(t), I(t), and R(t) for β=0.01 and γ=0.9.

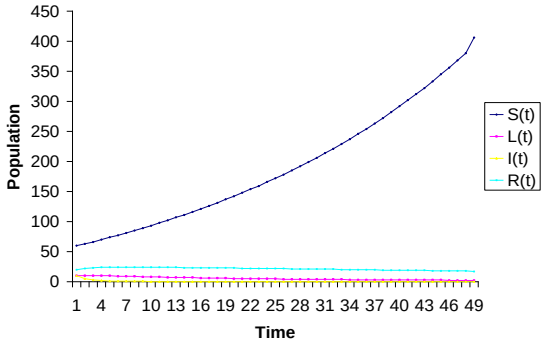

Figure 9: Graphical Profile of each Compartment for β=0.001 and γ=0.5.

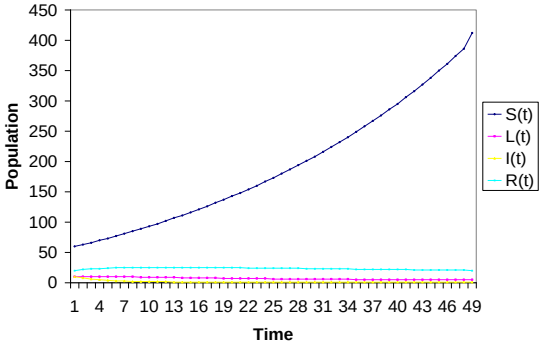

Figure 7: Graphical Profile of each Compartment for β=0.001 and γ=0.23.

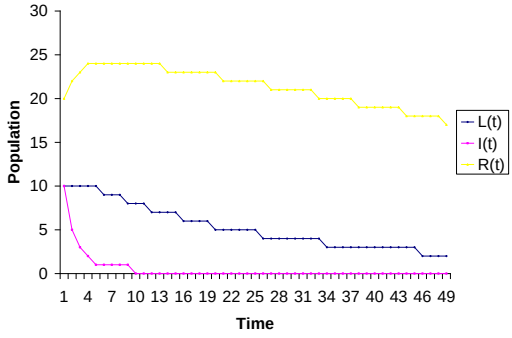

Figure 10: Closer View of L(t), I(t), and R(t) for β=0.001 and γ=0.5.

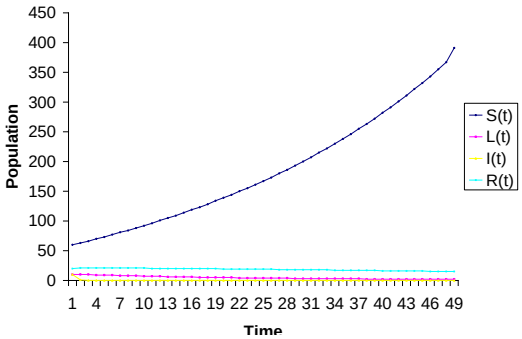

Figure 11: Graphical Profile of each Compartment for β=0.001 and γ=0.9.

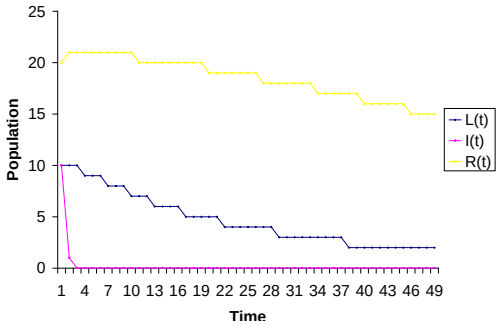

Figure 12: Closer View of L(t), I(t), and R(t) for β=0.001 and γ=0.9.

CONCLUSION

The existence of the Disease Free Equilibrium state implies that there is possibility of complete total eradication of Tuberculosis from Nigeria. The negativity of all the eigenvalues arising from the stability analysis carried out in section four shows that there will be no return to the Tuberculosis endemic state after eradication of Tuberculosis from Nigeria. The existence of the endemic equilibrium state in section three signifies the possibility of Nigeria remaining a Tuberculosis endemic Nation. The numerical simulations show that total eradication of tuberculosis using this method of control is achievable within three years if the rate of infection is reduced to 0.1% alongside of a 90% recovery rate shown in Figure 12. However, achieving this will be very difficult in reality. A better alternative method of control will be to introduce Latent Tuberculosis treatment alongside of the current method.

REFERENCES

1. Benyah, F. 2008. “Introduction to Epidemiological Modelling”. 10th Regional College on Modelling, Simulation and Optimization. University of Cape Coast, Ghana, 13-19 April 2008.

2. Dye, C. 2006. “Global Epidemiology of Tuberculosis”. *The Lancet*. 367(9514):938–940.

3. Enagi, A.I. 2011. “Modelling the Effect of Anti-Retroviral Therapy and Latent Tuberculosis Treatment in Controlling the Spread of Tuberculosis in Nigeria. Science Alert”. *International Journal of Current Research in Tuberculosis*. USA. (On line) ISSN 1819-3366, 3(1):9-15.

4. National Population Commission, Abuja. 2008. “Report of Live Births, Deaths & Stillbirths In Nigeria (1994-2007)” http://www.population.gov.ng/images/stories/Report%20on%20Birth-Death-Stillbirth-Registration.pdf

5. National Tuberculosis and Leprosy Control Programme, Abuja. 2008. “Annual Report”. http://www.ntbltc.org/reports/Annual%20Report%202008%20NTBLCP.pdf

6. Sanchez, M.A. and S.M. Blower. 1997. “Uncertainty and Sensitivity Analysis of the Basic Reproductive Rate: Tuberculosis as an Example”. *Am. J. Epidemiol*. 145(12):1127–1137.

7. Sowumi, C.O. 1997. “Stability of Steady State and Boundedness of a 2 Sex Population Model.” *Nonlinear Analysis*. 39(2000):693-709.

8. World Health Organization (WHO). 2006a. “Frequently Asked Questions About TB and HIV”. WHO: Geneva, Switzerland. Retrieved September 9, 2006, http ://www .who.int/ tb/hiv/faq/en/ index.htrnl.

9. World Health Organization (WHO). 2006b. “Tuberculosis Fact Sheet”. WHO: Geneva, Switzerland. Retrieved September 9, 2006, http://www.who. int/mediacentre/factsheets/fs 104/en/index.html.

10. Yusuf , T. 2008. “Mathematical Model to Simulate Tuberculosis Disease Population Dynamics”. *American Journal of Applied Sciences*. 5(4):301-306. 2008 ISSN 1546-9239 2008.

ABOUT THE AUTHOR

Dr. Abdullah Idris Enagi, is a Lecturer in the Department of Mathematics and Statistics, Federal University of Technology, Minna, Nigeria. He holds a Ph.D. degree in Applied Mathematics from Usmanu Danfodiyo University Sokoto, Nigeria. His research interest is in mathematical modelling of infectious diseases.

SUGGESTED CITATION

Enagi, A.I. 2013. "A Deterministic Compartmental Model of Tuberculosis Control Strategy Adopted by the National Tuberculosis and Leprosy Control Programme in Nigeria". *Pacific Journal of Science and Technology*. 14(1):342-348.

Pacific Journal of Science and Technology