Lime Shaft Kilns: Modeling and Simulation.

R.O. Olayiwola, Ph.D.[*1]; A.O. Adesanya, Ph.D.[2]; A.M. Okedoye, Ph.D.[3]; and A.O. Popoola, Ph.D.[4]

[1]Department of Mathematics and Statistics, Federal University of Technology, PMB 65, Minna, Nigeria.
[2]Department of Pure and Applied Mathematics, Ladoke Akintola University of Technology, Ogbomoso, Nigeria.
[3]Department of Mathematics, Covenant University, Ogun State, Nigeria.
[4]Department of Mathematical and Physical Sciences, Osun State University, Osogbo, Nigeria.

E-mail: olayiwolarasaq@yahoo.co.uk*

ABSTRACT

This study presents a mathematical model to predict the heat transfer in a lime kiln. We assume the reaction is not well-stirred. We prove the existence of unique solution of the time-dependent problems. We also examine the properties of solution under certain conditions. The time-dependent temperature profiles are obtained through analytical method. It is discovered that to increase the furnace productivity depends on the parameters involved.

(Keywords: lime shaft kiln, lime production, calcination, preheat zone, burning zone, cooling zone.)

INTRODUCTION

The term calcination refers to the process of limestone thermal decomposition into quicklime and carbon dioxide. The following chemical reaction takes place in the kiln with dolomitic limestone:

$$CaCO_3 + MgCO_3 + HEAT \rightarrow CaO + MgO + 2CO_2 \qquad \textbf{(1)}$$

Heat is created in the kiln by burning pulverized coal, natural gas or oil. Kilns are normally operated at temperatures of 1100°C or higher to drive carbon dioxide from the limestone.

A lime shaft kiln is basically a moving bed reactor with the upward-flow of hot gases passing counter-current to the downward-flow of a feed consisting of limestone particles undergoing calcination. A kiln basically has three operating sections: the preheating, the burning and the cooling zone. The preheating zone is that part of the kiln where the limestone is heated to its dissociation temperature. The burning zone is that part of the kiln in which reaction of the burden takes place. The cooling zone is that part of the kiln in which the lime emerging from the burning zone is cooled before discharge.

The most common fuels used in shaft kilns are coke, natural gas, weak gas and pulverized lignite [1].

The majority of shaft furnaces for limestone calcination operate with counter-current flows of burden materials and gases (Boynton [2], Terruzzi [3], Tabunshikov [4] and Monastirev and Aleksandrov [5]). The furnace incorporates three technological zones: preheating, calcination and cooling (from top to bottom).

Gordon et al. [6] developed the multi-dimensional mathematical model to optimize the furnace design and the process parameters. The developed mathematical model belongs to the group of essentially non-linear models. According to them, it is not possible to develop an analytical solution of the problem. The finite element method was used to provide a solution.

Olayiwola et al. [7] developed a mathematical model of calcination process. The developed model took into account the Arrhenius heat generation and chemical reaction. They provided an analytical solution of the model and investigated the effects of activation energy and Frank-Kamenetskii parameters on the gas and material temperatures.

In this paper we extend the model developed in [7] to account for a situation where the reaction is not well-stirred. We prove the existence and uniqueness of solution of the time-dependent problems. We also examine the properties of solution under certain conditions. The equations are solved analytically and investigated for a wide range of parameters involved.

MATHEMATICAL MODEL

Here, we assume that the reaction is not well-stirred, so that, change depends on both time and space variable. Then the primary dependent variables are the material temperature, $T_m(x,t)$ and the gas temperature, $T_g(x,t)$. Under this assumption, we arrived at the following transient energy equation

For Material:

$$\rho_m(1-\varepsilon)C_m\frac{\partial T_m}{\partial t}=\lambda_m\frac{\partial^2 T_m}{\partial x^2}-\alpha_v\left(T_g-T_m\right)+q_m+QAe^{-\frac{E}{RT_g}}=0 \quad \textbf{(2)}$$

For Gas:

$$\rho_g\varepsilon C_g\frac{\partial T_g}{\partial t}=\lambda_g\frac{d^2 T_g}{dx^2}+\alpha_v\left(T_g-T_m\right)+q_g+QAe^{-\frac{E}{RT_g}}=0 \quad \textbf{(3)}$$

The initial and boundary conditions are:

$$T_m(x,0)=T_0,\ \ T_m(0,t)=T_0,\ \ T_m(L,t)=T_0 \quad \textbf{(4)}$$

$$T_g(x,0)=T_0,\ \ T_g(0,t)=T_0,\ \ T_g(L,t)=T_0 \quad \textbf{(5)}$$

Here, ρ is density, E is activation energy, R is gas constant, q is heat source, α is heat transfer coefficient, λ is thermal conductivity, C is heat capacity, ε is porosity, T is temperature, t is time, x is position, Q is heat of reaction, A is pre-exponential factor, g is gas, m is material.

METHOD OF SOLUTION

Let $\theta=\frac{E}{RT_0^2}\left(T_g-T_0\right),\quad \phi=\frac{E}{RT_0^2}\left(T_m-T_0\right),$

$$t'=\frac{t}{t_0},\qquad x'=\frac{x}{L},\qquad \in=\frac{RT_0}{E}$$

Then (2) - (5) becomes:

$$\frac{\partial\phi}{\partial t}=\lambda_1\frac{\partial^2\phi}{\partial x^2}-\alpha_1(\theta-\phi)+\beta_1+\delta_1\exp\left(\frac{\theta}{1+\in\theta}\right) \quad \textbf{(6)}$$

$$\frac{\partial\theta}{\partial t}=\lambda_2\frac{\partial^2\theta}{\partial x^2}+\alpha_2(\theta-\phi)+\beta_2+\delta_2\exp\left(\frac{\theta}{1+\in\theta}\right) \quad \textbf{(7)}$$

together with the initial and boundary conditions,

$$\phi(x,0)=0,\qquad \phi(0,t)=0,\qquad \phi(1,t)=0 \quad \textbf{(8)}$$

$$\theta(x,0)=0,\qquad \theta(0,t)=0,\qquad \theta(1,t)=0 \quad \textbf{(9)}$$

where,

$$\alpha_1=\frac{\alpha_v t_0}{\rho_m(1-\varepsilon)C_m},$$

$$\beta_1=\frac{q_m t_0}{\in T_0\rho_m(1-\varepsilon)C_m},$$

$$\alpha_2=\frac{\alpha_v t_0}{\rho_g\varepsilon C_g},$$

$$\beta_2=\frac{q_g t_0}{\in T_0\rho_g\varepsilon C_g},$$

$\delta_1=\dfrac{QAt_0\exp\left(-\frac{E}{RT_0}\right)}{\in T_0\rho_m(1-\varepsilon)C_m}$ is the Frank-Kamenetskii parameter for material

$\delta_2=\dfrac{QAt_0\exp\left(-\frac{E}{RT_0}\right)}{\in T_0\rho_g\varepsilon C_g}$ is the Frank-Kamenetskii parameter for gas

$\lambda_1 = \dfrac{\lambda_m t_0}{\rho_m (1-\varepsilon) C_m L^2}$ is the scaled thermal conductivity for material, and

$\lambda_2 = \dfrac{\lambda_g t_0}{\rho_g \varepsilon C_g L^2}$ is the scaled thermal conductivity for gas.

Existence and Uniqueness of Solution

Theorem 1 Let $\lambda_1 = \lambda_2 = \lambda$, $\delta_1 = \delta_2 = \delta$. Then, there exists a unique solution of (6) and (7) satisfy (8) and (9).

Proof: Let $\lambda_1 = \lambda_2 = \lambda$, $\delta_1 = \delta_2 = \delta$ and $u = \theta - \phi$. Then (6) - (9) becomes.

$$\frac{\partial u}{\partial t} = \lambda \frac{\partial^2 u}{\partial x^2} + \alpha u + \beta \quad \textbf{(10)}$$

$$u(x,0) = 0, \quad u(0,t) = 0, \quad u(1,t) = 0, \quad \textbf{(11)}$$

where,

$$\alpha = \alpha_2 + \alpha_1, \qquad \beta = \beta_2 - \beta_1$$

For the solution of (10), we assume,

$$u(x,t) = \sum_{n=1}^{\infty} u_n(t) \sin \frac{n\pi x}{L}, \quad \textbf{(12)}$$

where,

$$u_n(t) = \int_0^t e^{\left(\alpha - \lambda \frac{n^2\pi^2}{L^2}\right)(t-\tau)} F_n(\tau)\, d\tau + b_n e^{\left(\alpha - \lambda \frac{n^2\pi^2}{L^2}\right)t} \quad \textbf{(13)}$$

$$F_n(t) = \frac{2}{L}\int_0^L F(x,t) \sin \frac{n\pi x}{L}\, dx \quad \textbf{(14)}$$

$$b_n = \frac{2}{L}\int_0^L f(x) \sin \frac{n\pi x}{L}\, dx \quad \textbf{(15)}$$

Here, $f(x) = 0$, $F(x,t) = \beta$ and $L = 1$, so,

$$u(x,t) = \sum_{n=1}^{\infty} \frac{2\beta\left(1-(-1)^n\right)\left(e^{(\alpha - \lambda n^2\pi^2)t} - 1\right)}{n\pi\left(\alpha - \lambda n^2 \pi^2\right)} \sin n\pi x \quad \textbf{(16)}$$

Then, we obtain:

$$\phi(x,t) = \theta(x,t) - \sum_{n=1}^{\infty} \frac{2\beta\left(1-(-1)^n\right)\left(e^{(\alpha - \lambda n^2\pi^2)t} - 1\right)}{n\pi\left(\alpha - \lambda n^2 \pi^2\right)} \sin n\pi x \quad \textbf{(17)}$$

$$\theta(x,t) = \sum_{n=1}^{\infty} \frac{2\beta\left(1-(-1)^n\right)\left(e^{(\alpha - \lambda n^2\pi^2)t} - 1\right)}{n\pi\left(\alpha - \lambda n^2 \pi^2\right)} . \sin n\pi x + \phi(x,t) \quad \textbf{(18)}$$

Hence, there exists a unique solution of problem (6) and (7) satisfy (8) and (9). This completes the proof.

Properties of Solution

Theorem 2 Let $\in > 0$, $\lambda_1 = \lambda_2 = 1$ and $\alpha_1 = \alpha_2 = 0$ in (6) and (7). Then $\dfrac{\partial \phi}{\partial t} \geq 0$ and $\dfrac{\partial \theta}{\partial t} \geq 0$.

In the proof, we shall make use of following Lemma of Kolodner and Pederson [8].

Lemma (Kolodner and Pederson [8]) Let $u(x,t) = 0\left(e^{\alpha |x|^2}\right)$ be a solution on $R^n \times [0,t)$ of the differential inequality $\dfrac{\partial u}{\partial t} - \Delta u + K(x,t) u \geq 0$, where K is bounded from below. If $u(x,0) \geq 0$, then $u(x,t) \geq 0$ for all $(x,t) \in R^n \times [0,t_0)$.

Proof of Theorem 2: Let $\in > 0$, $\lambda_1 = \lambda_2 = 1$ *and* $\alpha_1 = \alpha_2 = 0$ in (6) and (7). We obtain

$$\frac{\partial \phi}{\partial t} - \frac{\partial^2 \phi}{\partial x^2} - \delta_1 e^{\frac{\theta}{1+\in\theta}} = \beta_1$$

$$\frac{\partial \theta}{\partial t} - \frac{\partial^2 \theta}{\partial x^2} - \delta_2 e^{\frac{\theta}{1+\in\theta}} = \beta_2$$

Differentiating with respect to t, we have

$$\frac{\partial}{\partial t}\left(\frac{\partial\phi}{\partial t}\right)-\frac{\partial^2}{\partial x^2}\left(\frac{\partial\phi}{\partial t}\right)-\left(\begin{matrix}\delta_1\left(\frac{1}{1+\in\theta}\right)^2\cdot\\ e^{\frac{\theta}{1+\in\theta}}\end{matrix}\right)\frac{\partial\theta}{\partial t}=0$$

$$\frac{\partial}{\partial t}\left(\frac{\partial\theta}{\partial t}\right)-\frac{\partial^2}{\partial x^2}\left(\frac{\partial\theta}{\partial t}\right)-\left(\begin{matrix}\delta_2\left(\frac{1}{1+\in\theta}\right)^2\cdot\\ e^{\frac{\theta}{1+\in\theta}}\end{matrix}\right)\frac{\partial\theta}{\partial t}=0$$

Let

$$u=\frac{\partial\phi}{\partial t}\ \text{ and }\ v=\frac{\partial\theta}{\partial t}$$

then

$$\frac{\partial u}{\partial t}-\frac{\partial^2 u}{\partial x^2}\geq 0$$
$$\frac{\partial v}{\partial t}-\frac{\partial^2 v}{\partial x^2}-\left(\delta_2\left(\frac{1}{1+\in\theta}\right)^2 e^{\frac{\theta}{1+\in\theta}}\right)v\geq 0$$

This can be written:

$$\frac{\partial u}{\partial t}-\frac{\partial^2 u}{\partial x^2}+K_1(x,t)u\geq 0,$$

$$\frac{\partial v}{\partial t}-\frac{\partial^2 v}{\partial x^2}+K_2(x,t)v\geq 0,$$

where

$$K_1(x,t)=0$$
$$K_2(x,t)=-\delta_2\left(\frac{1}{1+\in\theta}\right)^2 e^{\frac{\theta}{1+\in\theta}}$$

Clearly, K_1 is bounded everywhere and K_2 is bounded from below. Hence by Kolodner and Pederson's lemma $u(x,t)\geq 0$ and $v(x,t)\geq 0$ i.e., $\frac{\partial\phi}{\partial t}\geq 0$ and $\frac{\partial\theta}{\partial t}\geq 0$. This completes the proof.

Analytical Solution

Here, we consider equations (6) and (7) in the form:

$$\frac{\partial\phi}{\partial t}=\lambda_1\frac{\partial^2\phi}{\partial x^2}-\alpha_1 u+\beta_1+\delta_1\exp\left(\frac{\theta}{1+\in\theta}\right)\quad\textbf{(19)}$$

$$\frac{\partial\theta}{\partial t}=\lambda_2\frac{\partial^2\theta}{\partial x^2}+\alpha_2 u+\beta_2+\delta_2\exp\left(\frac{\theta}{1+\in\theta}\right)\quad\textbf{(20)}$$

Ayeni [9] has shown that $\exp\left(\frac{\theta}{1+\in\theta}\right)$ can be approximated as $1+(e-2)\theta+\theta^2$. In this paper we are going to take an approximation of the form:

$$\exp\left(\frac{\theta}{1+\in\theta}\right)\approx 1+(e-2)\theta\quad\textbf{(21)}$$

Then (19) and (20) can be written as:

$$\frac{\partial\phi}{\partial t}=\lambda_1\frac{\partial^2\phi}{\partial x^2}-\alpha_1 u+\beta_1+\delta_1(1+(e-2)\theta)\quad\textbf{(22)}$$

$$\frac{\partial\theta}{\partial t}=\lambda_2\frac{\partial^2\theta}{\partial x^2}+\alpha_2 u+\beta_2+\delta_2(1+(e-2)\theta)\quad\textbf{(23)}$$

We obtain the solution of (22) and (23) as:

$$\theta(x,t)=\sum_{n=1}^{\infty}\frac{2(\beta_2+\delta_2)(1-(-1)^n)}{n\pi(p_2-\lambda_2 n^2\pi^2)}\begin{pmatrix}e^{(p_2-\lambda_2 n^2\pi^2)t}\\ -1\end{pmatrix}\sin n\pi x+\sum_{n=1}^{\infty}\left(\begin{matrix}\frac{2\alpha_2\beta(1-(-1)^n)}{n\pi(\alpha-\lambda n^2\pi^2)}\\ \sum_{n=1}^{\infty}\left(\begin{matrix}\frac{e^{(p_2-\lambda_2 n^2\pi^2)t}-e^{(\alpha-\lambda n^2\pi^2)t}}{p_2-\lambda_2 n^2\pi^2+\alpha-\lambda n^2\pi^2}+\\ \frac{1-e^{(p_2-\lambda_2 n^2\pi^2)t}}{p_2-\lambda_2 n^2\pi^2}\end{matrix}\right)\end{matrix}\right)\sin n\pi x\quad\textbf{(24)}$$

$$
\begin{aligned}
\phi(x,t) = {} & \sum_{n=1}^{\infty} \frac{2(\beta_1+\delta_1)\left(1-(-1)^n\right)}{\lambda_1 n^4 \pi^4}\left(1-e^{-\lambda_1 n^2\pi^2 t}\right) \\
& \sin n\pi x + \\
& \sum_{n=1}^{\infty}\left(\frac{2p_1(\beta_2+\delta_2)\left(1-(-1)^n\right)}{n\pi\left(p_2-\lambda_2 n^2\pi^2\right)} \sum_{n=1}^{\infty}\left(\frac{e^{\left(p_2-\lambda_2 n^2\pi^2\right)t}-e^{-\lambda_1 n^2\pi^2 t}}{p_2-\lambda_2 n^2\pi^2+\lambda_1 n^2\pi^2} + \frac{e^{-\lambda_1 n^2\pi^2 t}-1}{\lambda_1 n^2\pi^2} \right)\right) \\
& \sin n\pi x + \\
& \sum_{n=1}^{\infty}\left(\sum_{n=1}^{\infty}\left(\frac{2p_1\alpha_2\beta\left(1-(-1)^n\right)}{n\pi\left(\alpha-\lambda n^2\pi^2\right)} \left(\frac{e^{\left(\alpha-\lambda n^2\pi^2\right)t}-e^{-\lambda_1 n^2\pi^2 t}}{S\left(\alpha+\lambda_1 n^2\pi^2-\lambda n^2\pi^2\right)} - \frac{e^{\left(p_2-\lambda_2 n^2\pi^2\right)t}-e^{-\lambda_1 n^2\pi^2 t}}{S\left(p_2-\lambda_2 n^2\pi^2+\lambda_1 n^2\pi^2\right)} + \frac{1-e^{-\lambda_1 n^2\pi^2 t}}{\lambda_1 n^2\pi^2\left(p_2-\lambda_2 n^2\pi^2\right)} - \frac{e^{\left(p_2-\lambda_2 n^2\pi^2\right)t}-e^{-\lambda_1 n^2\pi^2 t}}{\left(p_2-\lambda_2 n^2\pi^2+\lambda_1 n^2\pi^2\right)S_1} \right)\right)\right) \\
& \sin n\pi x - \sum_{n=1}^{\infty}\left(\sum_{n=1}^{\infty} \frac{2\beta\alpha_1\left(1-(-1)^n\right)}{n\pi\left(\alpha-\lambda n^2\pi^2\right)} \left(\frac{e^{\left(\alpha-\lambda n^2\pi^2\right)t}-e^{-\lambda_1 n^2\pi^2 t}}{\alpha-\lambda n^2\pi^2+\lambda_1 n^2\pi^2} + \frac{e^{-\lambda_1 n^2\pi^2 t}-1}{\lambda_1 n^2\pi^2} \right)\right) \quad \textbf{(25)} \\
& \sin n\pi x
\end{aligned}
$$

where,

$p_1 = \delta_1(e-2),\ p_2 = \delta_2(e-2)$,

$S = \alpha - p_2 + \lambda_2 n^2\pi^2 - \lambda n^2\pi^2$,

$S_1 = p_2 - \lambda_2 n^2\pi^2$

RESULTS AND DISCUSSION

We have proved the existence and uniqueness of solution of the Problem by actual solution. Also, we have shown that $\phi(x,t)$ and $\theta(x,t)$ are non-decreasing function of time, under certain conditions. The gas temperature and material temperature profiles are presented in Figures 1-12.

Figure 1 displays the graph of $\theta(x,t)$ against x and t for different values of δ_2.

Figure 2 displays the graph of $\theta(x,t)$ against x for different values of δ_2.

Figure 3 displays the graph of $\theta(x,t)$ against t for different values of δ_2.

From Figures 1-3 it is seen that gas temperature increases as Frank-Kamenetskii number increases.

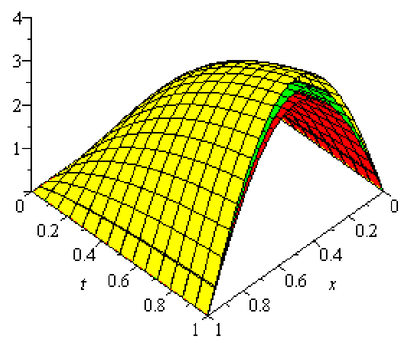

Figure 1: Plots of $\theta(x,t)$ against x and t for equation (20) for different values of δ_2 and $\delta_1 = 0.8$, $\lambda_1 = 0.3$, $\lambda_2 = 0.3$

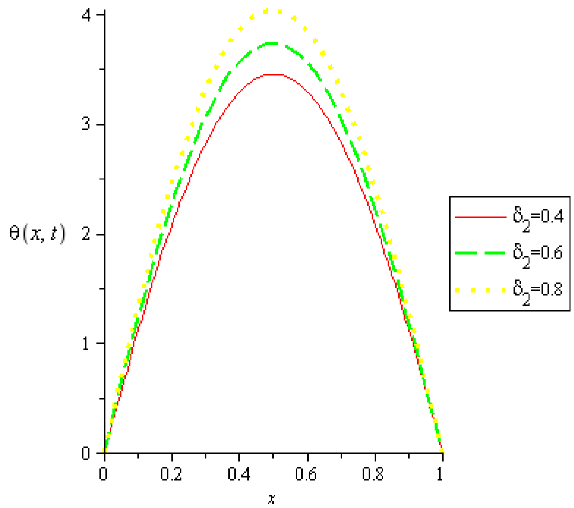

Figure 2: Plots of $\theta(x, t)$ against x for equation (20) for different values of δ_2 and $\delta_1 = 0.8$, $\lambda_1 = 0.3$, $\lambda_2 = 0.3$, $t = 1$

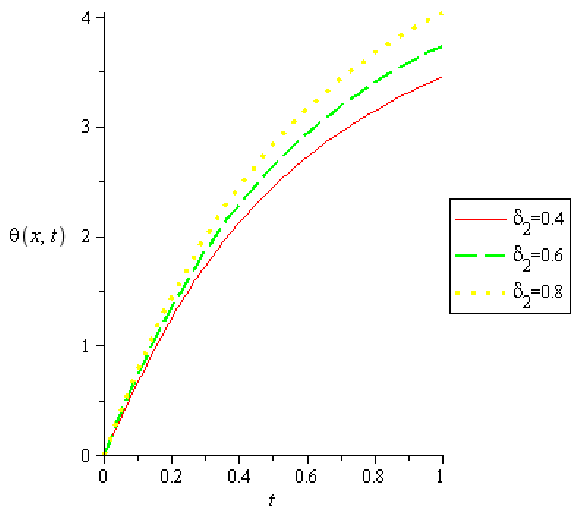

Figure 3: Plots of $\theta(x, t)$ against t for equation (20) for different values of δ_2 and $\delta_1 = 0.8$, $\lambda_1 = 0.3$, $\lambda_2 = 0.3$, $x = 0.5$

Figure 4 displays the graph of $\phi(x,t)$ against x and t for different values of δ_1.

Figure 5 displays the graph of $\phi(x,t)$ against x for different values of δ_1.

Figure 6 displays the graph of $\phi(x,t)$ against t for different values of δ_1.

From Figures 4-6 it is seen that material temperature increases as Frank-Kamenetskii number increases.

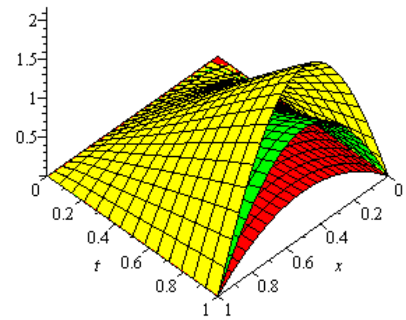

Figure 4: Plots of $\phi(x, t)$ against x and t for equation (19) for different values of δ_1 and $\delta_2 = 0.8$, $\lambda_1 = 0.3$, $\lambda_2 = 0.3$

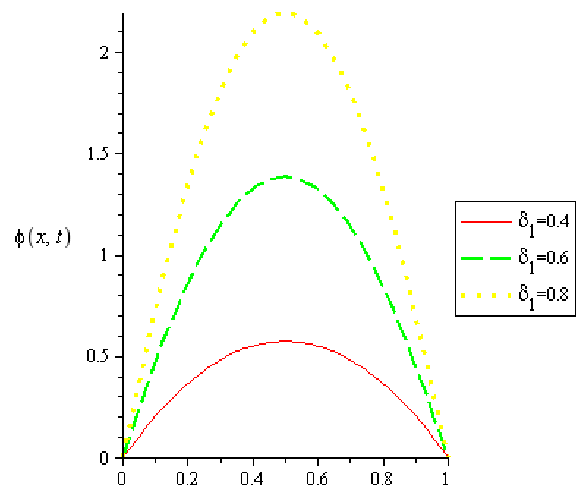

Figure 5: Plots of $\phi(x, t)$ against x for equation (19) for different values of δ_1 and $\delta_2 = 0.8$, $\lambda_1 = 0.3$, $\lambda_2 = 0.3$, $t = 1$

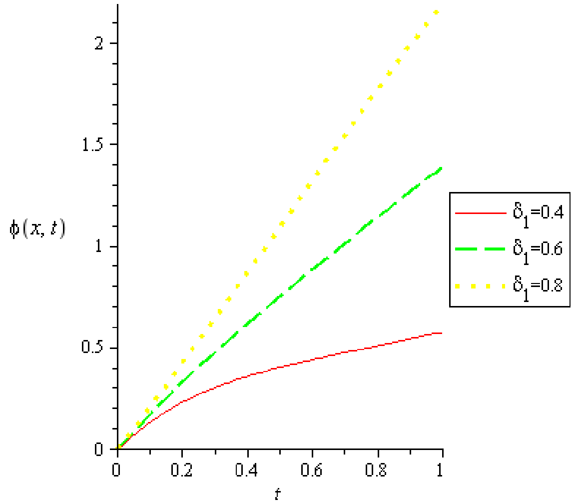

Figure 6: Plots of $\phi(x,t)$ against t for equation (19) for different values of δ_1 and $\delta_2 = 0.8$, $\lambda_1 = 0.3$, $\lambda_2 = 0.3$, $x = 0.5$

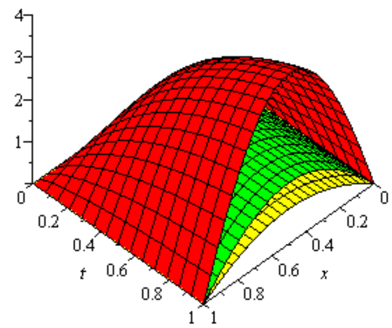

Figure 7: Plots of $\theta(x,t)$ against x and t for equation (20) for different values of λ_2 and $\delta_1 = 0.8$, $\delta_2 = 0.8$, $\lambda_1 = 0.3$

It is worth pointing out that the effects of δ_1 and δ_2 as shown in Figures 1-6 indicating that there is increase in heat of reaction Q. When the heat of reaction is high, the rate of conversion of limestone into quicklime and carbon dioxide is high. This is of great economic importance.

Figure 7 displays the graph of $\theta(x,t)$ against x and t for different values of λ_2.

Figure 8 displays the graph of $\theta(x,t)$ against x for different values of λ_2.

Figure 9 displays the graph of $\theta(x,t)$ against t for different values of λ_2.

From Figures 7-9 it is evident that gas temperature increases as scaled thermal conductivity decreases.

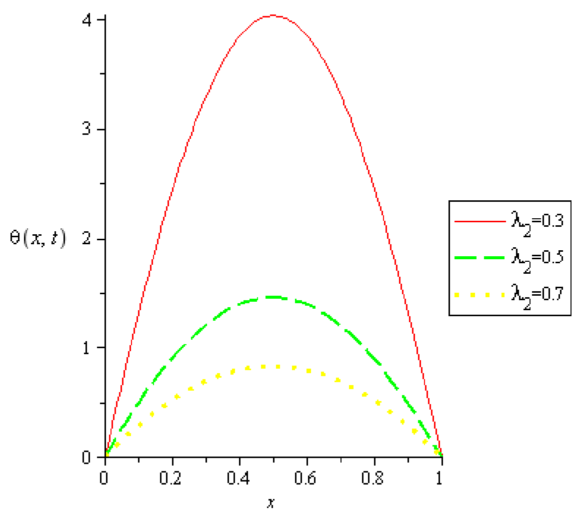

Figure 8: Plots of $\theta(x,t)$ against x for equation (20) for different values of λ_2 and $\delta_1 = 0.8$, $\delta_2 = 0.8$, $\lambda_1 = 0.3$, $t = 1$

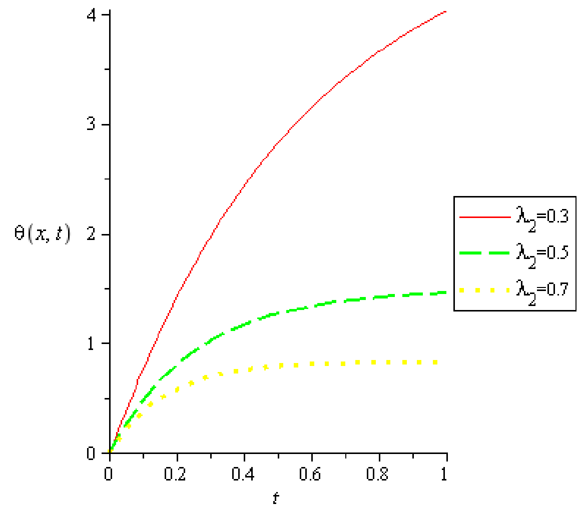

Figure 9: Plots of $\theta(x, t)$ against t for equation (20) for different values of λ_2 and $\delta_1 = 0.8$, $\delta_2 = 0.8$, $\lambda_1 = 0.3$, $x = 0.5$

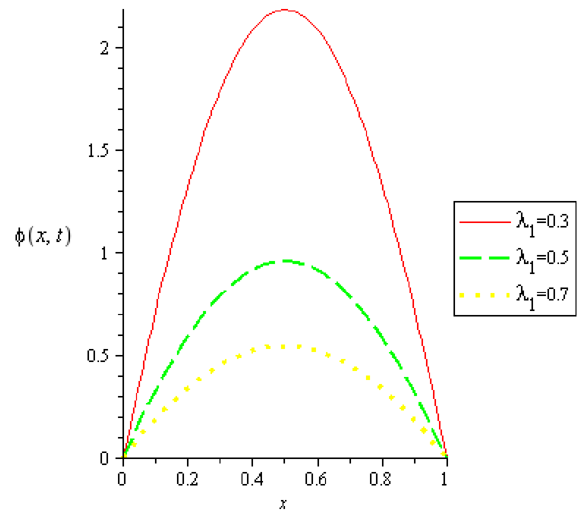

Figure 11: Plots of $\phi(x, t)$ against x for equation (19) for different values of λ_1 and $\delta_1 = 0.8$, $\delta_2 = 0.8$, $\lambda_2 = 0.3$, $t = 1$

Figure 10 displays the graph of $\phi(x,t)$ against x and t for different values of λ_1.

Figure 11 displays the graph of $\phi(x,t)$ against x for different values of λ_1.

Figure 12 displays the graph of $\phi(x,t)$ against t for different values of λ_1.

From Figures 10-12 it is evident that material temperature increases as scaled thermal conductivity decreases.

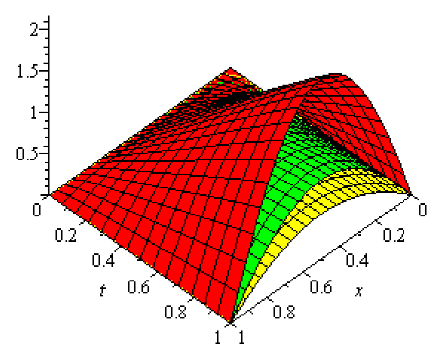

Figure 10: Plots of $\phi(x, t)$ against x and t for equation (19) for different values of λ_1 and $\delta_1 = 0.8$, $\delta_2 = 0.8$, $\lambda_2 = 0.3$

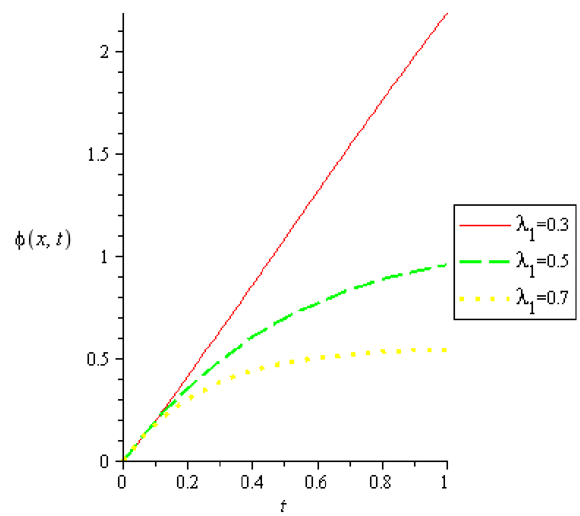

Figure 12: Plots of $\phi(x, t)$ against t for equation (19) for different values of λ_1 and $\delta_1 = 0.8$, $\delta_2 = 0.8$, $\lambda_2 = 0.3$, $x = 0.5$

CONCLUSION

For heat transfer in a lime kiln, analytical solution has been presented. The governing parameters of the problem are the scaled thermal conductivity for material (λ_1), scaled thermal conductivity for gas (λ_2), Frank-Kamenetskii number for material (δ_1) and Frank-Kamenetskii number for gas (δ_2). The heat transfer increases as Frank-Kamenetskii number increases and scaled thermal conductivity decreases. From the practical point of view, all these temperatures are favorable for formation of high quality quick lime since maintaining a high temperature of calcinations increases the furnace productivity. Therefore, our results showed that to increase furnace productivity depends on the parameters involved.

NOMENCLATURE

q : heat source
E : activation energy
R : gas constant
A : pre-exponential factor
Q : heat of reaction
C : heat capacity
T : temperature
t : time
x : position

Greek Letters

λ : thermal conductivity
α : heat transfer coefficient between gas and material
ρ : density
ε : porosity of the media
$\in$: dimensionless activation energy $\left\{= \dfrac{RT_0}{E}\right\}$
θ : dimensionless temperature for gas
ϕ : dimensionless temperature for material
λ_1 : scaled thermal conductivity for material $\left\{= \dfrac{\lambda_m t_0}{\rho_m (1-\varepsilon) C_m L^2}\right\}$
λ_2 : scaled thermal conductivity for gas $\left\{= \dfrac{\lambda_g t_0}{\rho_g \varepsilon C_g L^2}\right\}$
δ_1 : Frank-Kamenetskii number for material

$$\left\{= \frac{QAt_0 \exp\left(-\frac{E}{RT_0}\right)}{\in T_0 \rho_m (1-\varepsilon) C_m}\right\}$$

δ_2 : Frank-Kamenetskii number for gas

$$\left\{= \frac{QAt_0 \exp\left(-\frac{E}{RT_0}\right)}{\in T_0 \rho_g \varepsilon C_g}\right\}$$

Subscripts

g : gas
m : material
0 : initial

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ABOUT THE AUTHORS

Dr. R.O. Olayiwola, is a Lecturer in the Department of Mathematics and Statistics, Federal University of Technology, Minna, Nigeria. He holds a Ph.D. degree in Applied Mathematics from Ladoke Akintola University of Technology, Ogbomoso, Nigeria. His research interests are in areas of combustion, fluid mechanics, and mathematical modeling.

Dr. A.O. Adesanya, is a Senior Lecturer in the Department of Pure and Applied Mathematics, Ladoke Akintola University of Technology, Ogbomoso, Nigeria. He holds a Ph.D. degree in Applied Mathematics from Ladoke Akintola University of Technology, Ogbomoso, Nigeria. His research interests are in areas of fluid mechanics and computational fluid dynamics.

Dr. A.M. Okedoye, is a Senior Lecturer in the Department of Mathematics, Covenant University, Ogun State, Nigeria. He holds a Ph.D. degree in Applied Mathematics from Ladoke Akintola University of Technology, Ogbomoso, Nigeria. His research interests are in areas of fluid mechanics, computational fluid dynamics and mathematical modeling.

Dr. A. O. Popoola, is a Senior Lecturer in the Department of Mathematical and Physical Sciences, Osun State University, Osogbo, Nigeria. He holds a Ph.D. degree in Applied Mathematics from Ladoke Akintola University of Technology, Ogbomoso, Nigeria. His research interests are in areas of fluid mechanics, computational fluid dynamics and mathematical modeling.

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