# Dynamic Deflection of a Non-Uniform Rayleigh Beam when under the Action of Distributed Load.

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## ABSTRACT

The problem being investigated in this paper is that of the dynamic behavior of non-uniform beam on a constant elastic foundation. The elastic properties of the beam, the flexural rigidity, the mass per unit length, and the elastic modulus parameter are expressed as functions of the spatial variable x. However, an assumed mode technique is employed to simplify the displacement of the non-uniform system to second order ordinary differential equation and then solved via Laplace method.

The effects of damping, elastic foundation and rotatory inertia correction factors on the dynamic deflection of Rayleigh beams on distributed load are also investigated.

(Keywords: Rayleigh beam, elastic structure, distributed load, dynamic deflection)

## INTRODUCTION

Structural engineers, physicists, and mathematicians frequently encountered problems arising from deflection of beam on an elastic foundation. The complex one-dimensional structures such as non-uniform beam subjected to moving load have been a subject of investigation for decades by several authors [1-3].

This type of problem was first addressed by Kolouse [4]. He used normal mode analysis to address the topical problems. Chak and Seng [5] investigated the static response of beams on non-linear elastic foundation where the deformed shape structure was represented by a Fourier series, and thereafter, the fourth order governing equation is reduced to a set of a second order simultaneous equation using Galarkin's method.
In a similar manner, Oni [6] considered the response of a non-uniform thin beam resting on a constant elastic foundation to several moving masses. He used the versatile technique of Galarkin to reduce the complex governing fourth order partial differential equation with variable and singular coefficients to a set of ordinary differential equation with variable coefficients.
Later, Oni and Awodola [7] investigated the problem of vibration under concentrated moving mass of a non-uniform Rayleigh beam resting on a variable elastic foundation. Numerical results in plotted curves are presented.

However, these researchers either considered beams with prismatic materials under harmonic and concentrated loads, or beams on a non-linear elastic foundation under moving loads.

In this paper, the problem of vibrations under partially distributed moving masses of a non-uniform Rayleigh beams on a constant elastic foundation is investigated. The dynamical analysis is carried out for various values of rotatory inertia correction factor, damping effects and foundation moduli.

## NON-UNIFORM RAYLEIGH BEAM ON ELASTIC FOUNDATION

The equation that governs the dynamic deflections of non-uniform beam is given as follows:

$$EJ\frac{\partial^4}{\partial x^4}Z(x,t)+M(x)\frac{\partial^2}{\partial t^2}Z(x,t)$$
$$-\frac{\partial}{\partial x}\left[M(x)R^o\frac{\partial^2}{\partial x\partial t}Z(x,t)\right]$$

$$+2W_oM_o\frac{\partial Z(x,t)}{\partial t}+K(x,t)=P(x,t) \quad \textbf{(1)}$$

By considering a Rayleigh beam resting on an elastic foundation and traverse by moving distributed load $P(x,t)$ in the right direction at a constant velocity $c$.

The non-uniformity of the beam indicates that its properties such as moment of inertia $J$ and the mass per unit length of the beam $M(x)$ vary along the span L of the beam. The equation of motion with damping considered is given by:

$$EJ\frac{\partial^4}{\partial x^4}Z(x,t)+M_o\left(1+Sin\frac{\pi x}{L}\right)\frac{\partial^2}{\partial t^2}Z(x,t)$$
$$-\frac{\partial}{\partial x}\left[M_o\left(1+Sin\frac{\pi x}{L}\right)R^o\frac{\partial^2}{\partial x\partial t}Z(x,t)\right]$$

$$+2W_oM_o\frac{\partial^2 Z(x,t)}{\partial t}+K(x,t)=P(x,t) \quad \textbf{(2)}$$

Where E is the Young's modulus , $Z(x,t)$ is the transverse displacement, $J$ is the moment of inertia, $K$ is the foundation constant, $R^o$ is the measure of rotatory inertia effect and $x$ , $t$ are respectively spatial and the time coordinates.

The possible boundary condition for simply supported Rayleigh beam are taken to be:

$$Z(x,t)=0=Z(L,t),$$
$$\frac{\partial}{\partial x}Z(x,t)=0=\frac{\partial^2}{\partial^2 x}Z(L,t) \quad \textbf{(3)}$$

## SOLUTION PROCEDURE

To simplify the governing Equation (1), an assumed mode technique is employed to obtain the dynamic deflection $Z(x,t)$ of the non-uniform vibrating beam. It is defined as:

$$Z(x,t)=\sum_{m=1}^{N}W_m(t)U_m(x) \quad \textbf{(4)}$$

where $W_m(t)$ are coordinates in modal space and $U_m(x)$ is the normal mode of vibration of the beam and when the non-uniform Rayleigh have simple supports at both ends, $W_m(t)$

$$U_m(x)=\sin\frac{m\pi x}{L} \quad \textbf{(5)}$$

substituting Equation (4) into Equation (5), one obtains:

$$\sum_{m=1}^{n}\left[\frac{EI}{M_o}U_m{}^{IV}(x)W_m(t)+\left(U_m(x)\right.\right.$$
$$+U_m(x)\sin\frac{\pi x}{L}\ddot{W}_m(t)+R^oU_m{}^{I}(x)\dot{W}_m(t)$$
$$+R^o\frac{\pi^2}{L^2}\sin\frac{\pi x}{L}U_m{}^{II}(x)\dot{W}_m+\frac{K_o}{M_o}U_m(x)W_m(t)$$
$$\left.+2W_oU_m(x)\dot{W}_m(t)\right]=\frac{P}{M_o}H(x-ct)$$
**(6)**

The non-uniform Rayleigh beam solution requires that the right hand side of Equation (6) be orthogonal to the function $U_k(x)$, so that Equation (6) can be written as:

$$\int_0^L\sum_{m=1}^{\infty}\left[\left(U_m(x)+\sin\frac{\pi x}{L}U_m(x)\right)\ddot{W}_m(t)\right.$$
$$+\left(R^o\left(U_m'(x)+\frac{\pi^2}{L^2}\sin\frac{\pi x}{L}U_m'''(x)\right)+2w_0U_m(x)\right)\dot{W}_m(t)$$
$$+\frac{EI}{M_o}U_m{}^{IV}(x)W_m(t)+\frac{k_0}{M_o}U_m(x)W_m(t)$$
$$\left.-\frac{P}{M_o}H(x-ct)\right]U_k(x)=0 \quad \textbf{(7)}$$

Further simplification of Equation (7) yields:

$$\sum_{m=1}^{\infty}\ddot{W}_m(t)+Q_{aa}\dot{W}_m(t)+Q_{ab}W_m(t) = Q_{ac}Cosqt \quad \textbf{(8)}$$

where

$$Q_{aa}=\frac{R^o\left(\frac{\pi}{L}H_3(x)+\frac{\pi^4}{L^4}H_4(x)\right)+2w_0U_m(x)}{H_1(x)+H_2(x)} \quad \textbf{(9)}$$

$$Q_{ab}=\frac{\frac{EI}{M_0}\frac{\pi^2}{L^2}H_6(x)+\frac{K_0}{M_0}H_6(x)}{H_1(x)+H_2(x)} \quad \textbf{(10)}$$

$$Q_{ac}=\frac{Mg(-q)}{M_0\left(H_1(x)+H_2(x)\right)} \quad \textbf{(11)}$$

And,

$$q=\frac{k\pi c}{L} \quad \textbf{(12)}$$

Now let us consider only the m$^{th}$ particle of the dynamical system, we have that:

$$\ddot{W}_m(t)+Q_{aa}\dot{W}_m(t)+Q_{ab}W_m(t)=Q_{ac}Cosqt \quad \textbf{(13)}$$

By subjecting second order ordinary differential Equation (13) to Laplace transform, taken into consideration the initial conditions defined as:

$$Z(x,0)=0=\frac{d}{dt}Z(x,0) \quad \textbf{(14)}$$

one obtains:

$$W_m(s)=Q_{ac}\left[\left(\frac{s}{s^2+q^2}\right)\times\frac{1}{(s-\eta_a)(s-\eta_b)}\right] \quad \textbf{(15)}$$

where,

$$\eta_a=\frac{-Q_{aa}+\sqrt{Q_{aa}^2-4Q_{ab}}}{2} \quad \textbf{(16)}$$

$$\eta_b=\frac{-Q_{aa}-\sqrt{Q_{aa}^2-4Q_{ab}}}{2} \quad \textbf{(17)}$$

and noting that,

$$\frac{1}{(s-\eta_a)(s-\eta_b)}=\frac{A_a}{(s-\eta_a)}+\frac{A_b}{(s-\eta_b)} \quad \textbf{(18)}$$

Further simplification of Equation (18) yields:

$$W_m(s)=\frac{Q_{ac}}{\eta_a-\eta_b}\left[\frac{s}{s^2+q^2}\cdot\frac{1}{(s-\eta_b)}-\frac{s}{s^2+q^2}\cdot\frac{1}{(s-\eta_a)}\right] \quad \textbf{(19)}$$

from Equation (19), the following representations were noted in order to obtain the Laplace inversion:

$$g_a(s)=\frac{s}{s^2+q^2},\quad f_a(s)=\frac{1}{(s-\eta_b)},\quad f_b(s)=\frac{1}{(s-\eta_a)} \quad \textbf{(20)}$$

Thus the Laplace inversion of Equation (19) is given by:

$$W_m(t)=\frac{Q_{ac}}{\eta_a-\eta_b}\left[\alpha_{jm}-\alpha_{jn}\right] \quad \textbf{(21)}$$

where

$$\alpha_{jm}=e^{\eta_b t}\int_0^L e^{-\alpha_b u}Cosqudu\,,\quad \alpha_{jn}=e^{\eta_a t}\int_0^L e^{-\alpha_a u}Cosqudu \quad \textbf{(22)}$$

solving Equation (22), Equation (21) becomes:

$$W_m(t) = \frac{Q_{ac}}{\eta_a - \eta_b}\left[\frac{qSinqt - \eta_b Cosqt}{\eta_b^2 - q^2}\right.$$

$$\left. - \frac{qSinqt - \eta_a Cosqt}{\eta_a^2 - q^2}\right] \quad \textbf{(23)}$$

which when inverted yields,

$$V(x,t) = \frac{Q_{ac}}{\eta_a - \eta_b}\sum_{m=1}^{n}\left[\frac{qSinqt - \eta_b Cosqt}{\eta_b^2 - q^2}\right.$$

$$\left. - \frac{qSinqt - \eta_a Cosqt}{\eta_a^2 - q^2}\right] \cdot \frac{Sinm\pi x}{L} \quad \textbf{(24)}$$

which gives the dynamic behavior of non-uniform Rayleigh beams traversed by moving distributed load on both constant elastic foundation at constant speed.

## NUMERICAL EXAMPLES AND DISCUSSION OF THE RESULTS

To illustrate the results obtained analytically for the problems of structurally damped Rayleigh beam transversed by moving distributed load, the beam is taken to be of length 12.192m, moving with velocity c as 8.128m/s, and flexural rigidity of 6068242m$^3$/s$^2$.

Also for the foundation moduli, the values are varied between 0N/m$^3$ and $2 \times 10^6 N/m^3$, and the mass per unit length of the beam is 4501.537g/m.

The transverse displacement response of a non-uniform Rayleigh beam resting on a constant elastic foundation under traveling distributed forces are calculated and plotted against time for varies values of foundation modulus. $k_f$, damping coefficient $w_0$ and rotatory inertia $R_0$.

Figure 1 depicts the rotatory inertia effect on the dynamic response of non-uniform Rayleigh beam on constant elastic foundation under moving distributed load for fixed values of foundation moduli $k_f$ and damping term $w_0$. The graph shows that the displacement response decreases as the rotatory inertia $R_0$ increases.

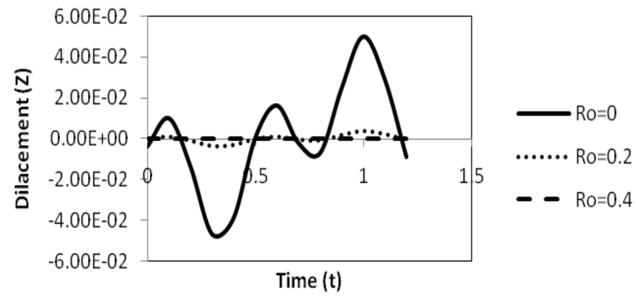


**Figure 1:** Effect of Rotary Inertia on Non-Uniform Beam Traversed by Moving Loads for Fixed Values of Wo and Kf and Different Values of Ro.

Figure 2 depicts the effect of damping on the dynamic response of non-uniform Rayleigh beam on constant elastic foundation under moving distributed load for fixed values of foundation modulus $k_f$ and rotatory inertia $R_0$. The graph show that the displacement Z(x,t) of the structure decreases as the damping term increases.

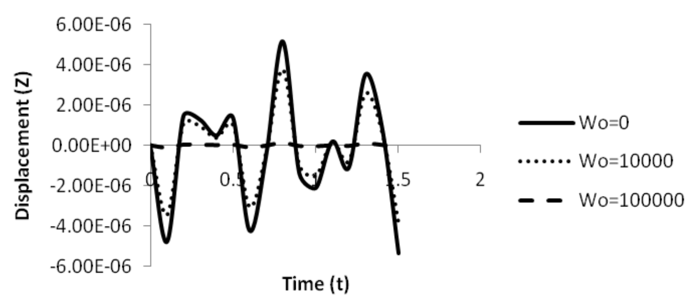


**Figure 2:** Effect of Damping on Non-Uniform Beam Traversed by Moving Loads for Fixed Values of Ro and Kf and Different Values of Wo.

The transverse response of non-uniform Rayleigh beam resting on constant elastic foundation under the action of moving distributed load for fixed values of rotatory inertia and damping decreases as the foundation modulus $k_f$ increases is given in Figure 3.

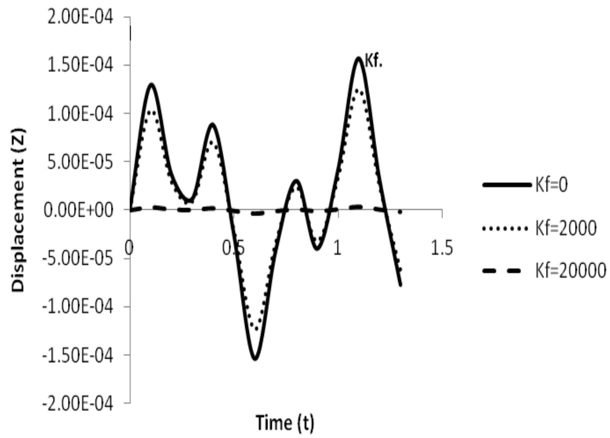


**Figure 3:** Effect of Foundation Modulus on Non-Uniform Beam Traversed by Moving Loads for Fixed Values of Ro and Wo and Different Values of Kf.

## CONCLUSION

In this paper the problem of dynamic response of a non-uniform Rayleigh beam transversed by moving distributed load has been investigated. Numerical analysis for the solution has been carried out and displays in plotted curve for the non-uniform Rayleigh beam under moving distributed load.

The effects of rotatory inertia, damping and foundation moduli have been established on the non–uniform Raylegh beam under moving distributed load. It is found out that as rotatory inertia is increased, the displacement response of the non-uniform Rayleigh beam decrease. A similar result goes for damping and foundation modulus.

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## SUGGESTED CITATION

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