Design and Construction of Low- and High-Pass Active Filters Using Butterworth and Chebychev Techniques.

O.A. Aremu[1]*; O.J. Iyiade[2]; and D.A. Ajadi[3]

[1]Department of Physics/Electronics. The Polytechnic, Ibadan, PMB 22, U.I. Post Office, Ibadan,
Oyo State, Nigeria.
[2]Department of Physical Sciences, Oduduwa University, Ipetumodu, PMB 5533, Ile Ife,
Osun State, Nigeria.
[3]Pure and Applied Physics Department, Ladoke Akintola University of Technology, PMB 4000,
Ogbomoso, Nigeria.

E-mail: aremurichard@yahoo.com*
Telephone: +2348032413944

ABSTRACT

A filter is a device that passes electric signals at certain frequency. In the communication system, it is essential to extract or enhance the useful information and remove any redundancy from a mix of conflicting information. In this paper, design and construction of low-pass and high-pass active filters using Butterworth and Chebychev techniques is presented. The gain and the frequencies of the device constructed were measured using oscilloscope and signal generator, respectively. The work shows the characteristics of each filter for a particular frequency specification. The tables for designing filters were presented and the equivalent circuit diagram was drawn. In this work, low pass Butterworth and Chebychev filters was designed to pass frequency from 0-1000Hz while the frequency above 1000Hz is attenuated. High pass Butterworth and Chebychev filters were also design to pass frequency from 100-5000Hz while the frequency below 100Hz is attenuated.

The result shows that Butterworth filters produce no ripple in the pass band and attenuate unwanted frequencies outside the band based on the specification while Chebychev filters attenuate unwanted frequencies with fewer components than Butterworth filters, but exhibit ripples in the pass band.

(Keywords: attenuation, Butterworth filters, Chebychev filters, pass band, ripples)

INTRODUCTION

Filter is a frequency selective circuit that allows a certain band of frequency to pass while attenuating the others frequencies. Filters are classified as analog and digital. Signal processing is an operation designed for extracting, enhancing, storing and transmitting useful information. The distinction between useful and unwanted information is situation and need dependent. Hence, signal processing is application and need dependent. An important element in digital processing is the filter. In circuit theory, a filter is an electrical circuit that manipulates the amplitude and /or phase characteristics of a signal with respect to frequency. In signal processing, the function of a filter is to remove unwanted parts of the signal such as noise, or selective extraction of the signal such as the components in a certain frequency range [1]. Hence, filter has a gain which depends on signal frequency.

Butterworth are termed maximally -flat-magnitude-response filters, optimized for gain flatness in the pass band and the attenuation is -3dB at the cut off frequency above the cut- off frequency the attenuation is -20dB/decade/order [y]. The transient response of a Butterworth filter to a pulse input shows mode rate overshoot and ringing. Chebyshev filters are designed to have ripple in the pass-band, but steeper roll off frequency. Cut off frequency is defined as the frequency at which the response falls below the ripple band. For a given filter order, a steeper cut off can be achieved by allowing more pass-band ripple.

The transient response of Chebyshev filter to a pulse input shows more overshoot and ringing that a Butterworth filter. The order n, of the transfer function should be as low as possible because low order filters have lower pole-Q factors than high-order filters [2]. This is particularly true for a Butterworth filter, which has “maximally flat” amplitude response and corresponds to the limit case of no ripple in the filter pass-band. Compared to Chebyshev filter of equal order, it has lower pole Qs. As was shown in [3], in order to realize a filter with low sensitivities to its component tolerance, the designer should choose a filter with the lowest possible pole Q-factor. For example, a Butterworth filter is always preferred to a Chebyshev filter and a low ripple Chebyshev filter is preferable to a Chebyshev filter with higher ripple when the sensitivity to component tolerances is to be held small. Thus, for low and high-pass filters of reasonably low order (n less or equal to 6), the use of single- amplifier filters can be advantageous in comparison to the cascade realization with 2nd and 3rd order section [3].

BUTTERWORTH ACTIVE FILTER POLYNOMIAL

Butterworth type polynomials were written with real coefficient by multiplying pole which is complex conjugates such as s_1 and s_n. The polynomials are normalized by setting $w_c=1$ [4]

For n even, $B_n(s) = \Pi_{k=1}^{\frac{n}{2}}\left[S^2 - 2S\cos\left(\frac{2k+n-1}{2n} - 1\right)\right]$ **(1)**

For n odd, $B_n(s) = (S+1)\Pi_{k=1}^{\frac{n-1}{2}}\left[S^2 - 2S\cos\left(\frac{2k+n-1}{2n} - 1\right)\right]$ **(2)**

The factors of polynomial $B_n(s)$ with the order n, for Butterworth filter is shown in Table 1.

Table 1: Factors of Polynomial for Butterworth Active Filter.

n	$B_n(s)$	Factors of polynomial
1	$B_1(s)$	$(s+1)$
2	$B_2(s)$	$(s^2+1.4142s+1)$
3	$B_3(s)$	$(s+1)\ (s^2+s+1)$
4	$B_4(s)$	$(s^2+0.7654s+1)\ (s^2+1.8478s+1)$
5	$B_5(s)$	$(s+1)\ (s^2+0.6180s+1)\ (s^2+0.6180s+1)$
6	$B_6(s)$	$(s^2+0.5176s+1)\ (s^2+1.4142s+1)\ (s^2+1.9319s+1)$
7	$B_7(s)$	$(s+1)\ (s^2+0.4450s+1)\ (s^2+1.2470s+1)\ (s^2+1.8019s+1)$
8	$B_8(s)$	$(s^2+0.3902s+1)\ (s^2+1.1111s+1)\ (s^2+1.6629s+1)\ (s^2+1.9616s+1)$

CHEBYSHEV ACTIVE FILTER POLYNOMIAL

The Chebyshev active filter has a transfer function built around a chebyshev polynomial and are given by the following Equation [4].

$$C_{n+1}\left(\frac{w}{w_p}\right) = 2\left(\frac{w}{w_p}\right)C_n\left(\frac{w}{w_p}\right) - C_{n-1}\left(\frac{w}{w_p}\right) \qquad \textbf{(3)}$$

If n=0, and w_p is normalized to unity, then

Cos $[ncos^{-1}(w)] = \cos h[ncosh^{-1}(w)] = 1\ \forall w$ **(4)**

Therefore, $C_o(w) = 1 \; \forall w$ **(5)**

If n =1, then, Equation (4) becomes:

$$C_1(w) = w \; \forall w \qquad \textbf{(6)}$$

For n less than 1, Equation (3) may be used for n=2:

$$C_2(w) = 2wC_1(w) - C_0(w) = 2w^2 - 1 \qquad \textbf{(7)}$$

The factor of polynomial with order n is as shown in Table 2.

Table 2: Chebyshev Polynomials for Several Values of n.

Order, n	Chebyshev Polynomial
1	w
2	$2w^2 - 1$
3	$4w^3 - 3w$
4	$8w^4 - 8w^2 + 1$
5	$16w^5 - 20w^3 + 5w$
6	$32w^6 - 48w^4 + 18w^2 - 1$
7	$64w^7 - 112w^5 + 56w^3 - 7w$
8	$128w^8 - 256w^6 + 160w^4 - 32w^2 + 1$

DESIGN SPECIFICATION

Both high pass Butterworth and Chebyshev filters were designed to pass frequency from 100 to 5000Hz using the following specifications:

$$Ap\ 3dB\ at\ Fp = 100Hz$$

$$As = 75dB\ at\ Fs = 25Hz$$

The same specification was also used for low-pass Butterworth and Chebyshev filters designed to pass frequency 0-1000Hz i.e. 0-999Hz will pass through, while frequency above 1000Hz will attenuate.

$$Ap\ 3dB\ at\ Fp = 1KHz$$

$$As = 40dB\ at\ Fs = 5KHz$$

Where Ap is the attenuation in the pass band; Fp is the frequency at which Ap occurs; As attenuation in the stop band and Fs is the frequency at which As occurs.

Block Diagram of Stage to Stage Design

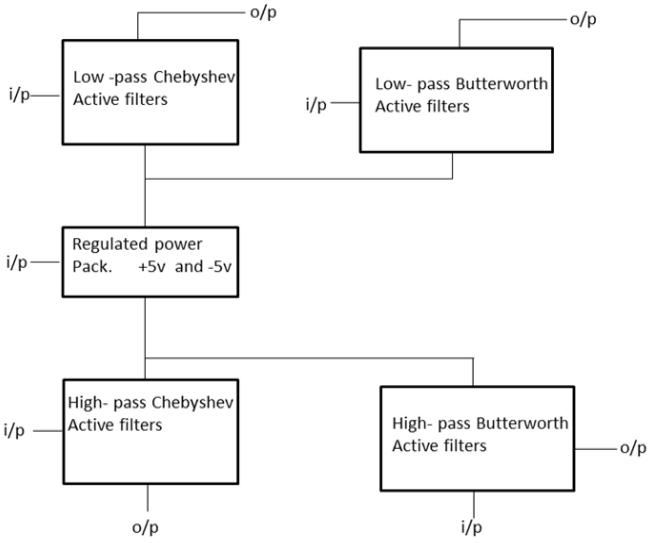

Figure 1: block schematic diagram of stage to stage design of Butterworth and Chebyshev filters.

Assembling Components of High-Pass Butterworth and Chebychev Filter

The operational amplifier (op-amp) used in the construction of this active filters is JRC 4558D, is a dual pack op-amps. They were constructed using this op-amp and five R-C networks respectively (5 poles). One op-amp JRC 4558D is used for first stage and the second op-amp for the second stage. Resistors and capacitor were coupled accordingly. The value of capacitor for Butterworth was assumed to be C= 0.15μF and the value of capacitor for Chebychev was assumed to be C=0.015μF, while the value of resistors varies.

In some cases, component were manipulated to provide an alternative, where the exact value of resistor required is not available in the market, two or three resistors were connected in series.

Assembling of Components Low-Pass Butterworth and Chebychev Filter

The op-amp used in the construction of these active filters is JRC45580D, it is a dual pack. Low pass Butterworth and Chebychev filter were constructed using this op-amp and three RC (three poles). The value of resistors used is 1KΩ, while the value of capacitors varies.

DESIGN PROCEDURE

Low-Pass Chebyshev Active Filter

Three poles; 0.1dB; 3rd order.
From the specification table for filter design.

Table 3: Specification Table [5] for Low-Pass Chebyshev Active Filter Design.

Filter Order, n	$\frac{C_1}{C}$ or $\frac{R}{R_c}$	$\frac{C_2}{C}$ or $\frac{R}{R_c}$	$\frac{C_3}{C}$ or $\frac{R}{R_3}$
2	1.638	0.695	
3	**6.653**	**1.825**	**0.1345**
4	1.900	1.241	
	4.592	0.241	
5	4.446	2.520	
	6.810	0.158	
6	2.553	1.776	
	3.487	0.492	
	9.531	0.111	
7	5.175	3.322	
	4.546	0.333	
	12.730	0.082	0.5693
8	3.270	2.323	
	3.857	0.689	
	5.773	0.240	
9	6.194	4.161	0.748
	4.678	0.466	
	7.170	0.181	
	4.011	2.877	
	4.447	0.876	
	5.603	0.333	
	8.727	0.142	
	25.320	0.040	

The value of capacitors used are determined by using the equation:

$$C_n = \frac{1}{2\pi F_p R} \tag{8}$$

For this work, F_P=1KHz; Fs =5KHz and R=1KΩ

$$C_n = \frac{1}{2x3.142x1000x1000}$$

C_n= 0.16μF

From the Table 3, the values that corresponds to 3 poles/order:
C_1/C_n = 6.653; C_2/C_n = 1.825; C_3/C_n = 0.1345

This yield the values of capacitors used for the construction, and the constructed circuit diagram is shown in Figure 2
.

C_1= (0.16) (6.653) = 1.06μF
C_1= (0.16) (1.825) = 0.292μF
C_1= (0.16) (0.1345) = 0.215μF

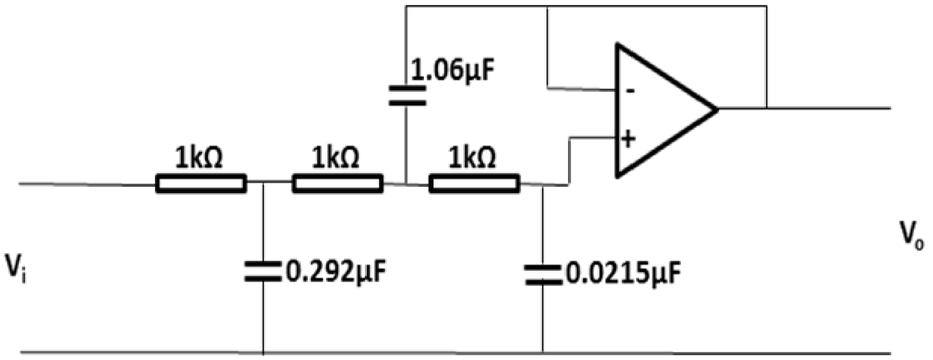

Figure 2: Equivalent Circuit Diagram for Low-Pass Chebyshev Active Filter Designed.

High-Pass Chebyshev Active Filter

It has 5poles, 0.5dB in 5th order.

Table 4: Specification Table for High-Pass Chebyshev Active Filter Design.

Filter Order, n	$\frac{C_1}{C}$ or $\frac{R}{R_1}$	$\frac{C_2}{C}$ or $\frac{R}{R_2}$	$\frac{C_3}{C}$ or $\frac{R}{R_3}$
2	1.950	0.6533	
3	11.230	2.2500	0.0895
4	2.582	1.3000	
	6.233	0.1802	
5	**6.842**	**3.317**	**0.3033**
	9.462	**0.1144**	
6	3.592	1.921	
	4.907	0.3743	
7	7.973	0.0790	0.4700
	6.446	0.057	
8	4.665	2.547	
	5.502	0.5303	
	8.237	0.1714	
	23.450	0.0441	
9	9.563	5.680	0.6260
	6.697	0.3419	
	10.260	0.1279	
	29.540	0.0347	
10	5.760	3.175	
	6.383	0.677	
	8.048	0.2406	
	12.530	0.099	
	36.360	0.0281	

From the table, the 5th order has two stages. For this work, C was chosen as 0.015μF and Fp =100Hz The value of resistors used are determined by using the equation:

$$R = \frac{1}{2\pi FpC} \qquad \textbf{(9)}$$

$$hence, R = 1.06x10^5 \Omega$$

The value for each resistor to be used is:

R_1 =R/6.842 = $1.06x10^5/6.842$ = 15.5KΩ

R_2 =R/3.317 = $1.06x10^5/3.317$ = 32.0KΩ

R_3 =R/0.3033 = $1.06x10^5/0.3033$ = 349KΩ

$R_{1`}$ =R/9.462 = $1.06x10^5/9.462$ = 11.2KΩ

$R_{2`}$=R/0.1144 = $1.06x10^5/0.1144$ = 927KΩ

The equivalent circuit constructed with the specification above is shown in Figure 3.

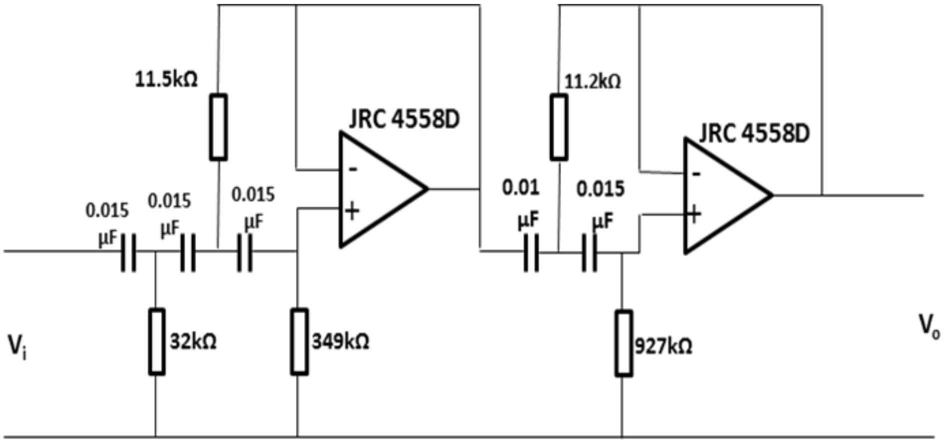

Figure 3: Equivalent Circuit Diagram for High-Pass Chebyshev Active Filter Designed.

Low-Pass Butterworth Active Filters

The parameters used for the design and construction are shown in the table 5 below. It is a three poles order.

For this work, Fp=1.0KHz and R=1.0KΩ

The value of capacitors used are determined by using the equation

$$C_n = \frac{1}{2\pi F_p R} \qquad \textbf{(10)}$$

Hence Cn=0.16µF

Table 5: Specification Table for Low-Pass Butterworth Active Filter Design.

Order, n	$\frac{C_1}{C}$ or $\frac{R}{R_1}$	$\frac{C_2}{C}$ or $\frac{R}{R_2}$	$\frac{C_3}{C}$ or $\frac{R}{R_3}$
2	1.414	0.7071	
3	**3.546**	**1.392**	**0.2024**
4	1.082	0.9241	
	2.613	0.3825	
5	**1.753**	**1.354**	**0.4214**
	3.235	**0.309**	
6	1.035	0.966	
	1.414	0.7071	
	3.864	0.2588	
7	1.5331	1.336	0.4885
	1.604	0.6235	
	4.493	0.2225	
8	1.020	0.9809	
	1.202	0.8813	
	1.800	0.5557	
	5.125	0.1950	
9	1.455	1.327	0.5170
	1.305	0.7661	
	2.000	0.5000	
	5.758	0.1736	
10	1.012	0.9874	
	1.220	0.8908	
	1.414	0.7071	
	2.202	0.4540	
	6.390	0.1562	

From the Table 5 the values correspond to third order/poles are used to determine the value of capacitance used for the constructions. The equivalent circuit diagram constructed is shown in Figure 4.

$C_1/C_n = 3.546$; $C_2/C_n = 1.392$; $C_3/C_n = 0.2024$

These yield capacitors of values:
C_1 =(0.16) (3.546) = 0.56μF
C_1 =(0.16) (1.392) = 0.22μF
C_1 =(0.16) (0.2024) = 0.03μF

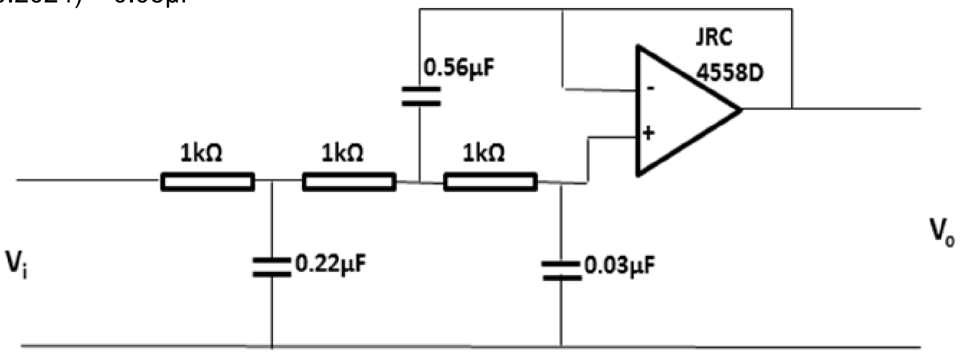

Figure 4: Equivalent Circuit Diagram for Low-Pass Butterworth Active Filter Designed.

High-Pass Butterworth Active Filters

With reference to Table 5, the values used for the design and construction of high- pass Butterworth active filter.

It has five poles and 5th order with two stages.

First stage:

$$\frac{R}{R_1} = 1.753 \qquad \frac{R}{R_2} = 1.354 \qquad \frac{R}{R_3} = 0.4214$$

Second stage:

R/R_1`=3.235 $R/R_2$`=0.309
Fp = 100Hz and C= 0.15µF

From, $R = \frac{1}{2\pi FpC}$ **(11)**

Hence, $R = 1.06 \times 10^4 \Omega$

The value for each resistor to be used is:

$R_1 = \frac{R}{1.753}$ $R_1 = 78.3\Omega$

$R_2 = \frac{R}{1.354}$ $R_2 = 60.5\Omega$

$R_3 = \frac{R}{0.4214}$ $R_3 = 251.5\Omega$

R_1`= $\frac{R}{2.235}$ $R_1$` =32.8Ω

R_2`= $\frac{R}{0.309}$ $R_2$` =343Ω

The equivalent circuit diagram for the values is shown in Figure 5.

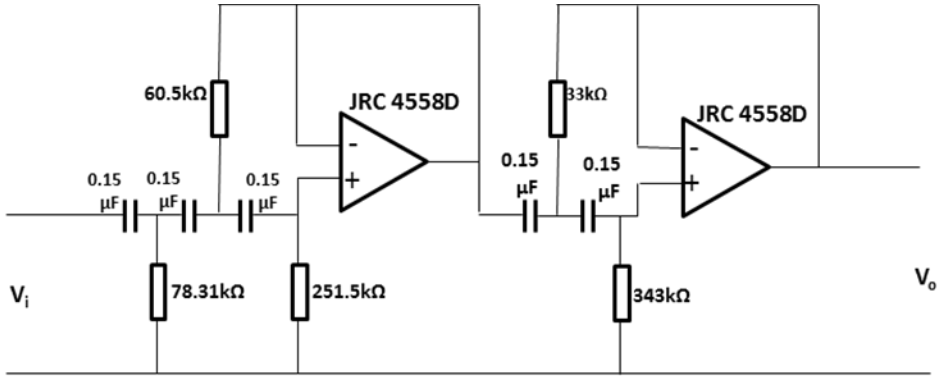

Figure 5: Equivalent Circuit Diagram for High-Pass Butterworth Active Filter Designed.

Performance Test

The device worked perfectly as expected when performance test was carried out in the laboratory. This was done by connecting it with signal generator and oscilloscope. Sinusoidal waveform was selected from the signal generator and applied to the input of the device and oscilloscope to the output. The waveform was observed at the input and at the output by varying the frequency within the pass-band and outside the pass band.

RESULTS

Tables 6 to 9 show the results obtained from the performance test and specifications used.

Table 6: Low-Pass Chebyshev Results.

$F(Hz)$	V_{in}	V_{out}	$A = \frac{V_{out}}{V_{in}}$	$A(dB) = 20log\frac{V_{out}}{V_{in}}$
200.00	0.88	0.84	0.95	-0.44
400.00	0.92	0.76	0.82	-1.72
600.00	0.96	0.64	0.66	-3.60
800.00	1.00	0.52	0.52	-5.68
900.00	1.00	0.52	0.52	-5.68
1000.00	1.02	0.50	0.49	-6.20
2000.00	0.68	1.00	1.50	-3.52
3000.00	0.68	0.40	0.60	-4.44
4000.00	0.68	0.00	0.00	0.00

Table 7: High-Pass Chebyshev Results.

$F(Hz)$	V_{in}	V_{out}	$A = \frac{V_{out}}{V_{in}}$	$A(dB) = 20log\frac{V_{out}}{V_{in}}$
20.00	1.20	0.00	0.00	0.00
40.00	1.32	0.00	0.00	0.00
60.00	1.36	0.00	0.14	0.00
80.00	1.42	0.20	0.30	-17.00
100.00	1.50	0.40	0.98	-10.50
500.00	0.96	0.92	0.96	-0.35
1000.00	0.96	0.92	0.98	-0.35
2000.00	0.92	0.90	0.98	-0.16
3000.00	0.90	0.88	0.98	-0.16
4000.00	0.90	0.86	0.95	-0.35
5000.00	0.86	0.82	0.95	-0.45

Table 8: Low-Pass Butterworth Results.

$F(Hz)$	V_{in}	V_{out}	$A = \frac{V_{out}}{V_{in}}$	$A(dB) = 20log\frac{V_{out}}{V_{in}}$
200.00	0.88	0.84	0.95	-0.44
400.00	0.72	0.62	0.86	-1.31
600.00	0.68	0.56	0.82	-1.68
800.00	0.54	0.48	0.88	-1.11
900.00	0.52	0.40	0.76	-2.28
1000.00	0.50	0.34	0.68	-3.34
2000.00	0.70	0.40	0.57	-4.88
3000.00	0.70	0.00	0.00	0.00
4000.00	0.70	0.00	0.00	0.00

Table 9: High-Pass Butterworth Results.

$F(Hz)$	V_{in}	V_{out}	$A = \frac{V_{out}}{V_{in}}$	$A(dB) = 20log\frac{V_{out}}{V_{in}}$
20.00	0.96	0.00	0.00	0.00
40.00	1.08	0.00	0.00	0.00
60.00	1.16	0.00	0.00	0.00
80.00	1.22	0.40	0.32	-10.00
100.00	1.28	0.80	0.63	-4.00
500.00	1.00	0.92	0.96	-0.72
1000.00	1.00	0.92	0.96	-0.72
2000.00	0.98	0.92	0.98	-0.63
3000.00	0.98	0.94	0.98	-0.44
4000.00	1.00	0.94	0.96	-0.53
5000.00	0.96	0.94	0.96	-0.26

CONCLUSION

These active filters has provided solution to electronic and electrical circuit because of the active component used in the construction of this filter such as op-amp along with passive components like capacitors and resistors. Considering the result above, the designer can now design and construct low pass and high pass filters with response at any frequencies. Bands pass and band stop filters can now be implemented with dual op-amps and a minimum number of passive components.

From the results presented, it was clearly shown that the Butterworth filters produce no ripples in the pass band and attenuate unwanted frequencies outside the band based on the specification. Chebychev filters attenuate unwanted frequencies with fewer components than Butterworth filters, but exhibit ripples in the band pass.

REFERENCES

1. Fernandez, A. and G. Jovanovich. 2006. “A New Method for the Design of IIR Filters with Flat Magnitude Response”. *IEEE Transactions on Circuits and Systems: Regular Papers*. 53(8).

2. Sallen, R.P. and E.L. Key. 1955. “A Practical Method of Designing RC Active Filters”. *IRE Transactions on Circuit Theory*. CT-2,1955, 78-85.

3. Moschytz, G.S. and P. Horn. 1981. *Active Filter Design Handbook*. Wiley: Chihester, UK.

4. Aikens, R.S. and W.J. Kerwin. 1972. “Single Amplifier, Minimal RC, Butterworth, Thompson and Chebychev Filters to 6th Order”. *Proc. of Intern. Filt. Symposium*. Santa Monica, CA. April 1972, 81-82.

5. Lakeradn, K.R. and M.S Ghausi. 1975. “Statistical Multi-Parameter Sensitivity: A Valuable Measure for CAD”. *Proc. of the IEEE Int. Symp. on Circuits and Sytems*. ISCAS. 1975,April 1975,333-336

SUGGESTED CITATION

Aremu, O.A., O.J. Iyiade, and D.A. Ajadi. 2013. “Design and Construction of Low- and High-Pas Active Filters Using Butterworth and Chebychev Techniques”. *Pacific Journal of Science and Technology*. 14(1):139-149.

Pacific Journal of Science and Technology