Effects of Some Thermophysical Properties on the Propagation of Forward and Opposed Smouldering Combustion.

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ABSTRACT

In this paper, we study one-dimensional, transient, governing equations for smouldering combustion in a porous fuel. We assume that there is a perfect contact between gas and solid phases so that one can make the hypothesis of local thermal equilibrium between the phases, $T_g = T_s = T$. We prove the existence and uniqueness of solution of the problems by actual solution. The time-dependent temperature and species mass fraction profiles are obtained through analytical method.

(Keywords: smouldering, self-sustaining smouldering, energy sink, unburnt fuel, porous matrix, condensed-phase fuel)

INTRODUCTION

Smouldering is a slow, low-temperature, flameless form of combustion, sustained by the heat evolved when oxygen directly attacks the surface of a condensed-phase fuel [1] [2]. Smouldering is the leading cause of deaths in residential fires [3]. It is of interest both as a fundamental combustion problem and as a practical fire hazard.

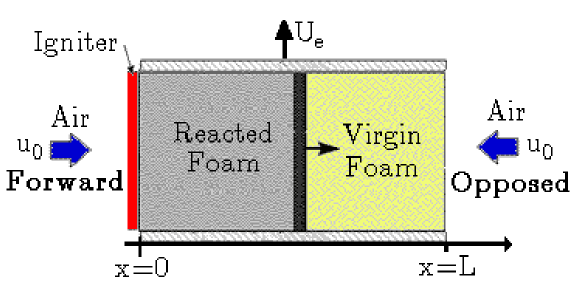

Figure 1: Computational Domain for Opposed and Forward Smouldering Combustion.

Smouldering is limited by the rate of oxygen-transport to the fuel's surface, resulting in a slower and lower temperature reaction than flaming. Importantly, smouldering can be self-sustaining (i.e., no energy input required after ignition) when the fuel is (or is embedded in) a porous medium. Self-sustaining smouldering occurs because the solid acts as an energy-sink and then feeds that energy back into the un-burnt fuel, creating a very energy efficient reaction [4]. Solid porous fuels such as polyurethane foam [5], cellulose [1] and charcoal are typical media that exhibit self-sustained smouldering.

While most research focuses on smouldering of solid fuels, there are several examples of combustion of a liquid fuel embedded in a porous matrix. Lagging fires occur inside porous insulating materials soaked in oils and other self-igniting liquids (Drysdale [6]). To enhance oil recovery, combustion fronts are initiated in petroleum reservoirs to drive oil toward extraction points (Greaves, et al. [7]). The reactions involved in enhanced oil recovery through in situ combustion are described as heterogeneous gas-solid and gas-liquid between oxygen and the heavy oil residue (Sarathi [8]).

Rein [9] synthesized a comprehensive view of smouldering combustion by bringing together contributions from diverse scientific disciplines.
In this paper, one-dimensional, transient, governing equation for smouldering combustion in a porous fuel is considered. We assume that there is a perfect contact between gas and solid phases. We consider the presure gradient to be parabolic. We prove the existence and uniqueness of solution. To simulate the flow analytically, we use asymptotics expansions.

MATHEMATICAL MODEL

The one-dimensional, transient, governing equations for smouldering combustion in a porous fuel is given by the equation of:

Conservation of Energy of Solid

$$\frac{\partial}{\partial t}\left(\rho_s C_{ps} T_s\right) = \frac{\partial}{\partial x}\left(k_s \frac{\partial T_s}{\partial x}\right) + \frac{h_{gs} A_{gs}}{V}\left(T_g - T_s\right) - \rho_0 \Delta h A e^{-\frac{E}{RT_s}} \quad \textbf{(1)}$$

Conservation of Energy of Gas

$$\frac{\partial}{\partial t}\left(\phi \rho_g C_{pg} T_g\right) = \frac{\partial}{\partial x}\left(\phi k_g \frac{\partial T_g}{\partial x}\right) + \frac{\partial}{\partial x}\left(\phi \rho_g \frac{K}{\mu}\frac{\partial P}{\partial x} C_{pg}\left(T_g - T_0\right)\right) - \frac{h_{gs} A_{gs}}{V}\left(T_g - T_s\right) \quad \textbf{(2)}$$

Conservation of Gas Species: Oxygen

$$\frac{\partial}{\partial t}\left(\phi \rho_g y_{o_2}\right) = \frac{\partial}{\partial x}\left(\phi \rho_g D \frac{\partial y_{o_2}}{\partial x}\right) + \frac{\partial}{\partial x}\left(\phi \rho_g \frac{K_x}{\mu}\frac{\partial P}{\partial x} y_{o_2}\right) - \rho_0 v_{o_2} A e^{-\frac{E}{RT_s}} \quad \textbf{(3)}$$

Smouldering Product

$$\frac{\partial}{\partial t}\left(\phi \rho_g y_{gp}\right) = \frac{\partial}{\partial x}\left(\phi \rho_g D \frac{\partial y_{gp}}{\partial x}\right) + \frac{\partial}{\partial x}\left(\phi \rho_g \frac{K_x}{\mu}\frac{\partial P}{\partial x} y_{gp}\right) + \rho_0 v_{gp} A e^{-\frac{E}{RT_s}} \quad \textbf{(4)}$$

The initial and boundary conditions were formulated as follows:

Initial condition:

At $t = 0$ and $\forall\ x$

$$\left.\begin{aligned} T_g &= T_0, & T_s &= T_0 \\ y_{o_2} &= 0, & y_{gp} &= 0 \end{aligned}\right\} \quad \textbf{(5)}$$

Boundary conditions:

$$\left.\begin{aligned} T_g\big|_{x=0} &= T_0, & T_g\big|_{x=L} &= T_0 \\ T_s\big|_{x=0} &= T_0, & T_s\big|_{x=L} &= T_0 \end{aligned}\right\} \quad \textbf{(6)}$$

$$\left.\begin{aligned} y_{o_2}\big|_{x=0} &= y_0, & y_{o_2}\big|_{x=L} &= 0 \\ y_{gp}\big|_{x=0} &= 0, & y_{gp}\big|_{x=L} &= 0 \end{aligned}\right\}, \quad \textbf{(7)}$$

where $\frac{A_{gs}}{V}$ is the ratio of surface area between gas and solid to volume, E is activation energy, R is the perfect gas constant, L is sample length, k is thermal conductivity, Δh is the enthalpy of reaction, C is specific heat, T is temperature, y is the mass fraction of gas species, t is time, x is position, h_{gs} is the heat transfer coefficient between gas and solid, K is permeability, μ is dynamic viscosity, P is pressure, ρ is density, ϕ is the porosity of the media.

We assume that there is a perfect contact between gas and solid phases so that one can make the hypothesis of local thermal equilibrium between the phases:

$$T_g = T_s = T \quad \textbf{(8)}$$

Adding (1) and (2), we obtain:

$$\frac{\partial}{\partial t}\left(\rho C_p T\right) = \frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial x}\left(\phi \rho_g \frac{k}{\mu}\frac{\partial P}{\partial x} C_{pg}\left(T - T_0\right)\right) - \rho_0 \Delta h A e^{-\frac{E}{RT}}, \quad \textbf{(9)}$$

where

$$\rho C_p = \rho_s C_{ps} + \phi \rho_g C_{pg}$$

$$\lambda = k_s + \phi k_g$$

are respectively, the overall thermal capacity per unit volume and the overall thermal conductivity.

METHOD OF SOLUTION

Here, we make the additional assumptions that $\rho,\ \rho_g,\ \rho_0,\ C_p,\ C_{pg},\ \ \phi,\ \lambda,\ D,\ K,\ K_x$ and μ are constant, and we consider the pressure gradient to be parabolic, i.e.:

$$\frac{\partial P}{\partial x} = f(x) = \frac{x}{L}\left(1 - \frac{x}{L}\right) \qquad \textbf{(10)}$$

These assumptions could be relaxed in the future. By introducing the following dimensionless variables:

$$\theta = \frac{E}{RT_o^2}(T - T_o),\ \ Y = \frac{y_{o_2}}{y_{o_2}^0},\quad Z = \frac{y_{gp}}{y_{gp}^0},$$

$$\in = \frac{RT_0}{E},\quad x' = \frac{x}{L},\quad t' = \frac{t}{t_0} \qquad \textbf{(11)}$$

Equations (3), (4) and (9) after dropping prime become

$$\frac{\partial \theta}{\partial t} = \lambda_1 \frac{\partial^2 \theta}{\partial x^2} + k_1 x(1-x)\frac{\partial \theta}{\partial x} + k_1(1-2x)\theta - \delta e^{\frac{\theta}{1+\in\theta}} \qquad \textbf{(12)}$$

$$\frac{\partial Y}{\partial t} = D_1 \frac{\partial^2 Y}{\partial x^2} + k_2 x(1-x)\frac{\partial Y}{\partial x} + k_2(1-2x)Y - \beta e^{\frac{\theta}{1+\in\theta}} \qquad \textbf{(13)}$$

$$\frac{\partial Z}{\partial t} = D_1 \frac{\partial^2 Z}{\partial x^2} + k_2 x(1-x)\frac{\partial Z}{\partial x} + k_2(1-2x)Z + \sigma e^{\frac{\theta}{1+\in\theta}} \qquad \textbf{(14)}$$

together with the initial and boundary conditions

$$\left.\begin{aligned} &\theta(x,0) = 0,\quad \theta(0,t) = 0,\quad \theta(1,t) = 0 \\ &Y(x,0) = 0,\ \ Y(0,t) = Y_0,\ \ Y(1,t) = 0 \\ &Z(x,0) = 0,\ \ Z(0,t) = 0,\ \ Z(1,t) = 0 \end{aligned}\right\}, \qquad \textbf{(15)}$$

where,

$$k_1 = \frac{\phi \rho_g K t_0 C_{pg}}{\rho C_p \mu L},\ \lambda_1 = \frac{t_0 \lambda}{\rho C_p L^2},$$

$$\delta = \frac{\Delta h \rho_0 A t_0 . e^{-\frac{E}{RT_0}}}{\in T_0 \rho C_p},\ D_1 = \frac{D t_0}{L^2},\ k_2 = \frac{K_x t_0}{\mu L},$$

$$\beta = \frac{\rho_0 A v_{o_2} t_0 . e^{-\frac{E}{RT_0}}}{\phi \rho_g y_{o_2}^0},\quad \sigma = \frac{\rho_0 A v_{gp} t_0 . e^{-\frac{E}{RT_0}}}{\phi \rho_g y_{gp}^0}$$

Existence and Uniqueness of Solution

Theorem 1 Let $k_1 = k_2 = k$, $D_1 = \lambda_1 = \lambda$ and $\beta = \sigma = \delta$. Then there exists a unique solution of problem (12), (13) and (14) satisfy (15).

Proof: Let $k_1 = k_2 = k$, $D_1 = \lambda_1 = \lambda$, $\beta = \sigma = \delta$ and $\phi = \left(Z + \frac{1}{2}(\theta + Y)\right)$.

Then (12) - (15) becomes,

$$\frac{\partial \phi}{\partial t} = \lambda \frac{\partial^2 \phi}{\partial x^2} + kx(1-x)\frac{\partial \phi}{\partial x} + k(1-2x)\phi \qquad \textbf{(16)}$$

$$\phi(x,0) = 0,\quad \phi(0,t) = \frac{1}{2}Y_0,\quad \phi(1,t) = 0 \qquad \textbf{(17)}$$

Equations (16) and (17) admit self-similar solution of the form:

$$\phi(x,t) = t^{-\alpha} f(\eta),\qquad \eta = xt^{-\alpha} \qquad \textbf{(18)}$$

such that Equations (16) and (17) become

$$\lambda f'' + k\eta(1-\eta)f' + k(1-2\eta)f = 0 \quad \textbf{(19)}$$

$$f(0) = \frac{1}{2}Y_0, \qquad f(1) = 0 \quad \textbf{(20)}$$

The similarity solution exist if $\alpha = 0$. We adopt the Frobenius method of solution and we obtain the solution of Problem (19) in series form as (using the first few terms of the series):

$$f(\eta) = \frac{1}{2}Y_0\left(\begin{array}{l}1+\eta-\frac{k}{2\lambda}\eta^2+ \\ \frac{(k^2+2k\lambda)}{8\lambda^2}\eta^4\end{array}\right) - \frac{1}{2}\frac{Y_0\left(2-\frac{k}{2\lambda}+\frac{(k^2+2k\lambda)}{8\lambda^2}\right)}{\left(1-\frac{k}{12\lambda}+\frac{k^2}{15\lambda^2}\right)}. \left(\eta-\frac{k}{3\lambda}\eta^3+\frac{k}{4\lambda}\eta^4+\frac{k^2}{15\lambda^2}\eta^5\right) \quad \textbf{(21)}$$

and

$$\phi(x,t) = t^0\left(\begin{array}{l}\frac{1}{2}Y_0\left(\begin{array}{l}1+x-\frac{k}{2\lambda}x^2+ \\ \frac{(k^2+2k\lambda)}{8\lambda^2}x^4\end{array}\right) - \\ \frac{1}{2}\frac{Y_0\left(2-\frac{k}{2\lambda}+\frac{(k^2+2k\lambda)}{8\lambda^2}\right)}{\left(1-\frac{k}{12\lambda}+\frac{k^2}{15\lambda^2}\right)}. \\ \left(\begin{array}{l}x-\frac{k}{3\lambda}x^3+\frac{k}{4\lambda}x^4+ \\ \frac{k^2}{15\lambda^2}x^5\end{array}\right)\end{array}\right) \quad \textbf{(22)}$$

Then, we obtain:

$$\theta(x,t) = 2(\phi(x,t) - Z(x,t)) - Y(x,t) \quad \textbf{(23)}$$

$$Y(x,t) = 2(\phi(x,t) - Z(x,t)) - \theta(x,t) \quad \textbf{(24)}$$

$$Z(x,t) = \phi(x,t) - \frac{1}{2}(\theta(x,t) + Y(x,t)) \quad \textbf{(25)}$$

Hence, there exists a unique solution of problem (12) - (15). This completes the proof.

Analytical Solution

Here, we consider equations (12) - (15).

Ayeni [10] has shown that $\exp\left(\frac{\theta}{1+\in\theta}\right)$ can be approximated as $1+(e-2)\theta+\theta^2$. In this paper we are going to take an approximation of the form:

$$\exp\left(\frac{\theta}{1+\in\theta}\right) \approx 1+(e-2)\theta \quad \textbf{(26)}$$

Using the asymptotic expansion:

$$\theta = \theta_0 + \in\theta_1 + \in^2\ \theta_2 + h.o.t. \quad \textbf{(27)}$$

$$Z = Z_0 + \in Z_1 + \in^2\ Z_2 + h.o.t., \quad \textbf{(28)}$$

where $h.o.t.$ read "higher order terms in $\in$. In our analysis we are interested only in the first two terms.

Let $k_1 = \in k_0$ **(29)**

$k_2 = \in q_0$ **(30)**

and equate the powers of $\in$, we have the following set of non-homogeneous boundary value problems.

$O(1)$:

$$\frac{\partial\theta_0}{\partial t} = \lambda_1\frac{\partial^2\theta_0}{\partial x^2} - p\theta_0 - \delta \quad \textbf{(31)}$$

$$\theta_0(x,0) = 0, \qquad \theta_0(0,t) = 0, \qquad \theta_0(1,t) = 0$$

$$\frac{\partial Z_0}{\partial t} = D_1\frac{\partial^2 Z_0}{\partial x^2} + \sigma_0\theta_0 + \sigma \quad \textbf{(32)}$$

$$Z_0(x,0) = 0, \qquad Z_0(0,t) = 0, \qquad Z_0(1,t) = 0$$

$O(\in)$:

$$\frac{\partial \theta_1}{\partial t} = \lambda_1 \frac{\partial^2 \theta_1}{\partial x^2} - p\theta_1 + k_0 x(1-x)\frac{\partial \theta_0}{\partial x} + k_0(1-2x)\theta_0 \qquad \textbf{(33)}$$

$$\theta_1(x,0) = 0, \qquad \theta_1(0,t) = 0, \qquad \theta_1(1,t) = 0$$

$$\frac{\partial Z_1}{\partial t} = D_1 \frac{\partial^2 Z_1}{\partial x^2} + \sigma_0 \theta_1 + q_0 x(1-x)\frac{\partial Z_0}{\partial x} + q_0(1-2x)Z_0 \qquad \textbf{(34)}$$

$$Z_1(x,0) = 0, \qquad Z_1(0,t) = 0, \qquad Z_1(1,t) = 0\,,$$

where $p = \delta(e-2)$, $\beta_0 = \beta(e-2)$ and $\sigma_0 = \sigma(e-2)$

We obtain the solutions of Equations (31), (32), (33) and (34) respectively as:

$$\theta_0(x,t) = \sum_{n=1}^{\infty} \frac{2\delta\, a_n}{n\pi S}\left(1 - e^{-(p+\lambda_1 n\pi)t}\right)\sin n\pi x \qquad \textbf{(35)}$$

$$Z_0(x,t) = \sum_{n=1}^{\infty}\left(\sum_{n=1}^{\infty}\frac{2\sigma_0\delta\, a_n}{n\pi S}\left(\frac{e^{-St} - e^{-D_1 n\pi t}}{S - D_1 n\pi} + \frac{1 - e^{-D_1 n\pi t}}{D_1 n\pi}\right)\right)\sin n\pi x - \sum_{n=1}^{\infty}\frac{2\sigma\, a_n}{D_1 n^2\pi^2}\left(1 - e^{-D_1 n\pi t}\right)\sin n\pi x \qquad \textbf{(36)}$$

$$\theta_1(x,t) = \sum_{n=1}^{\infty}\left(\sum_{n=1}^{\infty}\frac{k_0\delta\, a_n b_n}{n^3\pi^3 S^2}\left(1 - (1 + St)e^{-St}\right)\right)\sin n\pi x \qquad \textbf{(37)}$$

$$\begin{aligned}
Z_1(x,t) = {} & -\sum_{n=1}^{\infty}\left(\sum_{n=1}^{\infty}\frac{\sigma\, q_0\, a_n\, b_n}{D_1^2 n^5 \pi^5}\left(1 - S_3\, e^{-D_1 n\pi t}\right)\right)\sin n\pi x + \\
& \sum_{n=1}^{\infty}\left(\sum_{n=1}^{\infty}\left(\frac{\sigma_0 \delta\, q_0 a_n\, b_n}{n^3\pi^3 S}\sum_{n=1}^{\infty}\left(-\frac{e^{-St} + S_1 e^{-D_1 n\pi t}}{(S - D_1 n\pi)^2} + \frac{1 - S_3 e^{-D_1 n\pi t}}{D_1^2 n^2 \pi^2}\right)\right)\right)\sin n\pi x + \\
& \sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{\sigma_0\delta\, k_0 a_n\, b_n}{n^3\pi^3 S^2}\sum_{n=1}^{\infty}\left(\frac{1 - e^{-D_1 n\pi t}}{D_1 n\pi} + \frac{e^{-St} - e^{-D_1 n\pi t}}{(S - D_1 n\pi)} + \frac{S\left(e^{-St} - e^{-D_1 n\pi t}\right)}{S_2} + \frac{S(S - D_1 n\pi)te^{-St}}{S_2}\right)\right)\sin n\pi x
\end{aligned} \qquad \textbf{(38)}$$

Therefore, we obtain:

$$\theta(x,t) = \sum_{n=1}^{\infty}\frac{2\delta\, a_n}{n\pi S}\left(1 - e^{-St}\right)\sin n\pi x + \sum_{n=1}^{\infty}\left(\sum_{n=1}^{\infty}\frac{k_1\delta\, a_n b_n}{n^3\pi^3 S^2}\left(1 - (1 + St)e^{-St}\right)\right)\sin n\pi x \qquad \textbf{(39)}$$

$$Z(x,t)=\sum_{n=1}^{\infty}\left(\sum_{n=1}^{\infty}\frac{2\sigma_0\delta\, a_n}{n\pi S}\left(\frac{e^{-St}-e^{-D_1 n\pi t}}{S-D_1 n\pi}+\frac{1-e^{-D_1 n\pi t}}{D_1 n\pi}\right)\right)\sin n\pi x-$$
$$\sum_{n=1}^{\infty}\frac{2\sigma\, a_n}{D_1 n^2\pi^2}\left(1-e^{-D_1 n\pi t}\right)\sin n\pi x-$$
$$\sum_{n=1}^{\infty}\left(\sum_{n=1}^{\infty}\frac{\sigma k_2 a_n b_n}{D_1^2 n^5\pi^5}\left(1-S_3 e^{-D_1 n\pi t}\right)\right)\sin n\pi x+$$
$$\sum_{n=1}^{\infty}\left(\sum_{n=1}^{\infty}\left(\sum_{n=1}^{\infty}\frac{\sigma_0\delta k_2 a_n b_n}{n^3\pi^3 S}\left(-\frac{e^{-St}+S_1 e^{-D_1 n\pi t}}{(S-D_1 n\pi)^2}+\frac{1-S_3 e^{-D_1 n\pi t}}{D_1^2 n^2\pi^2}\right)\right)\right)\sin n\pi x+$$
$$\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\left(\sum_{n=1}^{\infty}\frac{\sigma_0\delta k_1 a_n b_n}{n^3\pi^3 S^2}\left(\frac{1-e^{-D_1 n\pi t}}{D_1 n\pi}+\frac{e^{-St}-e^{-D_1 n\pi t}}{(S-D_1 n\pi)}+\frac{S\left(e^{-St}-e^{-D_1 n\pi t}\right)}{S_2}+\frac{S(S-D_1 n\pi)te^{-St}}{S_2}\right)\right)\sin n\pi x \qquad \textbf{(40)}$$

For the solution $Y(x,t)$, we use Equation (24) and obtain:

$$Y(x,t)=2t^0\left(\frac{1}{2}Y_0\left(1+x-\frac{k}{2\lambda}x^2+\frac{(k^2+2k\lambda)}{8\lambda^2}x^4\right)-\frac{1}{2}\frac{Y_0\left(2-\frac{k}{2\lambda}+\frac{(k^2+2k\lambda)}{8\lambda^2}\right)}{\left(1-\frac{k}{12\lambda}+\frac{k^2}{15\lambda^2}\right)}\left(x-\frac{k}{3\lambda}x^3+\frac{k}{4\lambda}x^4+\frac{k^2}{15\lambda^2}x^5\right)\right)-$$
$$\sum_{n=1}^{\infty}\left(\sum_{n=1}^{\infty}\frac{4\sigma_0\delta\, a_n}{n\pi S}\left(\frac{e^{-St}-e^{-D_1 n\pi t}}{S-D_1 n\pi}+\frac{1-e^{-D_1 n\pi t}}{D_1 n\pi}\right)\right)\sin n\pi x+$$
$$\sum_{n=1}^{\infty}\frac{4\sigma\, a_n}{D_1 n^2\pi^2}\left(1-e^{-D_1 n\pi t}\right)\sin n\pi x+$$
$$\sum_{n=1}^{\infty}\left(\sum_{n=1}^{\infty}\frac{2\sigma k_2 a_n b_n}{D_1^2 n^5\pi^5}\left(1-S_3\, e^{-D_1 n\pi t}\right)\right)\sin n\pi x-$$
$$\sum_{n=1}^{\infty}\left(\sum_{n=1}^{\infty}\left(\sum_{n=1}^{\infty}\frac{2\sigma_0\delta k_2 a_n b_n}{n^3\pi^3 S}\left(-\frac{e^{-St}+S_1 e^{-D_1 n\pi t}}{(S-D_1 n\pi)^2}+\frac{1-S_3 e^{-D_1 n\pi t}}{D_1^2 n^2\pi^2}\right)\right)\right)\sin n\pi x-$$
$$\sum_{n=1}^{\infty}\frac{2\delta a_n}{n\pi S}\left(1-e^{-St}\right)\sin n\pi x-$$
$$\sum_{n=1}^{\infty}\left(\sum_{n=1}^{\infty}\frac{k_1\delta a_n b_n}{n^3\pi^3 S^2}\left(1-(1+St)e^{-St}\right)\right)\sin n\pi x-$$
$$\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\left(\sum_{n=1}^{\infty}\frac{2\sigma_0\delta k_1 a_n b_n}{n^3\pi^3 S^2}\left(\frac{1-e^{-D_1 n\pi t}}{D_1 n\pi}+\frac{e^{-St}-e^{-D_1 n\pi t}}{(S-D_1 n\pi)}+\frac{S\left(e^{-St}-e^{-D_1 n\pi t}\right)}{S_2}+\frac{S(S-D_1 n\pi)te^{-St}}{S_2}\right)\right)\sin n\pi x \qquad \textbf{(41)}$$

where

$$a_n=\left((-1)^n-1\right)$$
$$b_n=\left((-1)^{2n}-1\right)$$
$$S=p+\lambda_1 n\pi$$

$$S_1 = ((S - D_1 n\pi)t - 1)$$
$$S_2 = S^2 + D_1^2 n^2 \pi^2 - 2D_1 n\pi S$$
$$S_3 = (1 + D_1 n\pi t)$$

RESULTS AND DISCUSSION

We have proved the existence and uniqueness of solution of the Problem by actual solution.

The temperature and concentration profiles are presented in Figures 1 - 12. Figure 1 displays the graph of $\theta(x,t)$ against x and t for different values of δ.

Figure 2 displays the graph of $\theta(x,t)$ against x for different values of δ.

Figure 3 displays the graph of $\theta(x,t)$ against t for different values of δ. From Figures 1-3 it is seen that temperature decreases as Frank-Kamenetskii number increases.

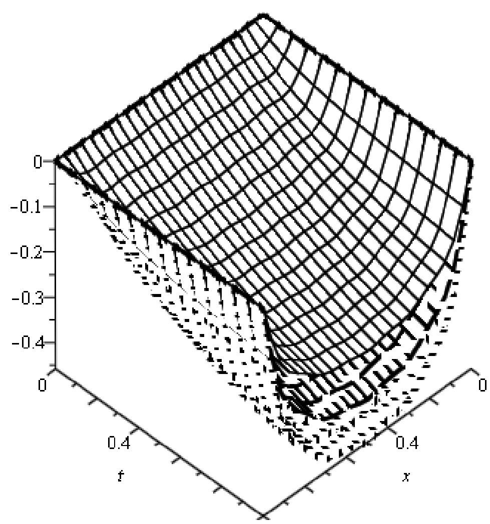

Figure 1: Plots of $\theta(x, t)$ against x and t for equation (3.3) for different values of δ and $\sigma = 0.2$, $\beta = 0.3$, $\lambda_1 = 0.3$, $D_1 = 0.3$, $e = 2.718$

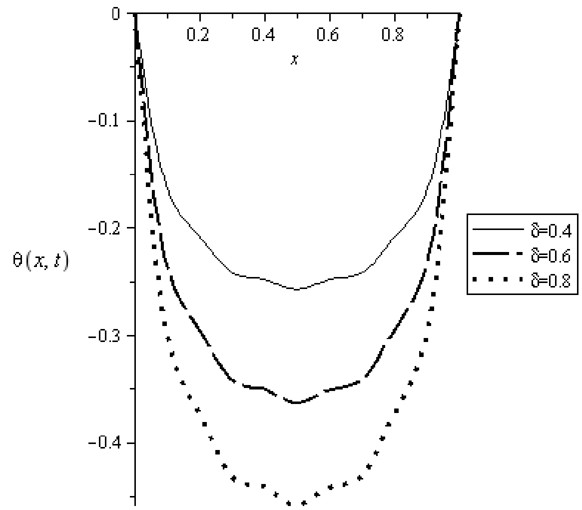

Figure 2: Plots of $\theta(x, t)$ against x for equation (3.3) for different values of δ and $\sigma = 0.2$, $\beta = 0.3$, $\lambda_1 = 0.3$, $D_1 = 0.3$, $e = 2.718$, $t = 1$

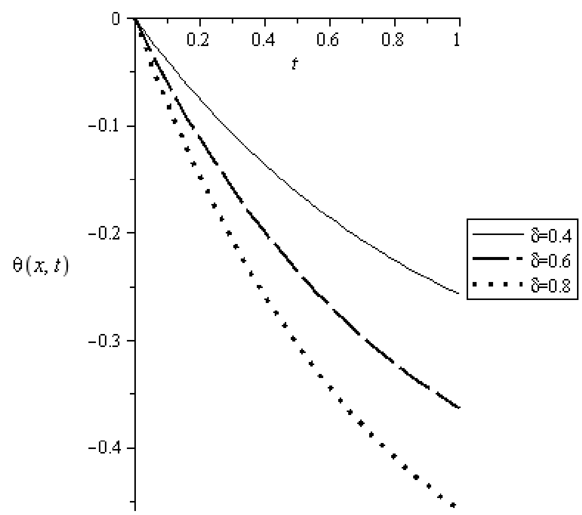

Figure 3: Plots of $\theta(x, t)$ against t for equation (3.3) for different values of δ and $\sigma = 0.2$, $\beta = 0.3$, $\lambda_1 = 0.3$, $D_1 = 0.3$, $e = 2.718$, $x = 0.5$

Figure 4 displays the graph of $\theta(x,t)$ against x and t for different values of λ_1. From Figure 4 it is evident that temperature decreases as scaled thermal conductivity decreases. Figure 5 displays the graph of $Y(x,t)$ against x and t for different values of D_1.

From Figure 5 it is seen that mass fraction of oxygen consumed increases as diffusion coefficient decreases.

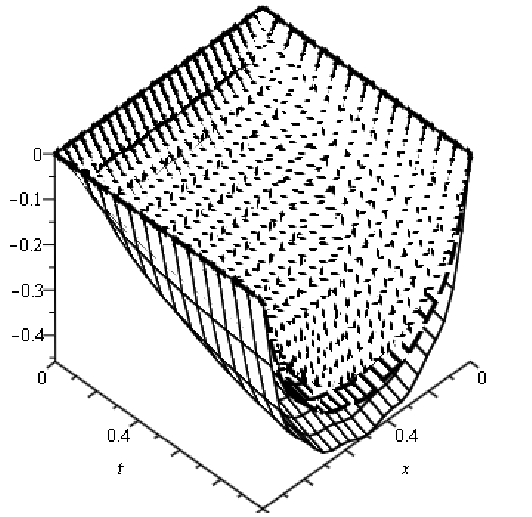

Figure 4: Plots of $\theta(x, t)$ against x and t for equation (3.3) for different values of λ_1 and $\sigma = 0.2$, $\beta = 0.3$, $\delta = 0.8$, $D_1 = 0.3$, $e = 2.718$

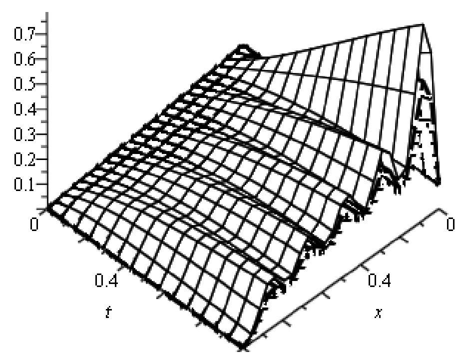

Figure 5: Plots of $Y(x, t)$ against x and t for equation (3.4) for different values of D_1 and $\sigma = 0.2$, $\beta = 0.3$, $\delta = 0.8$, $\lambda_1 = 0.3$, $e = 2.718$ $Y_0 = 0.1$, $k = 1$

Figure 6 displays the graph of $Z(x,t)$ against x and t for different values of D_1. From Figure 6 it is seen that mass fraction of smouldering product produced increases as diffusion coefficient decreases.

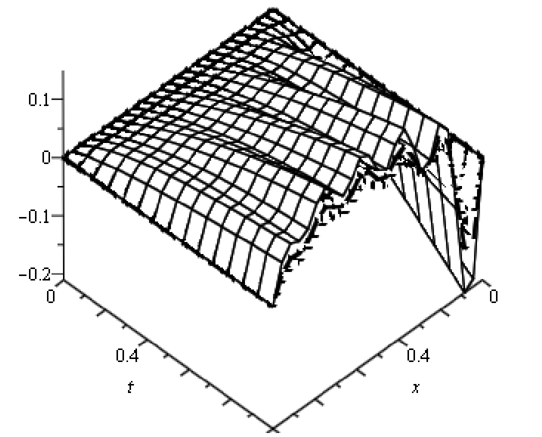

Figure 6: Plots of $Z(x, t)$ against x and t for equation (3.5) for different values of D_1 and $\sigma = 0.2$, $\beta = 0.3$, $\delta = 0.8$, $\lambda_1 = 0.3$, $e = 2.718$

Figure 7 displays the graph of $Y(x,t)$ against x and t for different values of δ.

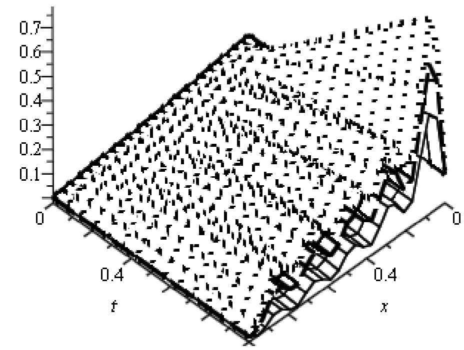

Figure 7: Plots of $Y(x, t)$ against x and t for equation (3.4) for different values of δ and $\sigma = 0.2$, $\beta = 0.3$, $\lambda_1 = 0.3$, $D_1 = 0.3$, $e = 2.718$, $Y_0 = 0.1$, $k = 1$

Figure 8 displays the graph of $Y(x,t)$ against x for different values of δ.

Figure 9 displays the graph of $Y(x,t)$ against t for different values of δ. From Figures 7-9 it is evident that mass fraction of oxygen consumed increases as Frank-Kamenetskii number increases.

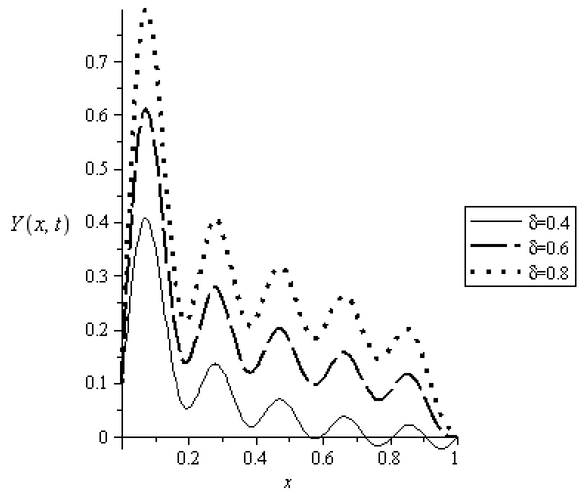

Figure 8: Plots of $Y(x, t)$ against x for equation (3.4) for different values of δ and $\sigma = 0.2$, $\beta = 0.3$, $\lambda_1 = 0.3$, $D_1 = 0.3$, $e = 2.718$, $t = 1$

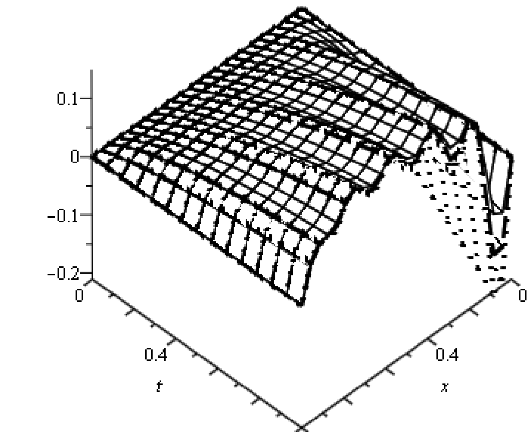

Figure 10: Plots of $Z(x, t)$ against x and t for equation (3.5) for different values of δ and $\sigma = 0.2$, $\beta = 0.3$, $\lambda_1 = 0.3$, $D_1 = 0.3$, $e = 2.718$

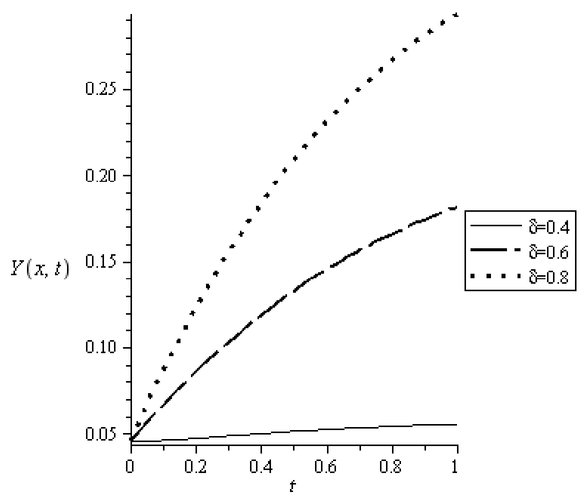

Figure 9: Plots of $Y(x, t)$ against t for equation (3.4) for different values of δ and $\sigma = 0.2$, $\beta = 0.3$, $\lambda_1 = 0.3$, $D_1 = 0.3$, $e = 2.718$, $x = 0.5$

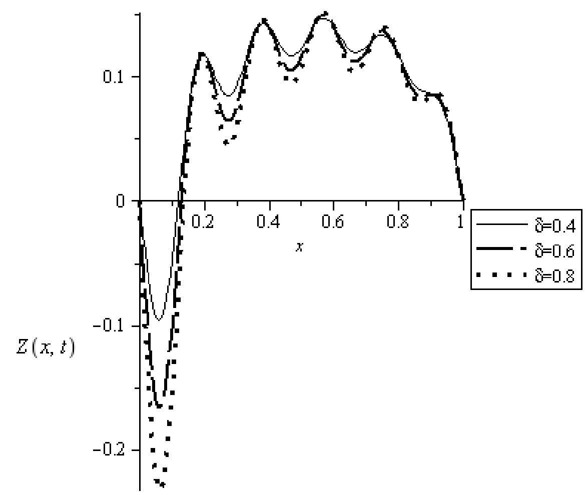

Figure 11: Plots of $Z(x, t)$ against x for equation (3.5) for different values of δ and $\sigma = 0.2$, $\beta = 0.3$, $\lambda_1 = 0.3$, $D_1 = 0.3$, $e = 2.718$, $t = 1$

Figure 10 displays the graph of $Z(x,t)$ against x and t for different values of δ.

Figure 11 displays the graph of $Z(x,t)$ against x for different values of δ.

Figure 12 displays the graph of $Z(x,t)$ against t for different values of δ. From Figures 10-12 it is evident that mass fraction of smouldering product produced increases as Frank-Kamenetskii number decreases.

It is worth pointing out that the effects observed in Figures 1 - 12 physically means that the temperature is decreased, the species is consumed and the smouldering product is produced. These occur as a result of the oxidizer flux to and the heat losses from the reaction zone.

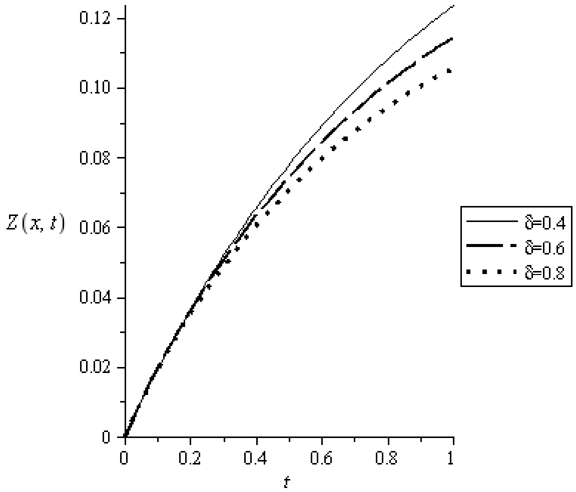

Figure 12: Plots of $Z(x,t)$ against t for equation (3.5) for different values of δ and $\sigma = 0.2$, $\beta = 0.3$, $\lambda_1 = 0.3$, $D_1 = 0.3$, $e = 2.718$, $x = 0.5$

CONCLUSION

For the slow, low-temperature, flameless form of combustion, sustained by the heat evolved when oxygen directly attacks the surface of a condensed-phase fuel, analytical solutions have been presented using the asymptotic expansion.

The governing parameters of the problem are Frank-Kamenetskii number (δ), scaled thermal conductivity (λ_1) and diffusion coefficient (D_1). The analytical method is used to search for transient state temperature and species profiles. The heat transfer decreased, species consumed and smouldering product produced.

NOMENCLATURE

$\frac{A_{gs}}{V}$: ratio of surface area between gas and solid to volume
E: activation energy
R: perfect gas constant
L: sample length
k: thermal conductivity
C: specific heat at constant pressure
T: temperature
y: mass fraction of gas species
t: time
x: position
h_{gs}: heat transfer coefficient between gas and solid
K: permeability
P: pressure
Y: dimensionless mass fraction of oxygen
Z: dimensionless mass fraction of smouldering product
D_1: dimensionless diffusion coefficient $\left\{= \frac{Dt_0}{L^2}\right\}$

Greek Letters:
Δh: enthalpy of reaction
μ: dynamic viscosity
ρ: density
ϕ: porosity of the media
$\in$: dimensionless activation energy $\left\{= \frac{RT_0}{E}\right\}$
θ: dimensionless temperature
λ: overall thermal conductivity
λ_1: scaled thermal conductivity $\left\{= \frac{t_0\lambda}{\rho C_p L^2}\right\}$
δ: Frank-Kamenetskii number $\left\{= \frac{\Delta h \rho_0 A t_0 . e^{-\frac{E}{RT_0}}}{\in T_0 \rho C_p}\right\}$

Subscripts:
g: gas
s: solid
p: constant pressure
0: initial

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SUGGESTED CITATION

Olayiwola, R.O. 2012. “Effects of Some Thermophysical Properties on the Propagation of Forward and Opposed Smouldering Combustion”. *Pacific Journal of Science and Technology*. 13(2):108-118.

Pacific Journal of Science and Technology