LMT Sludge Filtration Equation Using Dimensional Analysis.

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ABSTRACT

This study was completed by a combination of laboratory experimentation and extensive mathematical analysis. The volume of filtrate undergoing dewatering was expressed as a function of seven parameters which include: applied vacuum pressure, area of filtration, sludge solid content, viscosity, specific resistance of sludge, filtration time, and compressibility coefficient. The relationship between these parameters was determined using the Buckingham *π* method of dimensional analysis, a versatile tool reported to have been used in deriving many fluid flow equations. The generalized filtration equation thus obtained was calibrated by multiple linear regression using experimental data. The equation obtained resembles Carman's equation in the mode of parameter combination except the presence of compressibility coefficient in the denominator.

(Keywords: Buckingham π method, dimensional analysis, fluid flow equations, waste management)

INTRODUCTION

The high complex technology development in the form of industrialization has resulted in high percentage of waste generation. The problem of wastes has increased drastically due to rapid rate of industrialization and increase in population all over the world. Proper waste-management system should be established and enhanced in view of the obnoxious menace imposed on our community due to improper handling and disposal of wastes to our environment. Often, domestic and industrial wastes are being disposed by methods which create unhealthy environmental conditions.

The improper management and treatment of these wastes contribute to air, water and soil pollution. They also create breeding places for disease-carrying insects and rodents. These situations result in public nuisance, and adversely affect land values, and generally interfere with community lives.

Sludge generated in waste management process are difficult to handle and dispose off because of the high percentage of water content (Ademiluyi, 1984). Sludge is first digested, dewatered, dried and then incinerated. However, incineration is not an environmentally friendly option because of its implication for the global climate. A more sustainable approach is for the sludge to be used as soil conditioner or further treated and then used for bricks or other products. Recent research has revealed that despite the fact that sludge volume is usually less than 1% of the total plant influent, sludge handling costs are 21 – 50% of total plant operating and maintenance costs (Randal, 2001). Sludge contains solids, oil, fat, protein, phosphates, carbohydrate, nitrogen, water, etc with a specific gravity of 1.02 1.06 (for organic fraction) and 2.5 for inorganic fraction (Ekpobari, 2002). A comprehensive characterization of different kinds of sludge has been presented in Table 1.

The sludge generated by waste treatment must undergo dewatering before disposal to the environment. This can be achieved by filtration. Filtration is the process of separating a heterogeneous mixture of a fluid and particles of a solid by means of a filter medium which permits the passage of the filtrate but retains the particles during the process.

Carman (1934) proposed a filtration equation in which he assumed that the specific resistance is constant throughout the sludge cake thickness and that the cake is rigid. Ruth (1935) and Shirato (1972) challenged Carman's equation that the specific resistance parameter should be designated as an average value. Armenante (2002) assumed that the pressure across the cake has no impact on specific cake resistance.

Table 1: Properties of Different Kinds of Sludge.

Property		**Primary Sludge**	**Biological sludge**	**Biological sludge from clarified water**	**Mixed sludge**	**Digested sludge**
Dry matter (DM)	g/L	12	9	7	10	30
Volatile matter(VM)	%DM	65	67	77	72	50
pH		6	7	7	6.5	7
Carbon	% VM	51.5	52.5	53	51	49
Hydrogen	% VM	7	6	6.7	7.4	7.7
Oxygen	% VM	35.5	33	33	33	35
Nitrogen	% VM	4.5	7.5	6.3	7.1	6.2
Sulphur	% VM	1.5	1	1	1.5	2.1
Carbon/Nitrogen	-	11.4	7	8.7	7.2	7.9
Phosphorus	% DM	2	2	2	2	2
Chlorine	% DM	0.8	0.8	8.8	0.8	0.8
Potassium	“	0.3	0.3	0.3	0.3	0.3
Aluminium	“	0.2	0.2	0.2	0.2	0.2
Calcium	“	10	10	10	10	10
Iron	“	2	2	2	2	2
Magnesium	“	0.6	0.6	0.6	0.6	0.6
Fat	“	18	8	10	14	10
Protein	“	24	38	34	30	18
Fibres	“	16	7	10	13	10
Calorific V_a/m	kw/h/t DM	4,200	4,100	4,800	4,600	3,000

From Andersen (2001)

He stressed that the void fraction for most cakes can be significantly affected by pressure, because the cake is often compressible. He concluded that because the pressure drop changes with time the void fraction, can also be a function of time, at least in principle. Anazodo (1974) objected to Carman's equation on its formulation point of view. He argued that the approximation of compressible filter cakes to rigid bundles of capillary tubes or to non-compressible sand-beds did not make sense. He then developed another basic theoretical equation for the compressible sludge using FMTL$_x$ L$_y$L$_z$ dimensional analysis to arrive at Equation 1.

$$V^2 = \left[\frac{A^2}{Cr}\right]^{1/2}\left[\frac{A^{1/2}\ PC^{1/2}\ r^{1/2}t}{\mu}\right]^{f} \qquad (1)$$

In his equation Anazodo (1974) assumed f = ½ because the relationship between V and t was established to be parabolic. He finally arrived at:

$$V^2 = \frac{PA^{5/4}t}{\mu C^{1/2}\ r^{1/2}} \qquad (2)$$

The derivation of the dimensional equation was not acceptable to white and Gale (1975) because Anazodo(1974) did not justify the prediction that the volume of filtrate obtained after a fixed time is proportional to the filtration area to the power of 5/4 and that the exponent f of Equation 1 should be ½. They pointed out that Carman's equation which only predicts volume of filtrate to be just proportional to the area was preferable. Gale and White (1975) then suggested that Anazodo's partial equation should be written as:

$$V^2 = P\mu^{-1}A^{2b}\,t\,(C\,r)^{2b-3} \qquad (3)$$

Gale and White (1975) stressed that the relationship between V and A should be experimentally determined. At the end of the experiment if b=1 Carman's equation is implied and if b = 5/4, Anazodo's is preferable. White and Gale (1975) agreed with Anazodo that the determination of the correct value of 'b' based on theoretical or experimental considerations will guide the choice of filtration equation. To buttress further, Ademiluyi, Anazodo, and Egbuniwe (1982) carried out an investigation to determine experimentally the value of the exponent 'b' which relates to the volume of filtration to the area of filtration. The average value of 'b' was found to be 0.91 ± 0.02 when using effective area of filtration. At the end of the experiment, they suggested that the total area of filtration should be used in

Carman's equation while effective area of filtration should be used in Anazodo's dimensional equation for sludge filtration at constant pressure. The equations formulated by Ademiluyi and co-workers after the substitution of 'b' in the partial dimensional equation are:

$$V^2 = \frac{PA^{1.82}t}{\mu(cr)^{1.18}} \quad and \quad V^2 = \frac{PA_{eff}^{2.76}t}{\mu(cr)^{0.24}} \qquad \textbf{(4)}$$

It is worth noting that the traditional Carman's equation, Anazodo and Ademiluyi and his co-workers did not account for the compressibility coefficient in the formulation of their equations. The introduction of compressibility coefficient 'S' as an attribute of the new equation cannot be over emphasized. First, it eases the rigorous mathematical manipulation of maximum specific resistance as a means of determining the filterability of sludge cakes as formulated by Ademiluyi (1985). Secondly, since sewage sludge is compressible and not rigid as assumed by Carman, it is self-evident that compressibility attribute of sludge dewatering must be properly accounted for in the new equations.

Ademiluyi (1987) stated that in order obtain a valid sludge filtration equation, compressibility attribute of the sludge cake in question should be properly accounted for. The objective of this study is to develop a new filtration equation which incorporates the compressibility attribute as a measure of filterability index known as compressibility coefficient 'S'.

METHODOLOGY

Experimental Set Up

The traditional laboratory apparatus for sludge filtration is the Buchner funnel apparatus. The sludge sample was poured into a Buchner funnel and vacuum pressure was applied with the aid of a vacuum pump. The filtrate receiver was connected to a T-junction. A vacuum receiver was attached to the vacuum line, this aided in stabilizing the pressure at constant level. The pressure was measured by mercury manometer attached to the reservoir. A fine vacuum control knob was used for easy adjustment of pressure and this was connected to the vacuum reservoir. A stop-watch was used to measure the time of filtration. The apparatus described above can be applied for several purposes apart from laboratory evaluation of specific resistance. The volume of filtrate collected was determined by reading the lower meniscus of the measuring cylinder and time was also noted using a stop watch. The procedure above was carried out at pressures of 18.66KN/m^2, 32.52KN/m^2 and 52.51KN/m^2 for sludge solid contents of 87.6Kg/m^3, 70.08 Kg/$m^3$and 52.56 Kg/m^3.

Developing the New Filtration Equation

In order to derive the new sludge filtration equation the Buckingham's π-method of dimensional analysis was employed. The volume of the sludge *V* is assumed to be a function of filter paper area (*A*), time of filtration (*t*), mass of solids per unit volume of filtrate (*C*), net filtration pressure (*P*), viscosity of filtrate (μ), average specific resistance of filter cake (R) and compressibility coefficient (*S*). This is mathematically expressed as Equation 5. Table 2 is a summary of the relevant variables and their dimensions as applied in this derivation.

$$V = f\left(P, A, \mu, C, R, t, s\right) \qquad \textbf{(5)}$$

or $f\left(P, A, \mu, C, V, R, t, s\right) = 0$ **(6)**

The total number of variables (n) is eight while the number of fundamental dimensions (m) is three, hence the number of π - terms is n – m ➔ 8 – 3 = 5. Therefore, number of π-terms in the equation can be written as:

$$f\ (\pi_1\ \ \pi_2\ \ \pi_3\ \pi_4\ \pi_5) \quad = \quad 0 \qquad (7)$$

$$\pi_1 = P^a\ A^b\ \mu^C\ V \qquad (8)$$

$$\pi_2 = P^a\ A^b\ \mu^C\ C \qquad (9)$$

$$\pi_3 = P^a\ A^b\ \mu^C\ R \qquad (10)$$

$$\pi_4 = P^a\ A^b\ \mu^C\ t \qquad (11)$$

$$\pi_5 = P^a\ A^b\ \mu^C\ S \qquad (12)$$

Table 2: Summary of LMT Dimensions of Parameters.

Physical Variable	Symbol	Dimension
Volume of filtrate	V	L^3
Filtration Area	A	L^2
Time for Filtration	t	T
Mass of cake dry solids per unit volume of filtrate	C	ML^{-3}
Net filtration Pressure	P	$ML^{-1}T^{-2}$
Viscosity of filtrate	μ	$ML^{-1}T^{-1}$
Average specific resistance of filter cake	R	LM^{-1}
Compressibility coefficient	S	$M^{-1}LT^2$

Where π_1 to π_5 are dimensionless terms while *a, b,* and *c* are exponents to be determined by dimensional analysis.

Considering π_1 – Term

By replacing the right hand side of Equation 8 with the corresponding dimensions of the variables and the dimensionless term on the left hand side with $M^0L^0T^0$, Equation 13 is obtained.

$$M^o L^o T^o = \left(ML^{-1} T^{-2}\right)^a \left(L^2\right)^b \left(ML^{-1} T^{-1}\right)^C \left(L^3\right) \quad \textbf{(13)}$$

Equating the exponents of M, L and T on the left hand side to the corresponding exponents on the right hand side, we obtain that *a* = *c* = 0 and $b = -3/2$.

Substituting the values of *a, b* and *c* in Equation 8, we obtain Equation 14

$$\pi_1 = \frac{V}{A^{3/2}} \quad \textbf{(14)}$$

By repeating the above procedure for the remaining dimensionless terms, we obtain Equations 15 to 18:

$$\pi_2 = \frac{\rho AC}{\mu^2} \quad \textbf{(15)}$$

$$\pi_3 = \frac{\mu^2 R}{P} \quad \textbf{(16)}$$

$$\pi_4 = \frac{pt}{\mu} \quad \textbf{(17)}$$

$$\pi_5 = PS \quad \textbf{(18)}$$

Substituting the specific expressions for the dimensionless terms $\pi_1,\ \pi_2,\ \pi_3,\ \pi_4\ and\ \pi_5$ into Equation (6), Equation (19) is obtained:

$$f\left(\frac{V}{A^{3/2}} \bullet \frac{PAC}{\mu^2} \bullet \frac{\mu^2 R}{P} \bullet \frac{Pt}{\mu} \bullet \ Ps \ \text{(13)}\right) = 0 \quad \textbf{(19)}$$

Equation 19 shows the relationship between the various parameters of interest and the different dimensionless combinations. However, the expression does not give the exact relationship between these parameters; this is only possible by the use of experimental data. Following Buckingham's π-method, any of the dimensionless terms of Equation 19 can be written as a function of the others as in Equation 20.

$$\frac{V}{A^{3/2}} = K\left(\frac{PAC}{\mu^2}\right)^a \left(\frac{\mu^2 R}{P}\right)^b \left(\frac{p\,t}{\mu}\right)^c (PS)^d \quad \textbf{(20)}$$

The exponents in Equation can be obtained by regression analysis using experimental data. First, Equation 20 is Log transformed to yield a linear expression for easy determination of the exponents.

$$Ln\,\frac{V}{A^{3/2}} \;=\; LnK + a\,Ln\,\frac{PAC}{\mu^2} + b\;Ln\,\frac{\mu^2 R}{p} + c\;Ln\,\frac{pt}{\mu} + d\;Ln\;PS \qquad \textbf{(21)}$$

Equation 21 can be written as:

$$Y \;= LnK + \;a\,x_1 + bx_3 + cx_3 + dx_4 \qquad \textbf{(22)}$$

Where,

$$Y \;=\; Ln\;\frac{V}{A^{3/2}};$$

$$X_1 \;=\; Ln\;\frac{PAC}{\mu^2}\;X_2 \;=\; Ln\;\frac{\mu^2\,R}{p};$$

$$X_3 \;=\; Ln\;\frac{P\,t}{\mu};X_4 \;=\; Ln\;\;PS$$

Using results obtained from the filtration experiment (data too large to reproduce) the values of a, b, c and d were obtained by regression (see Tables 3 and 4).

From Table 3 above, we have:

LnK = 8.022; a = -0.48; b = -0.689; c = 0.481; d = -0.000961

Hence K = $e^{8.022}$ = 3047.266

Substituting exponents a, b, c, d and the constant *K* in Equation 20 we have:

$$\frac{V}{A^{3/2}} = 3047.266\left(\frac{PAC}{\mu^2}\right)^{-0.48}\left(\frac{\mu^2 R}{P}\right)^{-0.68}\left(\frac{P\,t}{\mu}\right)^{-0.481}(PS)^{-0.000961} \qquad \textbf{(23)}$$

$$V \;=\; 3047.266\;\;A^{3/2}\left(\frac{PAC}{\mu^2}\right)^{-0.48}\left(\frac{\mu^2 R}{P}\right)^{-0.689}\left(\frac{P\,t}{\mu}\right)^{-0.481}(PS)^{-0.000961} \qquad \textbf{(24)}$$

Table 3: Model Coefficients.

Model	Constant(*LnK*)	Un-standardized		Standardize	Sig.
		B	Std. Error	Beta	
		8.022	.415		.000
	a	-0.480	.009	-1.136	.000
	b	-0.689	.026	-0.590	.000
	c	0.481	.021	0.607	.000
	d	-9.610E-04	.017	-0.001	.956

Table 4: Model Summary of LMT.

Model	R	R Square	Adjusted R	Std. Error of the
1	0.916	0.839	0.838	0.169

$$V = 3047.266 \quad \frac{A^{1.02} \quad P^{0.689} \quad t^{0.481}}{R^{0.689} \quad C^{0.48} \quad \mu^{0.889} \quad S^{0.00096}} \qquad \textbf{(25)}$$

Squaring both sides of the Equation (25):

$$V^2 = 9.29 \, x \, 10^6 \left[\frac{A^{2.04} \quad P^{1.378} \quad t^{0.962}}{R^{1.378} \; C^{0.96} \quad \mu^{1.798} \quad S^{0.002}} \right] \qquad \textbf{(26)}$$

Rounding off the exponents except that of *S* to two decimal places, we have:

$$V^2 = 9.29 \, x \, 10^6 \left[\frac{P^{1.38} \quad A^{2.04} \quad t^{0.96}}{R^{1.38} \quad C^{0.96} \quad \mu^{1.8} \quad S^{0.002}} \right] \qquad \textbf{(27)}$$

Equation 27 is the new sludge filtration equation obtained.

Expression for Specific Resistance

Given that $t/V = \phi V$ where ϕ is the slope of t/V versus V, we can transform Equation 27 to assume the same form, hence Equation 28:

$$\frac{t^{0.962}}{V^2} = \frac{R^{1.378} \; C^{0.96} \; \mu^{1.798} \; S^{0.002}}{9.29 \, x \, 10^6 \; A^{2.04} \; P^{1.378}} \qquad \textbf{(28)}$$

Approximating $t^{0.962} \approx t$ the following equation results:

$$\frac{t}{V} = V \left[\frac{R^{1.378} \; C^{0.96} \; \mu^{1.798} \; S^{0.002}}{9.29 \, x \, 10^6 \; A^{2.04} \; P^{1.378}} \right] + i \qquad \textbf{(29)}$$

Plots of t/V versus *V* using experimental data yielded the graph in Figure 1. The figure shows that within the limits of experimental error, t/*V* versus *V* is a linear relationship with slope (ϕ) = 0.27(*sec/ml*^{2}) and intercept (*i*) = 8.98(*sec/ml*).

The term $\left[\frac{R^{1.378} \; C^{0.96} \; \mu^{1.798} \; S^{0.002}}{9.29 \, x \, 10^6 \; A^{2.04} \; P^{1.378}} \right]$ represents the slope (ϕ) of the plot of *t/V* versus *V*, hence

$$\phi = \frac{R^{1.378} \; C^{0.96} \; \mu^{1.798} \; S^{0.002}}{9.29 \, x \, 10^6 \; A^{2.04} \; P^{1.378}} \qquad \textbf{(30)}$$

But slope (ϕ) = 0.27

$$0.27 = \frac{R^{1.378} \; C^{0.96} \; \mu^{1.798} \; S^{0.002}}{9.29 \, x \, 10^6 \; A^{2.04} \; P^{1.378}} \qquad \textbf{(31)}$$

From Equation 21 we obtain the expression for specific resistance (*R*) as shown in Equation 32.

$$R = \left[\frac{2508300 \; A^{2.04} \; P^{1.378}}{C^{0.96} \; \mu^{1.798} \; S^{0.002}} \right]^{0.725} \qquad \textbf{(32)}$$

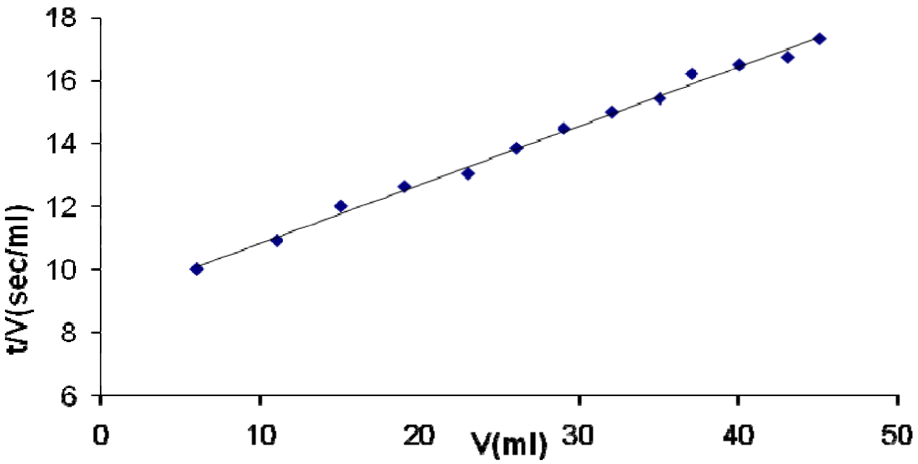

Figure 1: Plot of *t/V* versus *V.*

DISCUSSION

White and Gale (1975) disagreed with Anazodo's (1974) dimensional equation of sludge filtration on the grounds that the equation supposes that filtrate volume (*V*) is proportional to filtration area to the power of 5/4. The dimensional Equation (27) derived in this work shows that the square of filtrate volume is proportional to filtration area to the power of 2.04. Table 5 shows that for the areas of filtration used in the experimental work A^2 is approximately equal to $A^{2.04}$.

Table 5: Values of A^2, $A^{2.04}$ and $A^{0.91}$

A(m)	A^2	$A^{2.04}$	$A^{0.91}$
0.159	0.025281	0.023488	0.187617
0.3848	0.148071	0.142521	0.419338
0.6362	0.40475	0.397494	0.662629
0.9503	0.90307	0.901231	0.95467

Hence, it is obvious that A^2 is approximately equal to $A^{2.04}$ so that it is safe to say that the volume of filtrate is proportional to filtration area. The correctness of this has been verified by a plot of *V* versus *A* as shown in Figure 2.

Figure 2 suggests that *V* is roughly proportional to $A^{0.91}$. However, it has been shown in Table 5 that $A^{0.91} \approx A$ which should be acceptable within the limits of experimental error. From the foregoing, we see that *V* is proportional to *A* as the equation suggests and hence in agreement with the reasoning of White and Gale (1975). Equation 27 resembles Carman's equation in parameter combination and exponents except that the inclusion of compressibility coefficient (*S*) seems to have affected the exponent of pressure so that instead of P^2 we have $P^{1.38}$. The new filtration equation also provides for easy estimation of the compressibility coefficient of any sludge. Equation 32 shows that specific resistance of sludge is inversely proportional to its compressibility. However, this relationship between compressibility and specific resistance does not remain constant throughout the process of filtration. This is because at the start of filtration, the resistance offered by the sludge has two components ie the resistance offered by individual particles and that due to pore pressure (see Equation 33).

$$R = R_{pp} + R_{sp} \qquad \textbf{(33)}$$

R_{pp} = resistance due to pore pressure; R_{sp} = resistance due to sludge particles. As filtration progresses and water is drained out of the sludge, the particles become packed more closely together and the resistance due to solid particles increases while that due to pore pressure decreases but will not vanish completely because of the bound water which cannot be removed by mechanical means. From Equation 32, it can be seen that the expression also supposes that the filter medium resistance is negligible.

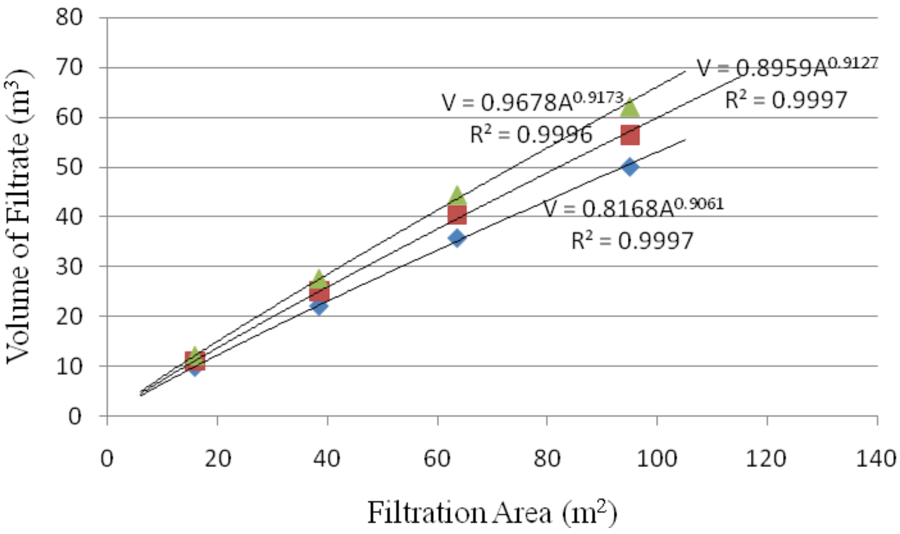

Figure 2: Plot of Filtrate Volume versus Filtration Area

CONCLUSION

Compressibility is a very important attribute of sludge hence the need to include it in sludge filtration equation. This new equation shows that compressibility of sludge is inversely proportional to specific resistance. With this new equation, it is easy to compute the compressibility coefficient of sludge.

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