Application of Network Analysis to Project Management.

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ABSTRACT

This paper considered the activities involved in a construction site as a network and proposed a network analytic technique namely the Critical Path Method (CPM) and a traditional method for all activities involved in the site to obtain the latest finish time (completion time) of the project. Results show that the CPM gave a shorter completion date than the traditional method since it gave a lower latest finish time and consequently, lower running and personnel costs.

(Keywords: network, network analytic technique, critical path management, CPM, latest finish time)

INTRODUCTION

Network analysis is the general name given to certain specific techniques which can be used for planning, management and control of project. It often acts as a network management tool for breaking down projects into components or individual activities and recording the result on a flow chart or network diagram. These results generally reveal information that is used to determine duration, resource limitations and cost estimates associated with the project.

It offers insight into what is occurring at each critical point of the network. Project management and efficient resource allocation are two critical aspects of the production and operations managers' responsibilities. Since a project is non-repetitive and temporal in nature, the mode of management differs from the usual job shop or other related types of scheduling.

A project consists of tasks with definite starting and ultimate ending points and hence a project manager is saddled with the responsibilities of getting job done on schedule within allowable cost and time constraint specified by the management. Typically all projects can be broken into:

- Separate activities – where each activity has an associated completion time (time from the start of the activity to its finish).
- Precedence relationships – which govern order in which we may perform the activities.

The main problem is to bring all these activities together in a coherent fashion to complete the project at a required time.

Apart from the traditional method of adding activity durations, these exist two different techniques for network analysis namely the PERT – Program Evaluation and Review Technique and CPM – Critical Path Management.

PERT has the ability to cope with uncertainty in activity completion times while CPM emphasized on the trade-off between cost of the project and its overall completion time.

The CPM has the advantage of decreasing completion times by probably spending more money.

Network analysis enables us to take a systematic quantitative structural approach to the problem of managing a project through to successful completion. Also, since it has a graphical representation, it can be easily understood and used by those with a less technical background.

MATERIALS AND METHODS

The following is the list of activities obtained from a construction firm involved in the construction of a shopping complex is sequential order.

- Clearing of site
- Setting out
- Excavation
- Concrete of column base
- Column reinforcement
- Casting of strips
- Form works
- Casting of columns
- Setting of block wall sub-structure
- Earth filling
- Hard core
- Electrical fixing I
- Plumbing fixing I
- Casting of ground floor slab
- Reinforcement of ground floor columns/lintel
- Electrical fixing II
- Plumbing fixing II
- Form work of ground floor columns/lintel
- Cast of ground floor columns/lintel
- Form work for first beam/first floor slab
- Setting of ground floor block wall
- Electrical fixing III
- Plumbing fixing III
- First floor slab reinforcement
- Casting of first floor slab
- Reinforcement of the first floor columns /lintel
- Electrical fixing IV
- Plumbing fixing IV
- Form work first floor columns/lintel
- Casting of first floor columns/lintel
- Form work for roof beam/reinforcement
- Casting roof beam/reinforcement
- Setting of first floor block wall
- Electrical fixing V
- Plumbing fixing V
- Roof/ceiling
- Door/windows
- Plastering
- Final electrical fixing
- Finishing
- Painting/decoration
- External works (Electrical/Plumbing)

The activities are tabulated in the succeeding tables.

Linear Programming Formulation

To ensure the formulation, we assume that our route is to be determined between nodes l *and* a where a also equals the total number of network modes. It will also be assumed that a single branch connects node i to node j (i.e. distances and branches associated with nodes that are not directly connected are ignored). Also, in the event of a directed branch, we do not consider flows in opposition to the direction.

Thus, we wish to find the set $x_{i,j}$ to:

$$\min imize \sum_{i=1}^{n}\sum_{j=1}^{n} d_{i,j} X_{i,j}$$

$$subject\ to \sum_{k=2}^{n} X_{i,k} = 1$$

$$\sum_{k=1}^{n-1} X_{k,n} = 1$$

$$\sum_{i=1}^{n} X_{i,k} - \sum_{j=1}^{n-1} X_{k,j} = 0 \quad for\ k = 2,\ldots,n-1$$

$$X_{i,k} = 0,1 \quad for\ all\ i, j$$

$$where\ d_{i,j} = dis\tan ce\ from\ node\ i\ to\ j\ (for\ directly\ connected\ nodes)$$

$$X_{i,j} = \begin{matrix} 1 \ if\ the\ branch\ from\ node\ i\ to\ j\ is\ connected \\ 0\ otherwise \end{matrix}$$

The first two constraints assure us that exactly one branch is taken from the first node whereas exactly one branch is taken into the final node. The third constraints simply force the number of branches into an intermediate node to equal those out of that node.

The Critical Path Composition

All activities have been enlisted in the path to completion. The CPM will be used to choose the path with the shortest completion time.

Table 1: List of Activities with Estimated Duration and Immediate Predecessor.

S/N	Duration (Days)	Activity	Immediate Predecessor
1	1	Clearing of site	-
2	1	Setting out	1
3	9	Excavation	2
4	2	Concrete of column base	3
5	5	Column reinforcement	3
6	3	Casting of strip	5, 4
7	4	Form work	6
8	5	Casting of columns	6,7
9	5	Setting of block wall sub-structure	8
10	5	Earth filling	9
11	4	Hard core	10
12	1	Electrical fixing I	10
13	2	Plumbing fixing I	10
14	3	Casting of ground floor slab	11, 12, 13
15	4	Reinforcement of ground floor columns/lintel	14
16	2	Electrical fixing II	15
17	3	Plumbing fixing II	15
18	3	Form work of ground floor columns/lintel	16, 17
19	3	Cast of ground floor columns/lintel	18
20	5	Form work for first beam/first floor slab	19
21	10	Setting of ground floor block wall	20
22	5	Electrical fixing III	19
23	5	Plumbing fixing III	19
24	4	First floor slab reinforcement	20
25	1	Casting of first floor slab	24
26	2	Reinforcement of the first floor columns /lintel	25
27	3	Electrical fixing IV	25
28	5	Plumbing fixing IV	26, 27
29	4	Form work first floor columns/lintel	28
30	4	Casting of first floor columns/lintel	29
31	3	Form work for roof beam/reinforcement	29
32	8	Casting roof beam/reinforcement	31
33	6	Setting of first floor block wall	32
34	8	Electrical fixing V	32
35	12	Plumbing fixing V	33, 34
36	4	Roof/ceiling	33, 34
37	15	Door/windows	35, 36
38	7	Plastering	37
39	7	Find electrical fixing	37
40	9	Finishing	37
41	14	Painting/decoration	37
42	21	External works	36
43	7	External works electrical	38, 39, 40
44	10	External works plumbing	38, 39, 40

Table 2: Activity Durations with Immediate Predecessor Activities.

Activity	Duration (Days)	Immediate Predecessor
[1, 2]	1	-
[2, 3]	1	[1, 2]
[3, 4]	9	[2, 3]
[4, 5]	2	[3, 4]
[4, 6]	5	[3, 4]
[5, 6]	0	[4, 5]
[6, 7]	3	[4, 5], [5, 6]
[7, 8]	4	[6, 7]
[8, 9]	5	[7, 8]
[9, 10]	5	[8, 9]
[10, 11]	5	[9, 10]
[11, 12]	4	[10, 11]
[11, 13]	1	[10, 11]
[11, 14]	2	[10, 11]
[12, 14]	0	[11, 12]
[13, 14]	0	[11, 13]
[14, 15]	3	[12, 14], [11, 14], [13, 14]
[15, 16]	4	[14, 15]
[16, 17]	2	[15, 16]
[16, 18]	3	[15, 16]
[17, 18]	0	[16, 17]
[18, 19]	3	[16, 18], [17, 18]
[19, 20]	3	[18, 19]
[20, 21]	5	[19, 20]
[20, 22]	5	[19, 20]
[20, 23]	5	[19, 20]
[21, 24]	10	[20, 21]
[21, 25]	4	[20, 21]
[22, 45]	0	[20, 23]
[23, 45]	0	[20, 23]
[24, 45]	0	[21, 24]
[25, 26]	1	[21, 25]
[26, 27]	2	[25, 26]
[26, 28]	3	[25, 26]
[27, 28]	0	[26, 27]
[28, 29]	5	[26, 28], [27, 28]
[29, 30]	5	[28, 29]
[30, 31]	4	[29, 30]
[31, 32]	3	[30, 31]
[32, 33]	8	[31, 32]
[33, 35]	8	[32, 33]
[34, 35]	0	[33, 34]
[35, 36]	12	[33, 35], [34, 35]
[35, 37]	4	[33, 35], [34, 35]
[36, 37]	0	[35, 36]
[37, 38]	21	[35, 37], [36, 37]
[37, 39]	15	[35, 37], [36, 37]
[38, 45]	0	[37, 38]
[39, 40]	7	[37, 39]
[39, 41]	14	[37, 39]
[39, 42]	7	[37, 39]
[39, 43]	9	[37, 39]
[40, 43]	0	[39, 40]
[41, 45]	0	[39, 41]
[42, 43]	0	[39, 42]
[43, 44]	8	[39, 43], [40, 43], [42, 43]
[43, 45]	10	[39, 43], [40, 43], [42, 43]
[44, 45]	0	[43, 44]

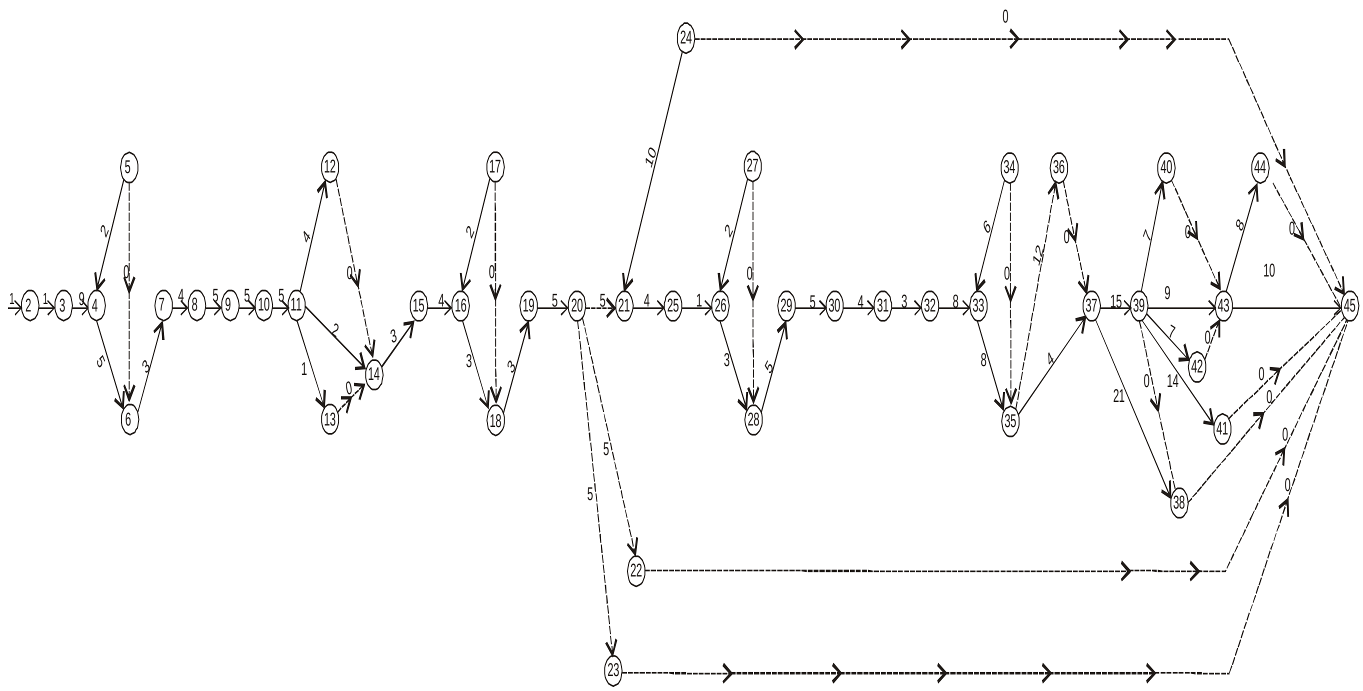

Figure 1: Network Diagram of Activities and Durations in the Site.

This is divided into:

- Forward Pass through the network,
- Backward Pass through network,
- Float time computation and
- Critical path determination.

Forward Pass through the Network

We compute the earliest occurrence of event i as:

$$ES_i = \max[ES_i + D_{i,j}] \; for \; all \; i, j$$

$where \; i = 1, 2, ..., n$ indicates the preceding event of the n activities that terminates at j

ES_i the earliest occurrence time of event i so that

$$when \; i = 1, \; ES_i = 0$$

Also, the earliest finish/completion time of event i is:

$$EF_i = ES_i + D_{i,j}$$

Backward Pass through the Network

The pass determine the latest occurrence time for each event. This is given by:

$$LF_n = ES_n \; at \; \sin k \;\; and$$

$$LF_{i,j} = \min[LF_j - D_{i,j}] \; for \; all \;\; i, j$$

The latest occurrence time of event i is:

$$LF_{ij} = LF_j - D_{ij}$$

$i = 1, 2, ..., n$ indicates the successor events of the n activities.

Determination of Final Time and Critical Path

Float time is activity can be delayed from its ES time without disturbing the critical activities that follow it. It is equal to LS minus ES or LF minus EF for activity i that is:

$$F_{ij} = LS_{ij} - ES_i \ or$$

$$= LF_j - EF_{ij}$$

The float time for event i is the difference between the earliest and latest occurrence time for the event. Note that all activities with zero float constitute the critical path.

RESULTS

Results are shown in Table 3.

The activities with zero float are:

[1,2], [2,3], [3,4], [4,6], [6,7], [7,8], [8,9], [9,10], [10,11], [11,12], [12,14], [14,15], [15,16], [16,18], [18,19], [19,20], [20,21], [21,25], [25,26], [26,28], [28,29], [29,30], [30,31], [31,32], [32,33], [33,35], [36,37], [37,39], [39,43] and [43,45].

Hence the critical path is:

$$1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6 \rightarrow 7 \rightarrow 8 \rightarrow 9 \rightarrow 10 \rightarrow 11 \rightarrow 12 \rightarrow 13 \rightarrow 14 \rightarrow 15$$
$$\rightarrow 16 \rightarrow 17 \rightarrow 18 \rightarrow 19 \rightarrow 20 \rightarrow 21 \rightarrow 22 \rightarrow 23 \rightarrow 24 \rightarrow 25 \rightarrow 26 \rightarrow 27 \rightarrow 28$$
$$\rightarrow 29 \rightarrow 30 \rightarrow 31 \rightarrow 32 \rightarrow 33 \rightarrow 34 \rightarrow 35 \rightarrow 36 \rightarrow 37 \rightarrow 38 \rightarrow 39 \rightarrow 43 \rightarrow 45$$

With total duration:

1 + 1 + 9 + 5 + 3 + 4 + 5 + 5 + 5 + 4 + 0 + 3 + 4 + 3 + 3 + 3 + 5
 + 4 + 1 + 3 + 5 + 4 + 4 + 3 + 8 + 8 + 12 + 0 + 15 + 9 + 10 = 149 Days

Tora optimization system was also used to analyze the data yielding the following output (Table 4):

Table 3: Results

Activity	D_{ij}	ES_i	EF_{ij}	LS_{ij}	LF_j	Float
[1, 2]	1	0	1	0	1	0
[2, 3]	1	1	2	1	2	0
[3, 4]	9	2	11	2	11	0
[4, 5]	2	11	14	14	16	2
[4, 6]	5	11	16	11	16	0
[5, 6]	0	13	13	16	16	3
[6, 7]	3	16	19	16	19	0
[7, 8]	4	19	23	19	23	0
[8, 9]	5	23	28	23	28	0
[9, 10]	5	28	33	28	33	0
[10, 11]	5	33	38	33	38	0
[11, 12]	4	38	42	38	42	0
[11, 13]	1	38	39	41	42	3
[11, 14]	2	38	40	40	42	2
[12, 14]	0	42	42	42	42	0
[13, 14]	0	39	39	42	42	3
[14, 15]	3	42	45	42	45	0
[15, 16]	4	45	49	45	49	0
[16, 17]	2	49	51	50	52	1
[16, 18]	3	49	52	49	52	0
[17, 18]	0	51	51	52	52	1
[18, 19]	3	52	55	52	55	0
[19, 20]	3	55	58	55	58	0
[20, 21]	5	58	63	58	63	0
[20, 22]	5	58	63	144	149	86
[20, 23]	5	58	63	144	149	86
[21, 24]	10	63	73	139	149	76
[21, 25]	4	63	67	63	67	0
[22, 45]	0	63	63	149	149	86
[23, 45]	0	63	63	149	149	86
[24, 45]	0	73	73	149	149	76
[25, 26]	1	67	68	67	68	0
[26, 27]	2	68	70	69	71	1
[26, 28]	3	68	71	68	71	0
[27, 28]	0	70	70	71	71	1
[28, 29]	5	71	76	71	76	0
[29, 30]	4	76	80	76	80	0
[30, 31]	4	80	84	80	84	0
[31, 32]	3	84	87	84	87	0
[32, 33]	8	87	95	87	95	0
[33, 34]	6	95	101	97	103	2
[33, 35]	8	95	103	95	103	0
[34, 35]	0	101	101	103	103	2
[35, 36]	12	103	115	103	115	0
[35, 37]	4	103	107	111	115	8
[36, 37]	0	115	115	115	115	0
[37, 38]	21	115	115	125	149	13
[37, 39]	15	115	115	115	130	0
[38, 45]	0	136	136	149	149	13
[39, 40]	7	130	137	130	139	12
[39, 41]	14	130	144	103	149	13
[39, 42]	7	130	137	132	139	2
[39, 43]	9	130	139	130	139	0
[40, 43]	0	137	137	139	139	2
[41, 45]	0	144	144	149	149	5
[42, 43]	0	137	137	139	139	2
[43, 44]	8	139	147	141	149	2
[43, 45]	10	139	149	139	149	0
[44,45]	0	147	147	149	149	2

Table 4: Solution Steps

FORWARD PASS			BACKWARD PASS		
STEP	NODE	EARLIEST TIME	STEP	NODE	LATEST TIME
1	1	0.00	46	45	149.00
2	2	1.00	47	22	149.00
3	3	2.00	48	24	149.00
4	4	11.00	49	38	149.00
5	5	13.00	50	41	149.00
6	6	16.00	51	44	149.00
7	7	19.00	52	43	139.00
8	8	23.00	53	40	139.00
9	9	28.00	54	42	139.00
10	10	33.00	55	39	130.00
11	11	38.00	56	37	115.00
12	12	42.00	57	36	115.00
13	13	39.00	58	35	103.00
14	14	42.00	59	34	103.00
15	15	45.00	60	33	95.00
16	16	49.00	61	32	87.00
17	17	51.00	62	31	84.00
18	18	52.00	63	30	80.00
19	19	55.00	64	29	76.00
20	20	58.00	65	28	71.00
21	21	63.00	66	27	71.00
22	22	63.00	67	26	68.00
23	23	63.00	68	25	67.00
24	24	73.00	69	21	63.00
25	25	67.00	70	23	67.00
26	26	68.00	71	20	58.00
27	27	70.00	72	19	55.00
28	28	71.00	73	18	52.00
29	29	76.00	74	17	52.00
30	30	80.00	75	16	49.00
31	31	84.00	76	15	42.00
32	32	87.00	77	14	42.00
33	33	95.00	78	13	38.00
34	34	101.00	79	12	33.00
35	35	103.00	80	11	28.00
36	36	115.00	81	10	23.00
37	37	115.00	82	9	28.00
38	38	136.00	83	8	23.00
39	39	130.00	84	7	19.00
40	40	137.00	85	6	16.00
41	41	144.00	86	5	16.00
42	42	139.00	87	4	11.00
43	43	131.00	88	3	2.00
44	44	147.00	89	2	1.00
45	45	149.00	90	1	0.00

At sink,
LF = ES = 149.00days after the 45th activity.

DISCUSSION

The network diagram (Figure 1) shows that activity [1, 2] has to wait for 2 days before activity [2, 3] can continue to [3, 4]. The latest start time for activity [1, 2] is zero and its latest time is 1 day. For total float, activity [2, 3] can be delayed without affecting the total project duration and so is activity [3, 4]. Activity [7, 8] can also be delayed for 4 days without affecting the overall duration while activity [4, 5] can be delayed for 2 days when all succeeding activities are completed as early as possible. The network diagram also reveals that the project will be completed after the 45th event.

Table 3 on float time was obtained from the network diagram. It reveals the critical path (the addition of all activities with zero float) with total a duration of 149 days.

Also inputting the data (Table 1) into the Tora system, the output gave a critical path with total duration of 149 days which coincided with the manual computation obtained earlier.

CONCLUSION AND RECOMMENDATION

A total of 149 days was obtained as a completion time for the project using the CPM technique whereas a time of 199 days would have been required if the traditional method of addition was adopted.

We conclude that 149 days is the required completion time for the project. This will save a period of 50 days in the construction process. It could also save running, personnel, and other resource costs. We therefore recommend that personnel relevant to the field of network analysis should be involved in planning of a project prior to its commencement. This will in turn give an idea of the project's shortest completion period.

Management should also seriously consider and implement the results of such personnel to avoid undue delay in the completion of any project.
Lastly, inefficient utilization of resources and increase in cost of utilities beyond reasonable proportions could lead to the abandonment of project works. In such cases, we recommend that additional resources such as overtime payments, bonuses and increased labour input should be incorporated so as to re-adjust the critical path.

REFERENCES

1. Taha, H.A. 1996. *Operations Research – An Introduction (Sixth Edition)*. Prentice-Hall: New York, NY.

2. Hillier, F.S. and Lieberman, G.J. 2002. *An Introduction to Operations Research.* McGraw Hill Higher Education: New York, NY.

3. Ignizio J.P. 1981. *Linear Programming in Single and Multiple-Objective Systems.* Prentice-Hall: New York, NY.

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