Improvement in Modified Weighted Symmetric Estimator for a Gaussian AR(1) Process with Additive Outliers.

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ABSTRACT

This paper presents two new estimators for a Gaussian first-order autoregressive (AR(1)) process with additive outliers. We apply the recursive mean and recursive median adjustment to the modified weighted symmetric estimator proposed by Fuller [1]. We consider the following estimators: the modified weighted symmetric estimator (MW), the recursive mean adjusted modified weighted symmetric estimator (RMW), the recursive median adjusted modified weighted symmetric estimator (RDMW). Using Monte Carlo simulations, we compare the mean square error (MSE) of estimators. Simulation results have shown that the RDMW estimator provides a MSE lower than those of the MW and RMW estimator for almost all situations.

(Keywords: parameter estimation, AR(1) process, recursive mean, recursive median, additive outliers)

INTRODUCTION

Business and economic time series are frequently affected by special events, for instance policy interventions, strikes, outbreaks of war, sudden changes in the market structure of a commodity, and so forth [2]. Such observations are usually referred to as outliers. Because outliers are known to wreck havoc on the parameter estimation, it is therefore important to have procedures that will deal such outlier effects.

The detection of time series outliers was first studied by Fox [3], who introduced two statistical models for times series contaminated by outliers, namely, additive outliers (AO) and innovations outliers (IO). Additive outliers correspond to the situation in which a gross error of observation or recording affects a single observation [3]. An innovations outlier affects not only the particular observation but also subsequent observations [3].

This paper focuses solely on the additive outliers as they are the most common type found in time series due to their association with human error such as typing and recording mistakes [4]. Furthermore, additive outliers are more harmful than innovations outliers [5]. A time series that does not contain any outliers is called an outlier-free series.

Suppose an unobservable outlier-free time series $X_t\,;\, t = 2,3,\ldots,n$ follows an AR(1) model:

$$X_t = \delta + \rho X_{t-1} + e_t\,, \qquad \textbf{(1)}$$

where $\delta = \mu(1-\rho)$, μ is the population mean, ρ is an autoregressive parameter, $\rho \in (-1,1)$, e_t are unobservable independent errors and identically $N(0,\sigma_e^2)$ distributed. For $|\rho| = 1$, the model (1) is called the random walk model, otherwise it is called a stationary AR(1) process when $|\rho| < 1$. For ρ close to one or near a non-stationary process, the mean and variance of this model change over time. Let the observed time series be denoted by Y_t. In the simple case when X_t has a single additive outlier at time point T $(1 < T < n)$, model (1) can be modified as:

$$Y_t = X_t + \nu I_t^{(T)}\,, \qquad \textbf{(2)}$$

where ν represents the magnitude of the additive outlier effect and $I_t^{(T)}$ is a time indicator signifying the time occurrence of the additive outlier such that,

$$I_t^{(T)} = \begin{cases} 1\ , & t = T, \\ 0\ , & t \neq T. \end{cases}$$

It is known that the ordinary least squares estimator of ρ, which is denoted by $\hat{\rho}_{OLS}$, for (1) is biased (e.g. Shaman and Stine [6]). Therefore, statisticians have suggested methods to reduce the bias. Park and Fuller [7] proposed the weighted symmetric estimator of ρ, which is denoted by $\hat{\rho}_W$.

Fuller [1] presented a modification of the weighted symmetric estimator (abbreviated, MW). So and Shin [8] applied a recursive mean adjustment to the OLS estimator (abbreviated, ROLS) and they concluded that the mean square error of the ROLS estimator, which is denoted by $\hat{\rho}_{OLS}$, is smaller than the OLS estimator for $\rho \in (0,1)$. They also showed that the $\hat{\rho}_{ROLS}$ estimator has a coverage probability which is close to the nominal value.

Recently, Niwitpong [9] applied the recursive mean adjustment to the weighted symmetric estimator of Park and Fuller [7] (abbreviated, RW). Panichkitkosolkul [10] proposed an estimator for an unknown mean Gaussian AR(1) process with additive outliers by applying the recursive median adjustment to the weighted symmetric estimator (abbreviated, RDW). He found that the $\hat{\rho}_{RDW}$ estimator provides mean square error lower than those of $\hat{\rho}_W$ and $\hat{\rho}_{RW}$ for almost all situations. We, therefore, apply recursive median adjustment to the modified weighted symmetric estimator (abbreviated, RDMW) for model (1) when there are additive outliers in time series data. Because the outliers do not affect the median values, we replace the mean adjustment by recursive median adjustment to the modified weighted symmetric estimator.

Further, we also apply recursive mean adjustment to the modified weighted symmetric estimator (abbreviated, RMW). Our aim in this paper is to compare three estimators, MW, RMW and RDMW, in terms of mean square error (MSE) of estimators.

The remainder of this paper is organized as follows. The details of the estimators MW, RMW, and RDMW are described in the next section. Simulation results obtained from Monte Carlo simulations are shown in the third section. In the forth section, all estimators are illustrated and compared through macro-economic real example. A discussion of the results and conclusions are presented in the final section.

METHODOLOGY

Fuller [1] proposed the modified weighted symmetric (MW) estimator of (ρ, δ) given by:

$$\hat{\rho}_{MW} = \hat{\rho}_W + c(\hat{\tau}_W)\left[\hat{V}(\hat{\rho}_W)\right]^{1/2}, \qquad \textbf{(3)}$$

and

$$\hat{\delta}_{MW} = \bar{Y}(1 - \rho_W^*), \qquad \textbf{(4)}$$

where

$$\hat{\rho}_W = \frac{\sum_{t=2}^{n}(Y_t - \bar{Y})(Y_{t-1} - \bar{Y})}{\sum_{t=3}^{n}(Y_{t-1} - \bar{Y})^2 + n^{-1}\sum_{t=1}^{n}(Y_t - \bar{Y})^2}, \qquad \textbf{(5)}$$

$$c(\hat{\tau}_W) = \begin{cases} -\hat{\tau}_W & \text{if } \hat{\tau}_W \geq -1.2, \\ 0.035672(\hat{\tau}_W + 7)^2 & \text{if } -7 < \hat{\tau}_W < -1.2, \\ 0 & \text{if } \hat{\tau}_W \leq -7, \end{cases}$$

$$\hat{\tau}_W = \left[\hat{V}(\hat{\rho}_W)\right]^{-1/2}(\hat{\rho}_W - 1),$$

$$\hat{V}(\hat{\rho}_W) = \frac{\hat{\sigma}_W^2}{\sum_{t=3}^{n}(Y_{t-1} - \bar{Y})^2 + n^{-1}\sum_{t=1}^{n}(Y_t - \bar{Y})},$$

$$\hat{\sigma}_W^2 = \frac{1}{n-2}\left\{ \begin{array}{l} \sum_{t=2}^{n} w_t\left[(Y_t - \bar{Y}) - \hat{\rho}_W(Y_{t-1} - \bar{Y})\right]^2 \\ + \sum_{t=1}^{n-1}(1 + w_{t+1})\left[(Y_t - \bar{Y}) - \hat{\rho}_W(Y_{t+1} - \bar{Y})\right]^2 \end{array} \right\},$$

$$w_t = n^{-1}(t-1),$$

and $$\rho_W^* = \begin{cases} \hat{\rho}_W & \text{if } |\hat{\rho}_W| < 1, \\ 1 & \text{if } \hat{\rho}_W \geq 1, \\ -1 & \text{if } \hat{\rho}_W \leq -1. \end{cases}$$

Niwitpong [9] replaces $\bar{Y}$ by $\bar{Y}_t = t^{-1}\sum_{i=1}^{t} Y_i$ in (5). The estimator of ρ obtained as a result of this recursive mean adjustment is:

$$\hat{\rho}_{RW} = \frac{\sum_{t=2}^{n}(Y_t - \bar{Y}_t)(Y_{t-1} - \bar{Y}_{t-1})}{\sum_{t=3}^{n}(Y_{t-1} - \bar{Y}_{t-1})^2 + n^{-1}\sum_{t=1}^{n}(Y_t - \bar{Y}_t)^2}. \quad \textbf{(6)}$$

Similarly, we apply the recursive mean adjustment to the modified weighted symmetric estimator in (3). The modified weighted symmetric estimator of (ρ,δ) obtained as a result of the recursive mean adjustment (RMW) is:

$$\hat{\rho}_{RMW} = \hat{\rho}_{RW} + c(\hat{\tau}_{RW})\left[\hat{V}(\hat{\rho}_{RW})\right]^{1/2}, \quad \textbf{(7)}$$

and

$$\hat{\delta}_{RMW} = \bar{Y}(1-\rho^{*}_{RW}), \quad \textbf{(8)}$$

where

$$c(\hat{\tau}_{RW}) = \begin{cases} -\hat{\tau}_{RW} & \text{if } \hat{\tau}_{RW} \geq -1.2, \\ 0.035672(\hat{\tau}_{RW}+7)^2 & \text{if } -7 < \hat{\tau}_{RW} < -1.2, \\ 0 & \text{if } \hat{\tau}_{RW} \leq -7, \end{cases}$$

$$\hat{\tau}_{RW} = \left[\hat{V}(\hat{\rho}_{RW})\right]^{-1/2}(\hat{\rho}_{RW}-1),$$

$$\hat{V}(\hat{\rho}_{RW}) = \frac{\hat{\sigma}^2_{RW}}{\sum_{t=3}^{n}(Y_{t-1} - \bar{Y}_{t-1})^2 + n^{-1}\sum_{t=1}^{n}(Y_t - \bar{Y}_t)},$$

$$\hat{\sigma}^2_{RW} = \frac{1}{n-2}\left\{\begin{array}{l} \sum_{t=2}^{n} w_t\left[(Y_t-\bar{Y}_t) - \hat{\rho}_{RW}(Y_{t-1}-\bar{Y}_{t-1})\right]^2 \\ +\sum_{t=1}^{n-1}(1+w_{t+1})\left[(Y_t-\bar{Y}_t) - \hat{\rho}_{RW}(Y_{t+1}-\bar{Y}_{t-1})\right]^2 \end{array}\right\},$$

and $$\rho^{*}_{RW} = \begin{cases} \hat{\rho}_{RW} & \text{if } |\hat{\rho}_{RW}| < 1, \\ 1 & \text{if } \hat{\rho}_{RW} \geq 1, \\ -1 & \text{if } \hat{\rho}_{RW} \leq -1. \end{cases}$$

When there are outliers in time series data, it affects the recursive mean $\bar{Y}_t$ in (6).

Panichkitkosolkul [10] replaced the recursive mean in (6) by the recursive median. The estimator of ρ obtained as a result of the recursive median adjustment is:

$$\hat{\rho}_{RDW} = \frac{\sum_{t=2}^{n}(Y_t - \tilde{Y}_t)(Y_{t-1} - \tilde{Y}_{t-1})}{\sum_{t=3}^{n}(Y_{t-1} - \tilde{Y}_{t-1})^2 + n^{-1}\sum_{t=1}^{n}(Y_t - \tilde{Y}_t)^2}, \quad \textbf{(9)}$$

where $\tilde{Y}_t = median(Y_1, Y_2, ..., Y_t)$.

We therefore apply the recursive median adjustment to the modified weighted symmetric estimator which it is similar to the approach described in Panichkitkosolkul [10]. As a result of the recursive median adjustment, the modified weighted symmetric estimator (RDMW) of (ρ,δ) is given by:

$$\hat{\rho}_{RDMW} = \hat{\rho}_{RDW} + c(\hat{\tau}_{RDW})\left[\hat{V}(\hat{\rho}_{RDW})\right]^{1/2}, \quad \textbf{(10)}$$

and

$$\hat{\delta}_{RDMW} = \bar{Y}(1-\rho^{*}_{RDW}), \quad \textbf{(11)}$$

where

$$c(\hat{\tau}_{RDW}) = \begin{cases} -\hat{\tau}_{RDW} & \text{if } \hat{\tau}_{RDW} \geq -1.2, \\ 0.035672(\hat{\tau}_{RDW}+7)^2 & \text{if } -7 < \hat{\tau}_{RDW} < -1.2, \\ 0 & \text{if } \hat{\tau}_{RDW} \leq -7, \end{cases}$$

$$\hat{\tau}_{RDW} = \left[\hat{V}(\hat{\rho}_{RDW})\right]^{-1/2}(\hat{\rho}_{RDW}-1),$$

$$\hat{V}(\hat{\rho}_{RDW}) = \frac{\hat{\sigma}^2_{RDW}}{\sum_{t=3}^{n}(Y_{t-1} - \tilde{Y}_{t-1})^2 + n^{-1}\sum_{t=1}^{n}(Y_t - \tilde{Y}_t)},$$

$$\hat{\sigma}^2_{RDW} = \frac{1}{n-2}\left\{\begin{array}{l} \sum_{t=2}^{n} w_t\left[(Y_t-\tilde{Y}_t) - \hat{\rho}_{RDW}(Y_{t-1}-\tilde{Y}_{t-1})\right]^2 \\ +\sum_{t=1}^{n-1}(1+w_{t+1})\left[(Y_t-\tilde{Y}_t) - \hat{\rho}_{RW}(Y_{t+1}-\tilde{Y}_{t-1})\right]^2 \end{array}\right\},$$

and $$\rho^{*}_{RDW} = \begin{cases} \hat{\rho}_{RDW} & \text{if } |\hat{\rho}_{RDW}| < 1, \\ 1 & \text{if } \hat{\rho}_{RDW} \geq 1, \\ -1 & \text{if } \hat{\rho}_{RDW} \leq -1. \end{cases}$$

In the next section, we present the Monte Carlo simulation results from estimating the mean square error (MSE) of these estimators, $(\hat{\rho}_{MW}, \hat{\delta}_{MW})$, $(\hat{\rho}_{RMW}, \hat{\delta}_{RMW})$ and $(\hat{\rho}_{RDMW}, \hat{\delta}_{RDMW})$.

RESULTS

In this section we examine−via Monte Carlo simulations−the performance of the proposed estimator for a Gaussian AR(1) process with additive outliers, with particular emphasis on comparisons between the new and existing approaches. Data are generated from a Gaussian AR(1) process[1] with additive outliers. The following parameter values were used; (μ, σ_e^2)= (0,1); $\rho = 0.1, 0.2, ..., 0.9, 0.95, 0.99$; sample sizes n = 25, 50, 100 and 250; the magnitude of the AOs effect $\nu = 5\sigma_e$; the percentage of additive outliers $p = 5\%$ and 10%. All simulations were performed using programs written in the R statistical software [11-12] with the number of simulation runs, $M = 10,000$. In addition, the additive outliers occurred randomly.

Our simulation results are shown in Tables 1 and 2. The following information is presented here: the estimated mean square error (MSE) of all estimators, $(\hat{\rho}_{MW}, \hat{\delta}_{MW})$, $(\hat{\rho}_{RMW}, \hat{\delta}_{RMW})$ and $(\hat{\rho}_{RDMW}, \hat{\delta}_{RDMW})$. As can be seen from Tables 1 and 2, the MSEs of the MW estimators, $(\hat{\rho}_{MW}, \hat{\delta}_{MW})$ are larger than the MSEs of the other estimators for almost all scenarios. The RDMW estimators, $(\hat{\rho}_{RDMW}, \hat{\delta}_{RDMW})$ provide the lowest MSE in all cases except when $p = 5\%$, the sample sizes are small and moderate (n = 25 and 50) and ρ is close to one. In addition, the MSEs of the RMW estimators, $(\hat{\rho}_{RMW}, \hat{\delta}_{RMW})$ are lowest MSE when $p = 5\%$, sample sizes are small and moderate and ρ is close to one.

The RDMW estimators, $(\hat{\rho}_{RDMW}, \hat{\delta}_{RDMW})$, in (10) and (11) dominate all estimators since the MSEs of the estimators are the lowest for almost all cases. We found that the RMW estimators are less MSE than the MW estimators. Furthermore, the MSE decreases when the sample increase. That is, for n = 25, $\rho = 0.1$ and $p = 5\%$, the MSE of $\hat{\rho}_{RDMW}$ is 0.0508 while it is 0.0056 for n = 250. Therefore, the Monte Carlo simulation results suggested that the RDMW estimators performed well, in sense of the less MSE. Thus, we recommend to use the RDMW estimators to estimate ρ and δ when there are additive outliers in time series data.

[1] We generate $Y_1 \sim N(0, \frac{\sigma_e^2}{1-\rho^2})$ and simulate the time series of length $n+50$ but the time series used in calculations are $\{Y_{51}, Y_{52}, ..., Y_{n+50}\}$.

An Example

In this section we applied the proposed estimators to a macro-economic time series. A real data set is the yearly real exchange rates between the USA and Sudan from ERS International Macroeconomic Data Set [13]. This series, which is showed in the Appendix, comes from 1970 to 2009 (Base year is 2005) giving a total of 40 observations.

The time series plot, the ACF and the PACF, as shown in Figures 1−3, suggest that an AR(1) model is suitable. We detect the additive outliers of this series by using an iterative detecting procedure proposed by Chang *et al.* [14] via the R statistical software (see i.e. Cryer and Chan [15]). We find that the time indices of potential AO are $t = 22$ and 23 (year 1991 and 1992) and we also construct all estimators, $(\hat{\rho}_{MW}, \hat{\delta}_{MW})$, $(\hat{\rho}_{RMW}, \hat{\delta}_{RMW})$ and $(\hat{\rho}_{RDMW}, \hat{\delta}_{RDMW})$ and the estimated variance of the estimators, $\hat{\rho}_{MW}$, $\hat{\rho}_{RMW}$ and $\hat{\rho}_{RDMW}$.

As presented in Table 3, the RDMW estimator, $\hat{\rho}_{RDMW}$, provides the estimated variance of the estimator less than those of the $\hat{\rho}_{MW}$ and $\hat{\rho}_{RMW}$ about 0.95% and 1.64%, respectively. The real example in this section confirms that the RDMW estimator $(\hat{\rho}_{RDMW}, \hat{\delta}_{RDMW})$ is much better than the other estimators.

DISCUSSION AND CONCLUSION

We have proposed two new estimators for a Gaussian AR(1) process with additive outliers.

Table 1: The Estimated Mean Square Error (MSE) of MW, RMW and RDMW Estimators when Percentage of Additive Outliers; $p = 5\%$.

n	ρ	$MSE(\hat{\rho})$			$MSE(\hat{\delta})$		
		MW	**RMW**	**RDMW**	**MW**	**RMW**	**RDMW**
25	0.1	0.0456*	0.0496	0.0508	0.0703	0.0642	0.0618*
	0.2	0.0441*	0.0445	0.0442	0.0730	0.0663	0.0652*
	0.3	0.0510	0.0465	0.0449*	0.0744	0.0672	0.0655*
	0.4	0.0597	0.0513	0.0498*	0.0794	0.0711	0.0698*
	0.5	0.0694	0.0578	0.0569*	0.0891	0.0793	0.0781*
	0.6	0.0816	0.0675	0.0673*	0.0994	0.0878	0.0874*
	0.7	0.0906	0.0743*	0.0751	0.1276	0.1116	0.1115*
	0.8	0.0967	0.0800*	0.0814	0.1797	0.1562*	0.1566
	0.9	0.0974	0.0818*	0.0832	0.3812	0.3254*	0.3313
	0.95	0.0970	0.0819*	0.0841	0.8002	0.6825*	0.7035
	0.99	0.0994	0.0855*	0.0889	4.6172	3.9669*	4.1728
50	0.1	0.0243	0.0235	0.0230*	0.0576	0.0554	0.0536*
	0.2	0.0295	0.0269	0.0256*	0.0578	0.0555	0.0538*
	0.3	0.0398	0.0354	0.0333*	0.0562	0.0538	0.0524*
	0.4	0.0504	0.0447	0.0426*	0.0576	0.0550	0.0538*
	0.5	0.0633	0.0564	0.0541*	0.0605	0.0574	0.0565*
	0.6	0.0727	0.0649	0.0629*	0.0631	0.0596	0.0588*
	0.7	0.0750	0.0672	0.0660*	0.0722	0.0678	0.0668*
	0.8	0.0710	0.0642	0.0641*	0.0945	0.0883	0.0881*
	0.9	0.0575	0.0522*	0.0525	0.1640	0.1522	0.1513*
	0.95	0.0469	0.0434*	0.0436	0.3067	0.2889*	0.2907
	0.99	0.0410	0.0386*	0.0392	1.8065	1.7103*	1.7434
100	0.1	0.0140	0.0131	0.0116*	0.0705	0.0694	0.0671*
	0.2	0.0244	0.0227	0.0197*	0.0667	0.0656	0.0634*
	0.3	0.0389	0.0364	0.0322*	0.0632	0.0621	0.0601*
	0.4	0.0572	0.0539	0.0493*	0.0610	0.0598	0.0581*
	0.5	0.0739	0.0700	0.0653*	0.0580	0.0567	0.0552*
	0.6	0.0868	0.0822	0.0783*	0.0569	0.0555	0.0542*
	0.7	0.0894	0.0851	0.0822*	0.0568	0.0552	0.0542*
	0.8	0.0793	0.0754	0.0733*	0.0676	0.0656	0.0641*
	0.9	0.0514	0.0491	0.0476*	0.0934	0.0905	0.0879*
	0.95	0.0334	0.0322	0.0315*	0.1588	0.1549	0.1516*
	0.99	0.0202	0.0199	0.0197*	0.7879	0.7827	0.7690*
250	0.1	0.0073	0.0069	0.0056*	0.0573	0.0569	0.0552*
	0.2	0.0160	0.0154	0.0127*	0.0529	0.0526	0.0510*
	0.3	0.0290	0.0281	0.0246*	0.0485	0.0482	0.0469*
	0.4	0.0458	0.0446	0.0405*	0.0450	0.0447	0.0435*
	0.5	0.0624	0.0611	0.0569*	0.0404	0.0401	0.0392*
	0.6	0.0756	0.0741	0.0704*	0.0368	0.0365	0.0357*
	0.7	0.0796	0.0781	0.0753*	0.0337	0.0334	0.0328*
	0.8	0.0682	0.0669	0.0650*	0.0322	0.0318	0.0313*
	0.9	0.0368	0.0361	0.0350*	0.0361	0.0356	0.0348*
	0.95	0.0174	0.0172	0.0166*	0.0480	0.0477	0.0465*
	0.99	0.0053	0.0054	0.0052*	0.1533	0.1580	0.1531*

* represents the minimum MSE.

Table 2: The Estimated Mean Square Error (MSE) of MW, RMW and RDMW Estimators when Percentage of Additive Outliers; $p = 10\%$.

n	ρ	$MSE(\hat{\rho})$			$MSE(\hat{\delta})$		
		MW	RMW	RDMW	MW	RMW	RDMW
25	0.1	0.0499*	0.0525	0.0542	0.1730	0.1592	0.1491*
	0.2	0.0520	0.0488	0.0469*	0.1731	0.1585	0.1495*
	0.3	0.0629	0.0552	0.0505*	0.1749	0.1599	0.1518*
	0.4	0.0827	0.0697	0.0635*	0.1735	0.1575	0.1492*
	0.5	0.1033	0.0864	0.0799*	0.1804	0.1628	0.1559*
	0.6	0.1315	0.1100	0.1027*	0.1982	0.1776	0.1700*
	0.7	0.1551	0.1308	0.1238*	0.2393	0.2132	0.2060*
	0.8	0.1757	0.1495	0.1437*	0.3309	0.2908	0.2824*
	0.9	0.1819	0.1557	0.1526*	0.7216	0.6277	0.6143*
	0.95	0.1789	0.1536	0.1517*	1.5009	1.2974	1.2806*
	0.99	0.1806	0.1558	0.1525*	8.8298	7.6505	7.3996*
50	0.1	0.0271	0.0258	0.0238*	0.2574	0.2487	0.2281*
	0.2	0.0404	0.0366	0.0296*	0.2537	0.2447	0.2256*
	0.3	0.0623	0.0561	0.0457*	0.2383	0.2294	0.2132*
	0.4	0.0903	0.0818	0.0676*	0.2291	0.2200	0.2043*
	0.5	0.1192	0.1086	0.0948*	0.2210	0.2116	0.1981*
	0.6	0.1494	0.1371	0.1221*	0.2158	0.2058	0.1930*
	0.7	0.1704	0.1568	0.1437*	0.2220	0.2106	0.1986*
	0.8	0.1754	0.1612	0.1511*	0.2547	0.2397	0.2268*
	0.9	0.1519	0.1399	0.1335*	0.4253	0.3966	0.3804*
	0.95	0.1335	0.1235	0.1188*	0.8200	0.7651	0.7351*
	0.99	0.1142	0.1065	0.1040*	5.1445	4.7848	4.6896*
100	0.1	0.0163	0.0153	0.0114*	0.2559	0.2523	0.2333*
	0.2	0.0321	0.0301	0.0217*	0.2423	0.2386	0.2215*
	0.3	0.0566	0.0537	0.0411*	0.2277	0.2242	0.2082*
	0.4	0.0875	0.0836	0.0685*	0.2150	0.2114	0.1971*
	0.5	0.1202	0.1152	0.0992*	0.2015	0.1978	0.1856*
	0.6	0.1514	0.1455	0.1303*	0.1855	0.1817	0.1717*
	0.7	0.1685	0.1620	0.1489*	0.1746	0.1705	0.1614*
	0.8	0.1636	0.1571	0.1471*	0.1765	0.1715	0.1636*
	0.9	0.1177	0.1129	0.1077*	0.2115	0.2047	0.1962*
	0.95	0.0812	0.0780	0.0747*	0.3562	0.3444	0.3300*
	0.99	0.0515	0.0501	0.0484*	1.9924	1.9421	1.8684*
250	0.1	0.0093	0.0089	0.0051*	0.2416	0.2403	0.2225*
	0.2	0.0242	0.0234	0.0149*	0.2283	0.2269	0.2103*
	0.3	0.0474	0.0462	0.0339*	0.2134	0.2121	0.1971*
	0.4	0.0767	0.0752	0.0607*	0.1973	0.1961	0.1833*
	0.5	0.1082	0.1064	0.0910*	0.1815	0.1803	0.1693*
	0.6	0.1377	0.1356	0.1212*	0.1623	0.1611	0.1526*
	0.7	0.1566	0.1544	0.1423*	0.1422	0.1410	0.1345*
	0.8	0.1496	0.1474	0.1395*	0.1174	0.1163	0.1119*
	0.9	0.0962	0.0946	0.0911*	0.1033	0.1020	0.0988*
	0.95	0.0514	0.0506	0.0485*	0.1279	0.1262	0.1215*
	0.99	0.0164	0.0165	0.0157*	0.4674	0.4691	0.4492*

* represents the minimum MSE.

Table 3: The Parameter Estimates and the Estimated Variance of Estimators of the US/Sudan of Real Exchange Rates Series.

Methods	Estimates		$\hat{V}(\hat{\rho})$
	$\hat{\rho}$	$\hat{\delta}$	
MW	0.8325	36.7266	0.02777
RMW	0.8380	35.5131	0.02796
RDMW	0.8432	34.3899	0.02751

Figure 1: The US/Sudan of Real Exchange Rates; Annual from 1970 to 2009.

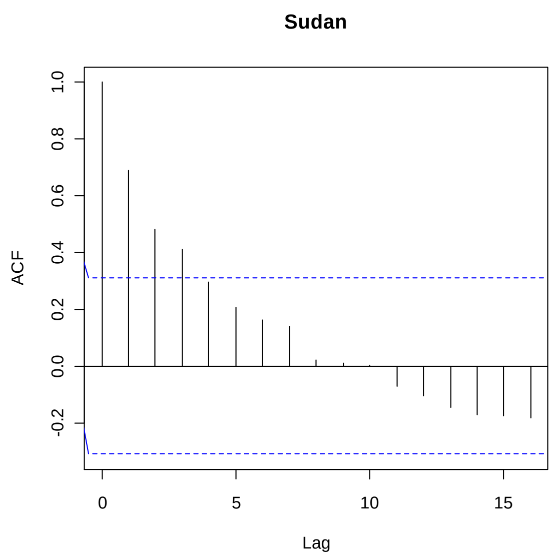

Figure 2: ACF of the US/Sudan of Real Exchange Rates.

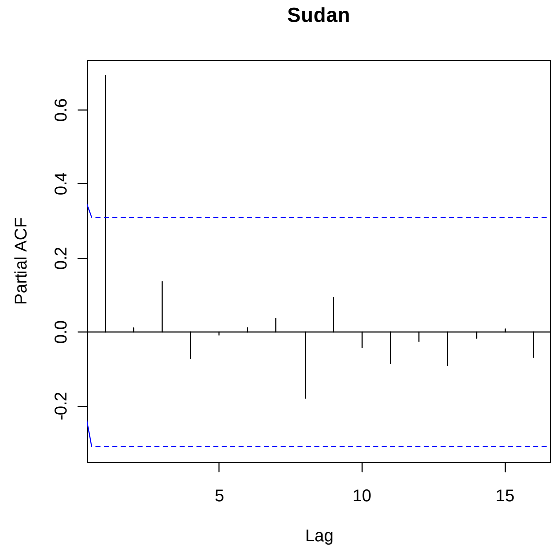

Figure 3: PACF of the US/Sudan of Real Exchange Rates.

This proposed estimators of (ρ, δ) are obtained by applying the recursive mean and the recursive median adjustment to the modified weighted symmetric estimator. The modified weighted symmetric estimator (MW), the recursive mean adjusted modified weighted symmetric estimator (RMW) and the recursive median adjusted modified weighted symmetric estimator (RDMW) are compared in this study. The RDMW estimator performs better than MW and RMW estimators in terms of the MSE for almost all scenarios. One reason behind this is that the additive outliers do not affect the median values. Moreover, the adjusted recursive median values, which are applied in the formula (11) and (12) for $\hat{\rho}_{RDMW}$ $\hat{\delta}_{RDMW}$, respectively, can also reduce the mean square error (MSE) of the estimators. Therefore, the RDMW estimator which is based on the recursive median adjustment is superior to the existing estimators.

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APPENDIX

The yearly exchange rates between the USA and Sudan from 1970 to 2009.

Year	Exchange rates	Year	Exchange rates
1970	230.01	1990	76.13
1971	236.70	1991	54.87
1972	215.30	1992	363.86
1973	198.47	1993	304.20
1974	174.59	1994	263.38
1975	153.71	1995	322.60
1976	159.88	1996	307.06
1977	145.34	1997	269.96
1978	142.25	1998	298.31
1979	137.28	1999	330.49
1980	144.82	2000	329.12
1981	143.38	2001	321.73
1982	206.33	2002	307.18
1983	222.59	2003	289.26
1984	173.00	2004	270.60
1985	218.39	2005	243.61
1986	194.09	2006	223.32
1987	200.19	2007	215.45
1988	189.79	2008	193.48
1989	119.30	2009	180.89

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