On Steady Free Convective Hydro Magnetic Flow of a Reactive Viscous Fluid in a Bounded Domain.

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ABSTRACT

A study of steady free convective hydro magnetic flow of a reactive viscous fluid in a bounded domain has been analyzed. The existence and uniqueness of the solution of momentum and energy equations are proved. By employing the Nachtsheim–Swigert iteration technique along with sixth order Runge-Kutta integration scheme, the momentum and energy equation are solved numerically. Results are presented for velocity and temperature profiles. It is shown that velocity profiles increases as Grashof number, Frank-Kamenetskii, and Prandtl number increases whereas the temperature profile increases as Frank-Kameneskii parameter increases.

(Keywords: hydro magnetic flow, Arrhineus kinetics, free convection, steady, numerical solution)

INTRODUCTION

Magneto hydrodynamics of an electrically conducting fluid is encountered in many problems in geophysics, astrophysics, engineering applications, and other industrial areas. Hydromagnetic free convection flows have a great significance for the applications in the field of stellar and planetary magnetospheres and aeronautics. Engineers employed magneto hydrodynamics principles in the design of heat exchangers pumps, in space vehicle propulsion, thermal protection, control and re-entry, and in creating novel power generating systems.

Hydro magnetic flow and heat transfer problems have also become more important, industrially. In many metallurgical processes involving the cooling of many continuous strips of filaments by drawing them through an electrically conducting fluid subject to a magnetic field, the rate of cooling can be controlled and final product of desired characteristics can be achieved. Another important application of hydromagnetics to metallurgy lies in the purification of molten metals from non-metallic inclusions by the application of a magnetic field [11].

Okedayo and Eletta [10] analyzed heat transfer characteristics of a steady hydro magnetic flow in horizontal plate. The effect of the Darcy number, Hartmann number, and the Brickman number were determined. Their results showed that there was an increase in temperature as the Hartmann number increases and an increase in the Darcy number led to a decrease in the temperature.

Adesanya and Ayeni [1] studied steady flow of a reacting temperature dependent viscous incompressible fluid past an infinite vertical, porous plate, with the flow generated by Newtonian heating and impulsive motion. The resulting momentum and energy equations were non-dimensionlized and the solution showed that the temperature increase as Prandtl and suction parameter increases while the reversed was the case for velocity.

Gideon and Eletta [5] presented similarity solutions to the flow of fluid through a porous bounded by a semi-infinite horizontal plate with viscous dissipation effect. The effect of Prandtl number and permeability parameter were considered. It was observed that as the permeability parameter increases the dimensionless temperature decreases. Also the dimensionless temperature increases as Prandtl number decreases.

Mahanti and Gaur [6] investigated the effects of linearly varying viscosity and thermal conductivity on steady free convective flow of a viscous incompressible fluid along an isothermal vertical plate in the presence of heat sink. The governing equations of continuity, momentum and energy were transformed into coupled and non-linear ordinary differential equations using similarity transformation and then solved using Runge-Kutta fourth order method with shooting technique. Skin-friction coefficient and Nusselt number at the plate were derived. They showed that the increase in thermal conductivity parameter ε increases the velocity and temperature of the fluid irrespective of the value of heat sink parameter S .And skin-friction coefficient increases while rate of heat transfer decreases with the increase in Prandtl number.

Mohamed et al. [8] considered the problem of steady, laminar, and two-dimensional buoyancy induced Darcy's flow along an isothermal vertical wavy surface embedded in a fluid saturated porous medium. The effect of the radiation parameter in the boundary layer adjacent to the vertical wavy plate was analyzed. Using the appropriate variables, the basic equations were transformed to convenient form and then solved numerically by shooting technique. The velocity distributions in the boundary layer and the rate of heat transfer coefficient in terms of Nusselt number were presented for selected values of the parameters. It was observed that the Nusselt number increases with the increase in both radiation and temperature ratio parameters. Moreover, greater fluctuations of the Nusselt number with increasing buoyancy parameter.

Chamkha et al. [3] reported natural convection due to solar radiation from a vertical plate with mass transfer. A local similarity solution has been obtained and boundary friction and heat transfer coefficients were discussed with various governing parameters .They observed that the boundary friction and Nusselt number were decreased as the porous medium parameter value was increased.

From the existing literature most of the previous studies considered only the case of free convection with heat transfer, but little has been done in the direction of reactive flow. In this work we shall consider a steady free convective hydro magnetic flow of a reactive viscous fluid in a bounded domain. In section two we provide the mathematical formulation of the problem and method of solution. In section three gives the properties of solution while section four gives the description of the numerical method employed and section five deals with discussion of results.

MATHEMATICAL FORMULATION

Consider a steady two-dimensional free convection flow of an incompressible electrically conducting viscous fluid on a finite plate and the temperature of the flow assume Arrhineus dependence. The x'-axis is along the plate in the upward direction and y'-axis normal towards it. A transverse constant magnetic field is assumed to be negligible. By assuming a very small magnetic Reynolds number the induced magnetic field is neglected. The appropriate governing equation is given as:

$$\frac{dv'}{dy'} = 0 \quad \textbf{(1)}$$

$$v'\frac{dv'}{dy'} = \nu\frac{d^2u'}{dy'^2} + g\beta'(T' - T'_\infty) - \frac{\sigma\beta_0'^2}{\rho}u' \quad \textbf{(2)}$$

$$v'\frac{dT'}{dy'} = \frac{k}{\rho c_p}\frac{d^2T'}{dy'^2} + \frac{\sigma\beta_0'^2}{\rho c_p}u'^2 + \frac{QAe^{-\frac{E}{RT}}}{\rho c_p} \quad \textbf{(3)}$$

The appropriate boundary conditions are:

$$T(y) = T_\infty \qquad y = \pm 1 \quad \textbf{(4)}$$

$$u(y) = 0 \qquad y = \pm 1 \quad \textbf{(5)}$$

where,

u', v' - Velocity along x,y coordinate
T'_∞ - Ambient temperature
ν - Kinematic viscosity
g - Acceleration due to gravity
T' - Fluid temperature
σ - Electrical conductivity
β'_0 - Coefficient of volumetric expansion
ρ - Density of the fluid
c_p - Specific heat capacity at constant pressure
k - Thermal conductivity
Q - Heat release per unit mass
A - Pre-exponent factor
E - Activation energy
R - Universal gas constant

Introducing the following non-dimensional quantities and assume v_0 =0

$$y=\frac{y^{'}u_0}{\nu},\ t=\frac{t^{'}u_0^{2'}}{\nu},\ u=\frac{u^{'}}{u_0},\ v=\frac{v_0^{'}}{u_0},\ \theta=\frac{(T-T_\infty^{'})}{T_W^{'}-T_\infty^{'}}$$
(6)

into Equations (2) and (3) one gets the following non-dimensional equations governing the flow:

$$\frac{d^2u}{dy^2}+G\theta-H_a^2u=0 \quad \textbf{(7)}$$

$$\frac{1}{\Pr}\frac{d^2\theta}{dy^2}+H_a^2Ec\theta u^2+\delta e^{\frac{\theta}{\alpha+\in\theta}}=0$$
(8)

The boundary conditions are:

$$u(\pm 1)=0 \qquad \theta(\pm 1)=0 \quad \textbf{(9)}$$

where the Prandtl number $\Pr=\frac{\rho c_p\nu}{k}$,

the Hartmann number $H_a=\beta_0 L\sqrt{\frac{\sigma}{\rho\nu}}$,

the Grashof number $\frac{g\beta^{'}(T_w^{'}-T_\infty^{'})\nu}{u_0^3}$,

the Eckert number $Ec=\frac{u_0^2}{c_p(T_w^{'}-T_\infty^{'})}$,

a constant $\alpha=\frac{\in T_\infty^{'}}{(T_w^{'}-T_\infty^{'})}$,

and the Frank-kamenetskii parameter

$$\delta=\frac{QA\nu e^{-\frac{E}{RT_\infty}}}{(T_w^{'}-T_\infty^{'})u_0^2\rho c_p}$$

Properties of Solution

Theorem 1: There exist a unique solution of (7) and (8) which satisfies (9)

Proof: Let

$x_1=y,\quad x_2=\theta,\quad x_3=u,\quad x_4=\theta^{'},\quad x_5=u^{'}$

Then,

$$x_1^{'}=1=f_1(x_1,\ldots\ldots,x_5),\qquad x_1(-1)=0$$

$$x_2^{'}=\theta^{'}=x_4=f_2(x_1,\ldots\ldots,x_5),\qquad x_2(-1)=0$$

$$x_3^{'}=u^{'}=x_5=f_3(x_1,\ldots\ldots,x_5),\qquad x_3(-1)=0$$

$$x_4^{'}=\theta^{''}=-H_a^2Ecx_2(x_3)^2\Pr-\delta\Pr e^{\frac{x_2}{1+\in x_2}}=f_4(x_1,\ldots\ldots,x_5),x_4(-1)=\gamma\ when\,\alpha=1$$

$$x_1^{'}=u^{''}=H_a^2x_3-Gx_2=f_5(x_1,\ldots\ldots,x_5),\qquad x_5(-1)=\lambda$$

where γ and λ is determine until boundary conditions are satisfied.

Clearly $\frac{\partial f_i}{\partial x_j}$, $i=1,\ldots.,5$ are bounded. Hence by theorem 11.2 of [4] problem (7) and (8) which satisfies (9) has unique solution. This completes the proof.

Theorem 2: Let θ and u satisfy (7)-(9), respectively, then:

(i) θ(-y) = θ(y)
(ii) u(-y) = u(y)

Proof: Let $\in=0$, $\alpha=0$,we replace y by –y and differentiate we obtain Equations (7)-(9).Hence θ(-y) = θ(y) and u(-y) = u(y). This completes the proof.

Numerical Computation

Equations (7) and (8) are non-linear ordinary differential equations. Hence it is solved numerically .The set of non-linear ordinary differential equations (7) and (8) with boundary conditions (9) are solved numerically by applying the Nachtsheim-Swigert [9] shooting iteration technique (for a detailed discussion of the method see Maleque and Sattar [7] and Alam et al. [2]) along with the sixth order Runge-Kutta integration scheme. A step size of $\Delta\eta$ =0.01 was selected to be satisfactory for a convergence criterion of 10^{-6} in all cases. The value of η_∞ was found at each iteration loop by the statement of $\eta_\infty = \eta_\infty + \Delta\eta$. The results of the integration are presented in Figures 1-12.

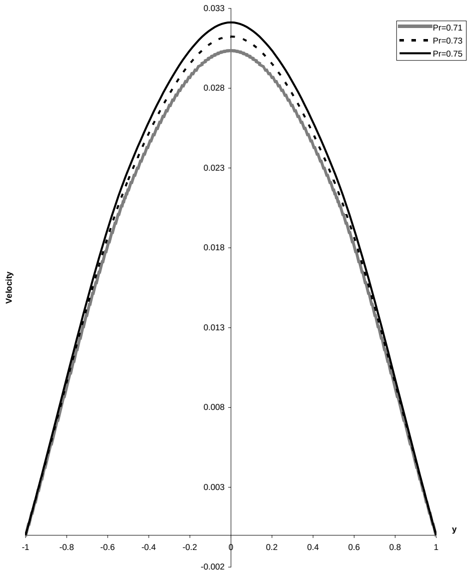

Figure 1: Velocity Profile when G=2.0, Ha=0.1, Ec=1.0, δ=0.1, ε=0.1

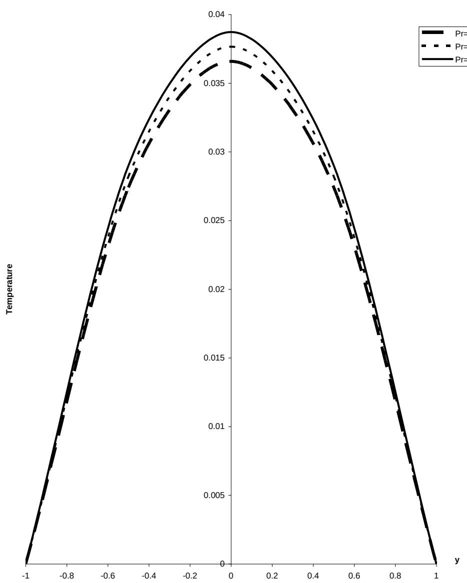

Figure 2: Temperature Profile when G=2.0, Ha=0.1, Ec=1.0, δ=0.1, ε=0.1

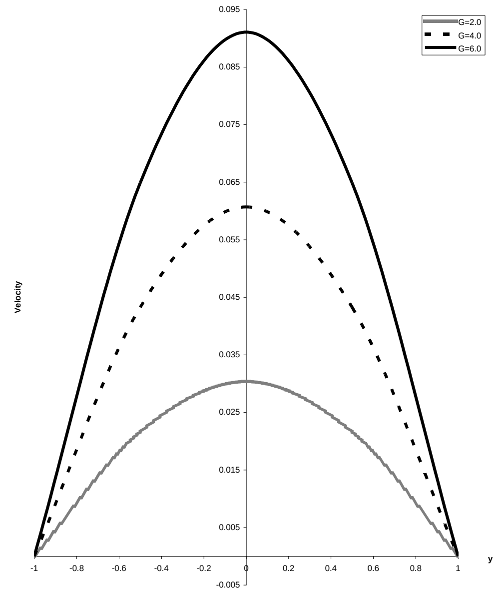

Figure 3: Velocity Profile when Pr=0.71, Ec=1.0, Ha=0.1, δ=0.1, ε=0.1

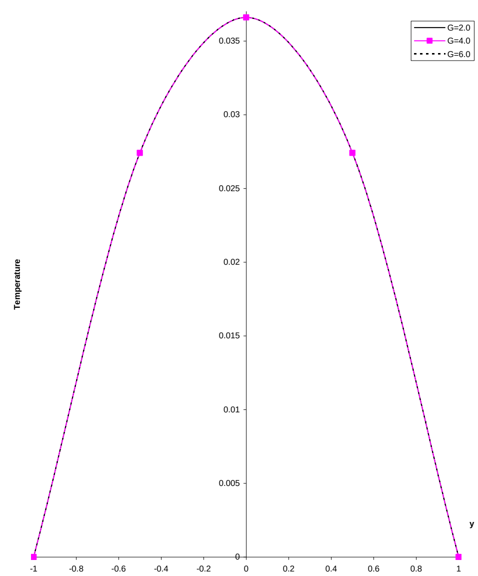

Figure 4: Temperature Profile when Pr=0.71, Ha=0.1, Ec=1.0, δ=0.1, ε=0.1

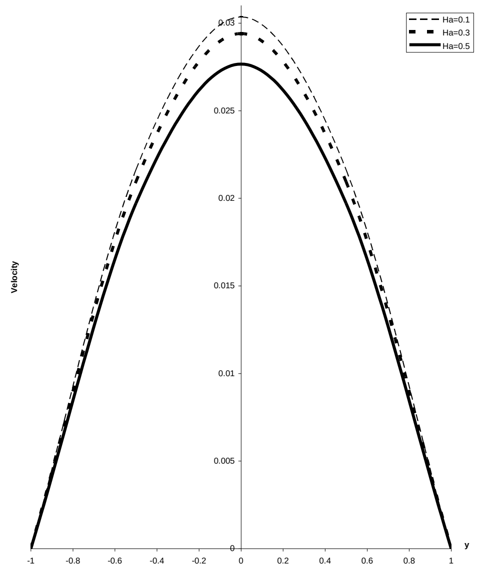

Figure 5: Velocity Profile when Pr=0.71, Ec=1.0, δ=0.1, ε=0.1

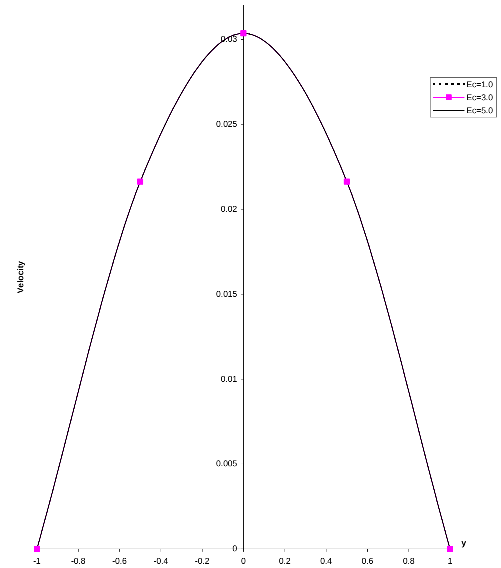

Figure 7: Velocity Profile when Pr=0.71, G=2.0, Ha=0.1, Ec=1.0, δ=0.1, ε=0.1

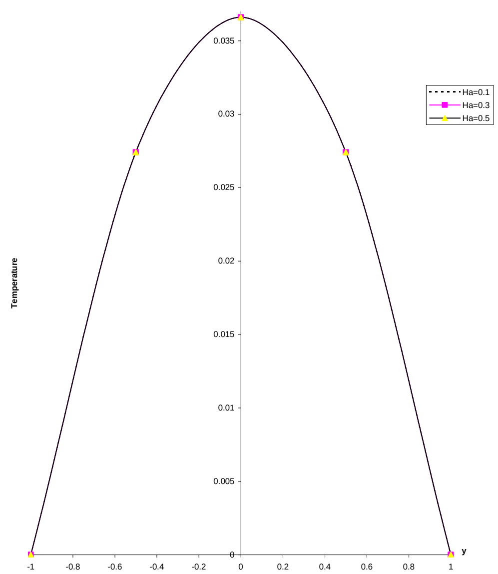

Figure 6: Temperature Profile when Pr=0.71, Ec=1.0, G=2.0, δ=0.1, ε=0.1

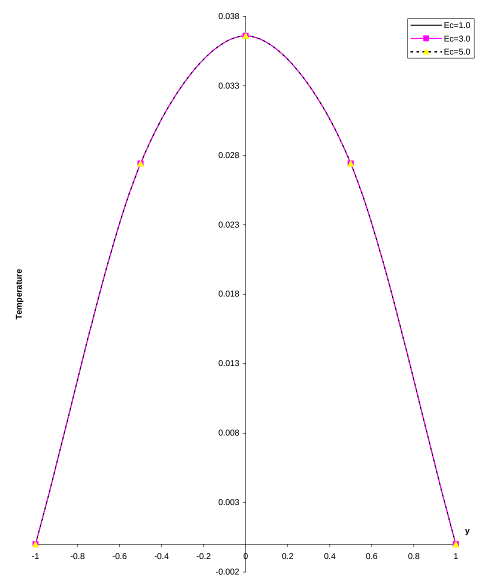

Figure 8: Temperature Profile when Pr=0.71, G=2.0, δ=0.1, ε=0.1

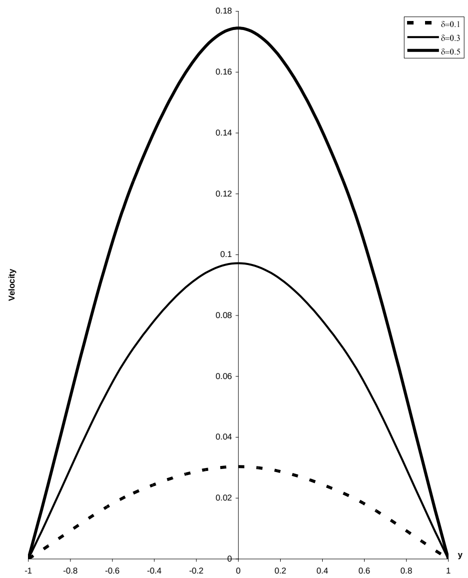

Figure 9: Velocity Profile when G=2.0, Ha=0.1, Ec=1.0, δ=0.1, ε=0.1, Pr=0.71

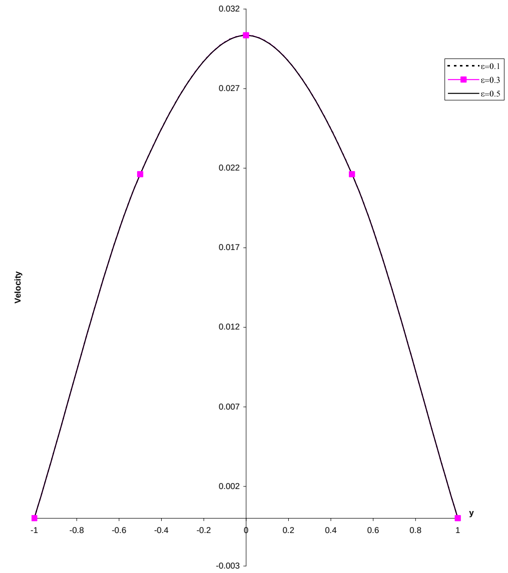

Figure 11: Velocity Profile when G=2.0, Pr=0.71, Ha=0.1, δ=0.1

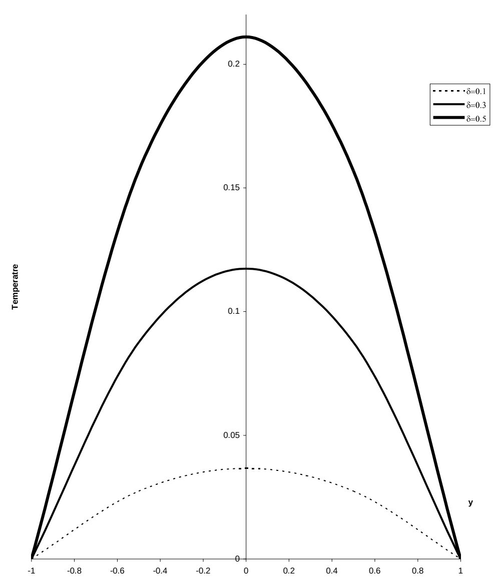

Figure 10: Temperature Profile when Pr=0.71, G=2.0, Ec=1.0, δ=0.1, ε=0.1

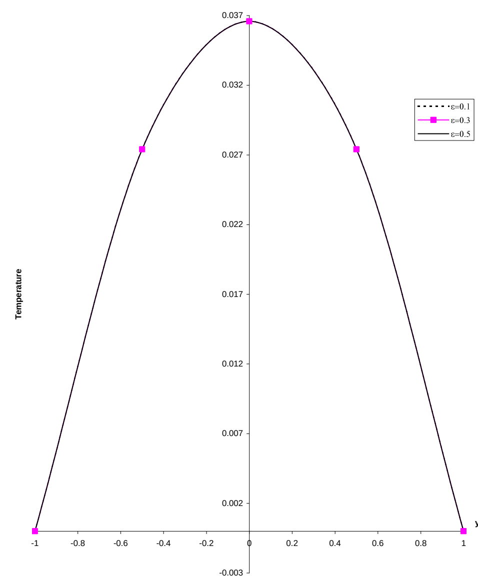

Figure 11: Temperature Profile when Pr=0.71, G=2.0, Ha=0.1, δ=0.1

RESULTS AND DISCUSSION

In this section, numerical computations are presented in the form of non-dimensional velocity and temperature profiles. Also, numerical computation have been carried out for different values of the parameters entering into the problem. The values of Grashof number are taken to be large from the physical point of view. The large Grashof number corresponds to the cooling problem. Figures 1 and 2 reveal the effect of Prandtl number on velocity and temperature profiles. It is observed that the velocity increases while temperature decreases as the Prandtl number increases. This is in agreement with Samad and Rahman [12] and Gideon and Eletta [5].

Figures 3 and 4 showed that the velocity increases and the temperature does not change as Grashof number increases.

Figures 5 and 6 showed the effect of Hartmann number on the velocity as well as temperature profiles. It is observed that the velocity decrease as the Hartmann number increases, it is in agreement with Samad and Rahman [12] while Hartmann number does not have noticeable effect on the temperature profile.

Figures 7, 8, 11, and 12 reveal that Eckert number and activation energy parameter does not have noticeable effect on both velocity and temperature profiles respectively.

Figures 9 and 10 depict the effects of Frank-Kamenetskii parameter on the velocity and temperature profile. It is shown that both velocity and temperature profiles increases as Frank-Kamenetskii parameter increases. Finally, it is observed in all that the flow is symmetric about the centre (i.e., y=0).

CONCLUSION

In this paper we derived the energy and momentum equations governing free convective hydro magnetic flow of a reactive viscous fluid. We have shown the existence and uniqueness of solutions, then presented numerical results for various non–dimensionless parameters graphically.

From the present study we can make the following conclusions:

(i) The velocity profiles increases whereas temperature profile does not have noticeable effect with an increase of the free convection current.

(ii) Using magnetic field we can control the flow characteristic and heat transfer.

(iii) The velocity and temperature profiles increase as the Frank-Kamenetskii parameter increases.

(iv) The velocity profile increases whereas the temperature profile decreases as the Prandtl number increases.

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