Model-Based Object Tracking of Moving Object: Double Pendulum.

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ABSTRACT

The goal of this article is to review the state-of-the-art tracking methods, classify them into different categories, and use one of them to track the motion of a double pendulum. Object tracking, in general, is a challenging problem. Difficulties in tracking objects can arise due to abrupt object motion, changing appearance patterns of the object and the scene, non-rigid object structures, object-to-object and object-to-scene occlusions, and camera motion. Tracking is usually performed in the context of higher-level applications that require the location and/or shape of the object in every frame. Typically, assumptions are made to constrain the tracking problem in the context of a particular application.

The usefulness of tracking cannot be over-emphasized, in coal production for instance, tracking can be used to detect the environmental conditions in which the coal to be mined is found. This can help prevent mishaps. In this investigation, two colored balls were attached to the joints of a double pendulum. A video of the motion of the double pendulum was made. A simple yet effective code was written in MATLAB® to track the motion of the balls in the double pendulum. The result shows that it is possible to track the double pendulum irrespective of the motion or speed using a simple method. These findings represent a breakthrough in applications such as mineral detection, crime detection, the study of natural phenomenon, the study of fluid flow, among others.

(Keywords: object tracking, double pendulum, MATLAB, motion, coal production)

INTRODUCTION

The concept of object tracking is of great importance because it will provide solutions to many significant problems, notable among them are: armed robbery, stolen vehicles, access to locations that cannot be accessed by humans, among others. In recent times it is not uncommon for crimes to be committed with the culprits apprehended; products failing without the major cause of the failure being detected; and fluid releases causing damages without the actual cause being observed. With this procedure, many such problems can be detected using simple but effective steps and codes to track down an image.

Image sequence analysis provides intermediate results for a conceptual description of events in a scene. A system that establishes such higher level descriptions based on tracking of moving objects in the image domain has been described in [Koller et al., 1991]. Here they introduced three-dimensional models of the structure and the motion of the moving objects as well as of the illumination of the scene in order to verify the hypotheses for object candidates and to robustly extract smooth trajectories of such objects.

Organisms from six-legged cockroaches to two-legged humans display similar dynamics when walking and running. Below a non-dimensional speed (the Froude number), dynamics can generally be modeled as an inverted pendulum [Goldman, 2010].

Tracking Methods

- Pick points by hand
- Brightest point(s) in image
- Track local maxima
- Centroid
- Shape based tracking

Double Pendulum: A double pendulum consists of two simple pendulums in tandem, where the second pendulum is attached to the mass of the first, as depicted in Figure 1. Beyond the assumption that our physical system may be modeled using the laws of classical mechanics, we make a few simplifying assumptions:

1. The mass of the rod connecting each pendulum mass to its respective pivot point is negligible (considered zero).

2. The two rods are perfectly rigid.

3. We may effectively treat the pendulum masses as point masses. This assumption is essentially true as long as the rods are perfectly rigid and of negligible mass, and the length of each rod is measured as the length between the pivot point and the center of mass of the pendulum bob.

4. The system is non-dissipative. That is, no energy loss occurs.

5. The gravitational force is approximately constant throughout the range of motion of the pendulum masses.

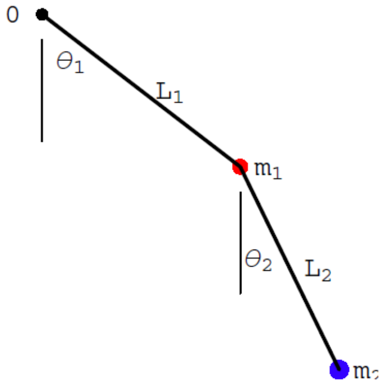

Figure 1: Free Body Diagram of the Double Pendulum.

RELATED INVESTIGATION

In this section we discuss related investigations about tracking and recognizing object models from image sequences. Gennery [1982] has proposed the first approach for tracking 3D-objects of known structure. A constant velocity six degrees of freedom (DOF) model is used for prediction and an update step similar to the Kalman filter – without addressing the nonlinearity – is applied. Edge elements closest to the predicted model line segments are associated as corresponding measurements. During the last ten years, Gennery's approach evolved and one can find in [Gennery, 1992] the most elaborate version of this approach in estimating the motion of a known object, with particular emphasis on a time efficient implementation of the recursive estimation and on the propagation of uncertainty. The used force- and torque-free motion model is the same as in [Gennery, 1982] and can be applied in robot activities in space.

Thompson and Mundy [1987] emphasize the object recognition aspect of tracking by applying a pose clustering technique. Candidate matches between image and model vertex pairs define points in the space of all transformations. Dense clusters of such points indicate a correct match.

Object motion can be represented by a trajectory in the transformation space. Temporal coherence then means that this trajectory should be smooth. Predicted clusters from the last time instant establish hypotheses for the new time instants which are verified as matches if they lie close to the newly obtained clusters. The images we have been working on did not contain the necessary vertex pairs in order to test this novel algorithm. Furthermore, we have not been able to show that the approach of [Thompson and Mundy, 1987] can be extended to the handling of parameterized objects.

Verghese et al. [1990] have implemented two approaches for tracking 3D-known objects in real-time. Their first method is similar to the approach of Thompson and Mundy [1987]. Their second method is based on the optical flow of line segments. Using line segment correspondences, of which initial (correct) correspondences are provided interactively at the beginning, a prediction of the model is validated and spurious matches are rejected.

Lowe [1991] has built the system that has been the main inspiration for our matching strategy. He does not enforce temporal coherence however, since he does not imply a motion model. Pose updating is carried out by minimization of a sum of weighted least squares including prior constraints for stabilization.

Line segments are used for matching but distances of selected edge points from infinitely extending model-lines are used in the minimization. Lowe [1990] uses a probabilistic criterion to guide the search for correct correspondences and a match iteration cycle.

A gradient-ascent algorithm is used by Worrall et al. [1991] in order to estimate the pose of a known object in a car sequence. Initial values for this iteration are provided interactively at the beginning. Since no motion model is used, the previous estimate is used at every time instant to initialize the iteration.

Marslin et al. [1991] have enhanced the approach by incorporating a motion model of constant translational acceleration and angular velocity. Their filter optimality, however, is affected by use of the speed estimates as measurements instead of the image locations of features.

Schick and Dickmanns [1991] use a generic parameterized model for the object types. They solve the more general problem of estimating both the motion and the shape parameters. The motion model of a car moving on a clothoid trajectory is applied including translational as well as angular acceleration. The estimation machinery of the simple EKF is used. So far, however, their approach has only been tested on synthetic line images.

A constant velocity model with six DOF is assumed by Wu et al. [1988], Harris and Stennet [1990], and Evans [1990], whereas Young and Chellappa [1990] use a processional motion model. A similar problem to Young and Chellappa [1990] with stereo images is solved by Zhang and Faugeras [1992] where closed form solutions for the prediction step are established using a constant angular velocity and constant translational acceleration motion model. By means of the motion estimates of the tracked 3D line segments Zhang and Faugeras [1992] obtain groupings of the 3D-line segments into single objects.

A quite different paradigm is followed by Murray et al. [1989]. They first try to solve the structure from motion problem from two monocular views. In order to accomplish this, they establish temporal correspondence of image edge elements and use these correspondences to solve for the infinitesimal motion between the two time instants and the depths of the image points. Based on this reconstruction, Murray et al. [1989] carry out a 3D-3D correspondence search. Their approach has been tested with camera motion in a laboratory set-up.

Further approaches exist for tracking moving objects in the image domain by using hypotheses about the change of the projections of the underlying 3D objects.

APPLICATION

Before now it was used in making of cartoons. In which case, colored objects are attached to the actress/actors to simulate the "cartoon character". These objects are then tracked and transform into computer to generate the cartoon character. This process can be used in crime detection (pipe vandalism, car theft, breaking and entering, facial detecting, among others), in fluid flow in which a little color can be added to the fluid to monitor and tracked down any action taking place in the fluid or the fluid carrying container. It can also be used in medicine to detect growth and regeneration. Also object tracking is important because it enables several important applications such as:

- security and surveillance – to recognize people, to provide better sense of security using visual information;
- medical therapy – to improve the quality of life for physical therapy patients and disabled people;
- retail space instrumentation – to analyze shopping behaviors of customers and enhance building and environment design;
- video abstraction – to obtain automatic annotation of videos, to generate object-based summaries;
- traffic management – to analyze flow, to detect accidents;
- video editing – to eliminate cumbersome human-operator interaction, to design futuristic video effects;
- and interactive games –provide natural ways of interaction with intelligent systems such as weightless remote control.

MATERIALS AND METHODS

Materials: This experiment makes use of an inexpensive high resolution camera capable of capturing and slowing down the speed of moving objects, double pendulum, and two colored balls (red and blue) for ease of tracking.

Procedure: Two colored pins were attached to a double pendulum as shown in Figure 1. The double pendulum was set in motion. A movie of the oscillation process of the pendulum was made. These pins were monitored and the movie was called into MATLAB® to read the movie and track the pins during the oscillation.

Coding: The following are the steps in tracking a picture or video be it static or mobile:

Step 1: call in the video or picture into the software, in this case MATLAB® or SCILAB®

Step 2: read and analyze the video or picture

Step 3: consult the expert (depending on the field of application – it could be a psychologist, researcher, etc.) to use the information

RESULTS AND DISCUSION

Parameters for Double Pendulum Model

The two pivots of the double pendulum were numbered; 1 for the pivot that is stationary, and 2 for the pivot that can move. Likewise, the two rectangular pieces were also numbered; 1 for the piece attached to both pivots and 2 for the piece attached only to pivot 2. We modeled the double pendulum, ignoring friction, with the following parameters:

- l: distance between the two pivots
- m_1, m_2: masses of the two rotating pieces
- r_1, r_2,: for each piece, distance from its pivot to its center of mass
- I_1, I_2,: moment of inertia for each piece about its pivot

We replaced all but l with the following dimensionless parameters, where k can be 1 or 2:

- ρ= m_2/m_1: mass ratio
- $\propto_k = r_k/\mathrm{l}$: relative distance to center of mass
- $\beta_k = I_k/(\mathrm{m_k}\, l^2)$: Moment of inertia normalized by the moment of a point l .

These five parameters, along with the ratio g/l where g is gravitational acceleration, are used in our model implementation. The model is thus non-dimensionalized, except for the time unit.

Model Equations

The state variables for the model are θ_1,θ_2,p_1, p_2, where θ_k, is the angle of piece k from the downward vertical, and p_k, is a generalized momentum.

$$\dot{\theta}_1 = \frac{\rho\beta_2 p_1 - \rho\alpha_2 \cos(\theta_1 - \theta_2) p_2}{(\beta_1 + \rho)\rho\beta_2 - \rho^2\alpha_2^2 cos^2(\theta_1 - \theta_2)}$$

$$\dot{\theta}_2 = \frac{(\rho + \beta_1) p_2 - \rho\alpha_2 \cos(\theta_1 - \theta_2) p_2}{(\beta_1 + \rho)\rho\beta_2 - \rho^2\alpha_2^2 cos^2(\theta_1 - \theta_2)}$$

$$\dot{p}_1 = -\frac{g}{l}(\propto_1 + \rho)\sin\theta_1 - \rho\alpha_2 \sin(\theta_1 - \theta_2)\dot{\theta}_1\dot{\theta}_2$$

$$\dot{p}_2 = -\frac{g}{l}\rho \propto_2 \sin\theta_2 + \rho\alpha_2 \sin(\theta_1 - \theta_2)\dot{\theta}_1\dot{\theta}_2$$

Kinematics of the Double Pendulum

- x = horizontal position of pendulum mass
- y = vertical position of pendulum mass
- θ = angle of pendulum (0 = vertical downwards, counter-clockwise is positive)
- L = length of rod (constant)

We placed the origin at the pivot point of the upper pendulum. We regard y as increasing upwards. We indicate the upper pendulum by subscript 1, and the lower by subscript 2. Begin by using simple trigonometry to write expressions for the positions x_1, y_1, x_2, y_2 in terms of the angles θ_1, θ_2.

$x_1 = L_1 \sin \theta_1$

$y_1 = -L_1 \cos \theta_1$

$x_2 = x_1 + L_2 \sin \theta_2$

$y_2 = y_1 - L_2 \cos \theta_2$

The velocity is the derivative with respect to time of the position.

$x_1' = \theta_1' L_1 \cos \theta_1$

$$y_1' = \theta_1' L_1 \sin\theta_1$$

$$x_2' = x_1' + \theta_2' L_2 \cos\theta_2$$

$$y_2' = y_1' + \theta_2' L_2 \sin\theta_2$$

The acceleration is the second derivative.

$$x_1'' = -\theta_1'^2 L_1 \sin\theta_1 + \theta_1'' L_1 \cos\theta_1 \qquad \textbf{(1)}$$

$$y_1'' = \theta_1'^2 L_1 \cos\theta_1 + \theta_1'' L_1 \sin\theta_1 \qquad \textbf{(2)}$$

$$x_2'' = x_1'' - \theta_2'^2 L_2 \sin\theta_2 + \theta_2'' L_2 \cos\theta_2 \qquad \textbf{(3)}$$

$$y_2'' = y_1'' + \theta_2'^2 L_2 \cos\theta_2 + \theta_2'' L_2 \sin\theta_2 \qquad \textbf{(4)}$$

Forces in the Double Pendulum

We treat the two pendulum masses as point particles. Begin by drawing the free body diagram for the upper mass and writing an expression for the net force acting on it. Define these variables:

- T = tension in the rod
- m = mass of pendulum
- g = gravitational constant

The forces on the upper pendulum mass are the tension in the upper rod T_1, the tension in the lower rod T_2, and gravity $-m_1g$. We write separate equations for the horizontal and vertical forces, since they can be treated independently. The net force on the mass is the sum of these. Here we show the net force and use Newton's law $F = m\,a$.

$$m_1x_1'' = -T_1 \sin\theta_1 + T_2 \sin\theta_2 \qquad \textbf{(5)}$$

$$m_1y_1'' = T_1 \cos\theta_1 - T_2 \cos\theta_2 - m_1g \qquad \textbf{(6)}$$

For the lower pendulum, the forces are the tension in the lower rod T_2, and gravity $-m_2g$.

$$m_2x_2'' = -T_2 \sin\theta_2 \qquad \textbf{(7)}$$

$$m_2y_2'' = T_2 \cos\theta_2 - m_2g \qquad \textbf{(8)}$$

In relating these equations to the diagrams, keep in mind that in the example diagram θ_1 is positive and θ_2 is negative, because of the convention that a counter-clockwise angle is positive.

Direct Method for Finding Equations of Motion

Now we do some algebraic manipulations with the goal of finding expressions for θ_1'', θ_2'' in terms of θ_1, θ_1', θ_2, θ_2'. Begin by solving Equations (7) and (8) for $T_2 \sin\theta_2$ and $T_2 \cos\theta_2$ and then substituting into Equations (5) and (6).

$$m_1x_1'' = -T_1 \sin\theta_1 - m_2x_2'' \qquad \textbf{(9)}$$

$$m_1y_1'' = T_1 \cos\theta_1 - m_2y_2'' - m_2g - m_1g \qquad \textbf{(10)}$$

Multiply Equation (9) by $\cos\theta_1$ and Equation (10) by $\sin\theta_1$ and rearrange to get:

$$T_1 \sin\theta_1 \cos\theta_1 = -\cos\theta_1\,(m_1x_1'' + m_2x_2'') \qquad \textbf{(11)}$$

$$T_1 \sin\theta_1 \cos\theta_1 = \sin\theta_1\,(m_1y_1'' + m_2y_2'' + m_2g + m_1g) \qquad \textbf{(12)}$$

This leads to the equation:

$$\sin\theta_1\,(m_1y_1'' + m_2y_2'' + m_2g + m_1g) = -\cos\theta_1\,(m_1x_1'' + m_2x_2'') \qquad \textbf{(13)}$$

Next, multiply Equation (7) by $\cos\theta_2$ and Equation (8) by $\sin\theta_2$ and rearrange to get:

$$T_2 \sin\theta_2 \cos\theta_2 = -\cos\theta_2\,(m_2x_2'') \qquad \textbf{(14)}$$

$$T_2 \sin\theta_2 \cos\theta_2 = \sin\theta_2\,(m_2y_2'' + m_2g) \qquad \textbf{(15)}$$

This leads to:

$$\sin\theta_2\,(m_2y_2'' + m_2g) = -\cos\theta_2\,(m_2x_2'') \qquad \textbf{(16)}$$

Next we use MATLAB® to solve Equations (13) and (16) for θ_1'', θ_2'' in terms of θ_1, θ_1', θ_2, θ_2'. Note that we also include the definitions given by Equations (1-4), so that we have 2 equations (13, 16) and 2 unknowns (θ_1'', θ_2''). The result is somewhat complicated, but is easy enough to program into the computer.

$$\theta_1'' = \frac{-g\,(2\,m_1 + m_2)\sin\theta_1 - m_2g\sin(\theta_1 - 2\,\theta_2) - 2\sin(\theta_1 - \theta_2)\,m_2\,(\theta_2'^2L_2 + \theta_1'^2L_1\cos(\theta_1 - \theta_2))}{L_1\,(2\,m_1 + m_2 - m_2\cos(2\,\theta_1 - 2\,\theta_2))}$$

$$\theta_2'' = \frac{2\sin(\theta_1 - \theta_2)\,(\theta_1'^2L_1\,(m_1 + m_2) + g(m_1 + m_2)\cos\theta_1 + \theta_2'^2L_2m_2\cos(\theta_1 - \theta_2))}{L_2\,(2\,m_1 + m_2 - m_2\cos(2\,\theta_1 - 2\,\theta_2))} \qquad \textbf{(17)}$$

These are the equations of motion for the double pendulum.

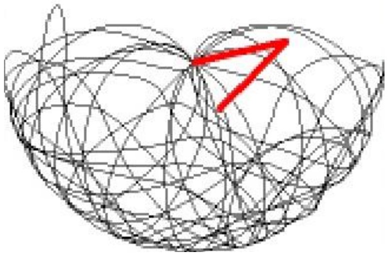

Figure 2: The Tracked Result of the Double Pendulum.

N.B: The double pendulum is the one shown with the red color.

Figure 3: Double Pendulum Tracked with LED Light.

CONCLUSION

The motion of the double pendulum was successfully tracked and the path modeled. In summary, For large motions the double pendulum is a chaotic system, but for small motions it is a simple linear system. For small angles, a pendulum behaves like a linear system. When the angles are small in the Double Pendulum, the system behaves like the linear Double Spring. For large angles, the pendulum is non-linear and the phase graph becomes much more complex.

This is a major breakthrough as many problem and motion hitherto that remain almost impossible to solve and track can now be successfully carried out using a similar approach.

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