Numerical Modeling of Spilled Oil Transport in Marine Environment.

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ABSTRACT

The environmental concern over marine oil spills has lead to the development of mathematical models that simulate the transport and fate of oil slicks. These models are used for spill response during accidents, environmental impact assessment, contingency planning, and response training. Spilled oil is transported, and its composition and character altered by a variety of physical, chemical, and biological processes. This paper presents a mass balanced oil spill transport model that uses hydrodynamic results obtained with a one-dimensional Crank-Nicolson numerical scheme to predict the mass concentration of spilled oil in time and space.

(Keywords: marine environment, oil spill, numerical scheme, spill transport, oil concentration, Navier-Stoke equation)

INTRODUCTION

Oil spill models are normally constructed by linking mathematical formulations or algorithms to represent oil transport and fate processes (Stolzenbach *et al*., 1977; Chen *et al*., 1997). Over the past three decades, intensive research has been directed toward the development of mathematical models to predict and forecast the fate and transport of spilled oil in marine environments (Reed *et al*., 1999; Spaulding, 1988; Blumberg and Mellor, 1987; Fay, 1971, 1969).

Fate processes include spreading, evaporation, dispersion, dissolution, emulsification, and sedimentation. While the transport aspect of spill determines the oil movement in space and time (Tkalich *et al*., 2003), the fate portion estimates the oil transport between various environmental components and the change in the physical characteristics of the oil density, water content, viscosity, etc.(Steinberg *et al*., 1997).

A number of spill models have been developed since the early sixties. The complete reviews of Huang (1983), Spaulding (1988), ASCE (1996), and Reed *et al*. (1999) summarize available models for the oil spill simulation. Earliest approaches used only semi-empirical formulae for the slick area evaluation. Fay (1969, 1971) developed a model for the prediction of oil spreading using a semi-empirical formula:

$$A = 2270\left(\frac{\Delta\rho}{\rho_0}\right)V^{2/3}t^{1/2} + 40\left(\frac{\Delta\rho}{\rho_w}\right)^{1/3}V^{1/3}U_{wind}^{4/3}t$$

where,

$\Delta\rho = \rho_w - \rho_0$

A = area of oil slick; V = total volume of the oil spill; ρ_w *= water density;* ρ_0 *= oil density;* U_{wind} *= wind speed; t = time.* **(1)**

Fay provided the most widely used spreading algorithm. However, field data show that Fay's theory greatly underestimates slick growth (Conomos, 1974; Murray, 1972). Experimental results from the Gulf region indicate that more realistic results were achieved in the gravity viscous phase when wind parameters were considered and when the assumption was made that oil spreads as an ellipse with the major axis in the direction of the wind (Johansen, 1987*a*, 1987b; Lehr *et al*., 1981; Lehr and Cekirge, 1980).

Later, Lehr *et al*. (1984) modified Fay's formula and considered oil spill as an ellipse with an area

$A = \left(\frac{\pi}{4}\right)QR$, where,

$$Q = c_1 \left\{ \frac{(\rho_w - \rho_0)}{\rho_0} \right\}^{1/3} V^{1/3} t^{1/4}; \qquad R = Q + c_2 W_i^{4/3} t^{3/4} \quad (2)$$

Here, $Q =$ *lengths of the minor axis (m);* $R =$ *lengths of the major axis (m);* $\rho_w =$ *water density (kg/m*^{3}*);* $\rho_0 =$ *oil density (kg/m*^{3}*);* $V =$ *initial volume of spill (barrels);* $W_i =$ *wind speed (knots); t = time (minutes); and* $c_1, c_2 = 1.7$*, 0.03 respectively, with appropriate dimensions.*

In recent years, with the development of computational science (Clark, 1996), new alternatives have appeared. Nowadays, an oil slick dynamics model can afford to routinely use such accurate and physically relevant information as the Navier-Stokes equation (Bermúdez, 1997). Tkalich *et al*. (2003) developed a multiphase oil spill model to simulate consequences of accidental oil spills in the marine environment. The multiphase model which was governed by the Navier-Stokes equations employed these modern approaches of environmental computational hydraulics to account for major phenomena of oil physics in an aquatic environment.

Du *et al.* (2005) used Random Walk Method in a Monte Carlo simulation framework to track the transport of oil due to the effects of waves, buoyancy, and turbulent diffusion. The method of moments was used to derive the spreading parameters of oil under regular waves. Wang *et al*. (2008) developed a three-dimensional numerical simulation model for transport of oil spills in sea. The amount of oil released at sea was distributed among a large number of particles tracked individually. A Lagrangian discrete particle algorithm was applied to simulate the transport and fate of the oil slick, which was treated as an ensemble of a large number of small particles whose discrete path and mass were followed and recorded as functions of time.

Mendes *et al.*(2009) conducted a study aiming at the estimation of dispersion through Ria de Aveiro of a offshore oil-spill that reaches the barrier/lagoon mouth. The approach used consisted of the Lagrangian modeling of passive particle emissions. The Lagrangian model was coupled to a hydrodynamic model, previously calibrated and validated for this lagoon. This model solves the shallow waters equations, obtained by vertical integration of the continuity and Navier-Stokes equations, representing the fundamental principles of mass and momentum conservation in a fluid (assumed Newtonian). Guo *et al*. (2009) proposes a numerical method to simulate oil spill trajectories, which are affected by the combination of advection, turbulent diffusion, and mechanical spreading process, based on a particle tracking algorithm. Thus, in modeling the diffusion process, a discrete method was employed for the generation of fractional Brownian motion (fBm) to illustrate super diffusive transport.

THE TRANSPORT MODEL

Let us consider the following pollutant fate and transport model whose governing equation is based on the law of conservation of mass with the Navier-Stokes system (Bermúdez, 1997):

$$-\nabla\left(VC(x,t)\right) + \nabla\left(D\nabla C(x,t)\right) + R + S = \frac{\partial C(x,t)}{\partial t} \quad (3)$$

where, $C(x,t) =$ *mass concentration of contaminants (spilled oil) in the system at point* x *and time t;* V *= fluid velocity (which may be obtained from the shadow water equation, but is here taken as a constant);* $\nabla =$ *gradient operator; D = turbulent diffusion tensor; S = sink or source term; and R = environmental factors.*

The diffusion tensor describes a change in concentration of the system resulting from the random motion of the pollutant in the transporting medium and can be attributed to both turbulent and molecular diffusion. It is assumed that molecular diffusion is negligible compared to turbulent diffusion. The source term includes the addition of contaminants due to discharge from external source, or losses due to processes such as evaporation, vertical mechanical dispersion, and emulsification.

When petroleum products enter the marine environment, the concentration of the pollutant is bound to decrease with time due to some weathering processes like evaporation, oxidation, etc. This can be described as a death process.

Usually, death rate is frequently modeled as a first order reaction. Thus, we modeled R as:

$$R = -kC(x,t) \quad \textbf{(4)}$$

where k is a kinetic constant.

We also assume that the source term is not known so that:

$$E(S) = 0 \quad \textbf{(5)}$$

Thus, (3) becomes:

$$\frac{\partial C(x,t)}{\partial t} = -\nabla(VC(x,t)) + \nabla \bullet (D\nabla C(x,t)) - kC(x,t) \quad \textbf{(6)}$$

We consider (6) in a bounded domain $\Omega \subset \Re^2$ with smooth boundary $\partial\Omega$:

$$\dot{u} + (u,\nabla)u - \nu\Delta u + \nabla p = kC(x,t), \qquad div\, u = 0 \quad ; x \in \Omega \quad \textbf{(7)}$$

Here, u = (u_1,u_2) is the velocity field of the fluid; $p = p(t,x)$ is the pressure; and $\nu > 0$ is the viscosity. Suppose that we supplement (7) with the Dirichlet boundary condition:

$$u\,|_{\partial\Omega} = 0 \quad \textbf{(8)}$$

Under some mild regularity assumptions on the right-hand side k, the Cauchy problem for (7), (8) has a unique solution defined on the half-line $t \geq 0$ (Vishik and Fursikov, 1988). Suppose that we introduce the space of measurable functions:

$$\Im = \left\{u \in L^2\left(\Omega, \Re^2\right): div\, u = 0, \qquad (u,n)|_{\partial\Omega} = 0\right\} \quad \textbf{(9)}$$

where n is the outward normal to $\partial\Omega$. Let $\hbar : L^2\left(\Omega, \Re^2\right) \to \Im$ be the orthogonal projection onto $\Im$. Applying formally the operator $\hbar$ to (7), we will obtain the following evolution equation in $\Im$:

$$\dot{u} + uLu + B(u) = kC(t) \quad \textbf{(10)}$$

Here, $L = -\hbar\Delta$ is the Stokes operator, $B(u) = \hbar(u,\nabla)u$ is the bilinear form resulting from the nonlinear term in (7). Considering as a stochastic process the term $kC(x,t)$ (or simply $k(x,t)$ where there is no possibility of confusion) and denoting by {e_i} an orthonormal basis in $\Im$ formed of the eigenvectors of the Navier-Stokes operator, we assume that:

$$kC(t) = \frac{\partial}{\partial t}\delta(t), \qquad \delta(t) = \sum_{i=1}^{\infty}\alpha_i\beta_i(t)e_i \quad \textbf{(11)}$$

where $\{\beta_i\}$ is a sequence of independent standard Brownian motions defined on a probability space $(\vartheta, \omega, \mathrm{P})$ and $\alpha_i \geq 0$ are some constants such that:

$$B := \sum_{i=1}^{\infty}\alpha_i^2 < \infty \quad \textbf{(12)}$$

Condition (12) ensures that almost every realization of $\delta(t)$ is a continuous function of time with range in $\Im$.

Let us give a more precise formulation of the problems we are dealing with. We wish to rather supplement (7) with the Neumann boundary condition:

$$u\,|^{\partial\Omega} = 0 \quad \textbf{(13)}$$

Given a positive real number T, we consider a cylinder:

$$Q_T = (\Omega \times (0,T)) \quad \textbf{(14)}$$

(or simply Q where there is no possibility of confusion). Introducing $C_0(Q)$ as the space of continuous functions with compact support in Ω, and $Z = \partial\Omega \times (0,T)$ as the lateral boundary of Q, (2.4) becomes:

$$
\left.\begin{array}{llll}
\frac{\partial C(x,t)}{\partial t}+V\frac{\partial C(x,t)}{\partial x}-\frac{\partial^2 C(x,t)}{\partial x^2}+kC(x,t)=0 & & in & Q \\
C(x,0) & =C_0 & in & \Omega \\
\frac{\partial C(x,t)}{\partial n} & =0 & on & Z
\end{array}\right\} \quad (15)
$$

Since it has been proven that there exists a unique solution for the state equation (3) (Agusto, 2008), thus, the solution of (15) can be obtained using the Crank-Nicolson finite difference scheme (Ancona, 2002). The finite difference representation of (15) for any point i, a generic point x on the grid at $t \geq 0$ is:

$$
\frac{C_i^{j+1}-C_i^j}{\Delta t}+\frac{V}{2}\left\{\frac{C_{i+1}^{j+1}-C_i^{j+1}}{2\Delta x}+\frac{C_{i+1}^j-C_i^j}{2\Delta x}\right\}-
\frac{D}{2}\left\{\frac{C_{i+1}^{j+1}-2C_i^{j+1}+C_{i-1}^{j+1}}{\Delta x^2}+\frac{C_{i+1}^j-2C_i^j+C_{i-1}^j}{\Delta x^2}\right\}+kC_i^{j+1}=0 \quad (16)
$$

Then we have,

$$
-\lambda C_{i-1}^{j+1}+(2-\tau+2\lambda-2k)C_i^{j+1}+(\tau-\lambda)C_{i+1}^{j+1}= \lambda C_{i-1}^j+(2+\tau-2\lambda)C_i^j+(\lambda-\tau)C_{i+1}^j \quad (17)
$$

where , $\tau=\frac{V\Delta t}{2\Delta x}$ *and* $\lambda=\frac{D\Delta t}{\Delta x^2}$

Consider the boundary and initial conditions needed to solve (15):

$$
C(x,0)=C_0 \ and \quad \frac{\partial C}{\partial x}=h(t) \quad (18)
$$

These conditions can be stated without lost of generality as (Agusto 2008):

$$
(1-u_1)C(0,t)-u_1C_x(0,t)=h_1(t) \quad (19)
$$

$$
(1-u_2)C(a,t)+u_2C_x(a,t)=h_2(t) \quad (20)
$$

Here, if $u_1=0$, we have the Dirichlet boundary (8). But here, it is considered to be regular in order to be able to apply the finite difference method of numerical approach to the solution. If on the other hand, $u_2=0$, we have the Neumann boundary (13).

To evaluate (17) at $i=1$ and $i=n$ therefore, the boundary values C_0^j and C_{n+1}^j are required. So we develop expressions for these values using a two-point forward difference for the derivative in (19), and a two-point backward difference for (20). This converts the boundary condition constraints into the following difference equations:

$$
(1-u_1)C_0^j-u_1\left(\frac{C_1^j-C_0^j}{\Delta x}\right)=h_1(t_j) \quad (21)
$$

$$
(1-u_2)C_{n+1}^j+u_2\left(\frac{C_{n+1}^j-C_n^j}{\Delta x}\right)=h_2(t_j) \quad (22)
$$

Solving (21) and (22) for the boundary values yield:

$$
C_0^j=\frac{h_1(t_j)\Delta x+u_1C_1^j}{u_1+(1-u_1)\Delta x} \quad (23)
$$

$$
C_{n+1}^j=\frac{h_2(t_2)\Delta x+u_2C_n^j}{u_2+(1-u_2)\Delta x} \quad (24)
$$

The Crank-Nicolson equations can be expressed in vector form by letting $C^j=\left[C_1^j,C_2^j,...,C_n^j\right]^T$ to denotes the solution at time t_j for $0\leq j\leq n$. Suppose that both boundary conditions are Neumann, and setting $n=5$ we have the following implicit linear algebraic systems:

$$\begin{bmatrix} (2-\tau+2\lambda+2k) & (\tau-\lambda) & 0 & 0 & 0 \\ -\lambda & (2-\tau+2\lambda+2k) & 0 & 0 & 0 \\ 0 & -\lambda & (2-\tau+2\lambda+2k) & (\tau-\lambda) & 0 \\ 0 & 0 & -\lambda & (2-\tau+2\lambda+2k) & (\tau-\lambda) \\ 0 & 0 & 0 & -\lambda & (2-\tau+2\lambda+2k) \end{bmatrix} C^{j+1}$$

$$\begin{bmatrix} (2+\tau-2\lambda) & (\lambda-\tau) & 0 & 0 & 0 \\ \lambda & (2+\tau-2\lambda) & (\lambda-\tau) & 0 & 0 \\ 0 & \lambda & (2+\tau-2\lambda) & (\lambda-\tau) & 0 \\ 0 & 0 & \lambda & (2+\tau-2\lambda) & (\lambda-\tau) \\ 0 & 0 & 0 & \lambda & (2+\tau-2\lambda) \end{bmatrix} C^{j} + \begin{bmatrix} \lambda\left(C_0^{j+1}+C_0^{j}\right) \\ 0 \\ 0 \\ 0 \\ (\tau-\lambda)\left(C_{n+1}^{j+1}+C_{n+1}^{j}\right) \end{bmatrix} \tag{25}$$

The stability of the methods is controlled by the dispersion and advection currant number given by:

$$C_{adv} = \frac{V\Delta t}{\Delta x} \qquad and \qquad C_{disp} = \frac{D\Delta t}{\Delta x^2} \tag{26}$$

By implementing the Crank-Nicolson numerical scheme using varying parametric hypothetical values for the decay rate k, the velocity term V and the diffusive term D, and for any time $t \geq 0$, we observe in general, a decrease in the concentration of spilled oil in the system (Figure 1).

CONCLUSION

Under favorable conditions, tele-detection can be used to monitor the affected zones and the extent of the spills in marine environment. However, numerical models are intrinsically capable of predicting the evolution and behavior of the oil spilled at sea. In this work, a mass balanced marine oil spill transport model was used to predict the mass concentration of pollutant in water bodies for any time $t \geq 0$. It was observed that given enough time, the combined actions of weathering and biodegradation can eliminate most of the spilled oil. But the implication is that, leaving the spill for longer time without cleanup effort may wash up the contaminant on beaches or into biologically sensitive tidal areas or estuaries, causing toxic effect to marine biota. Base on the stated result, we make the following recommendations:

- Consideration should be given to long-term monitoring of sensitive habitats even after response to oil spill to assess chronic effects and long-term recovery.

- Higher cleanup efforts should be adopted to lower the probability of pollution exceeding x. That is to say, $F(x \mid c_1) < F(x \mid c_2)$ if $c_2 > c_1$, where $F(x)$ is the cumulative distribution of pollution and c is cleanup effort.

- Research should be conducted in laboratory and wave-tank systems to investigate those parameters that control oil diffusibility.

- Studies should also be conducted to quantify horizontal and vertical diffusivities and the rate of energy dissipation in the field under a variety of sea states that can be used for contingency planning.

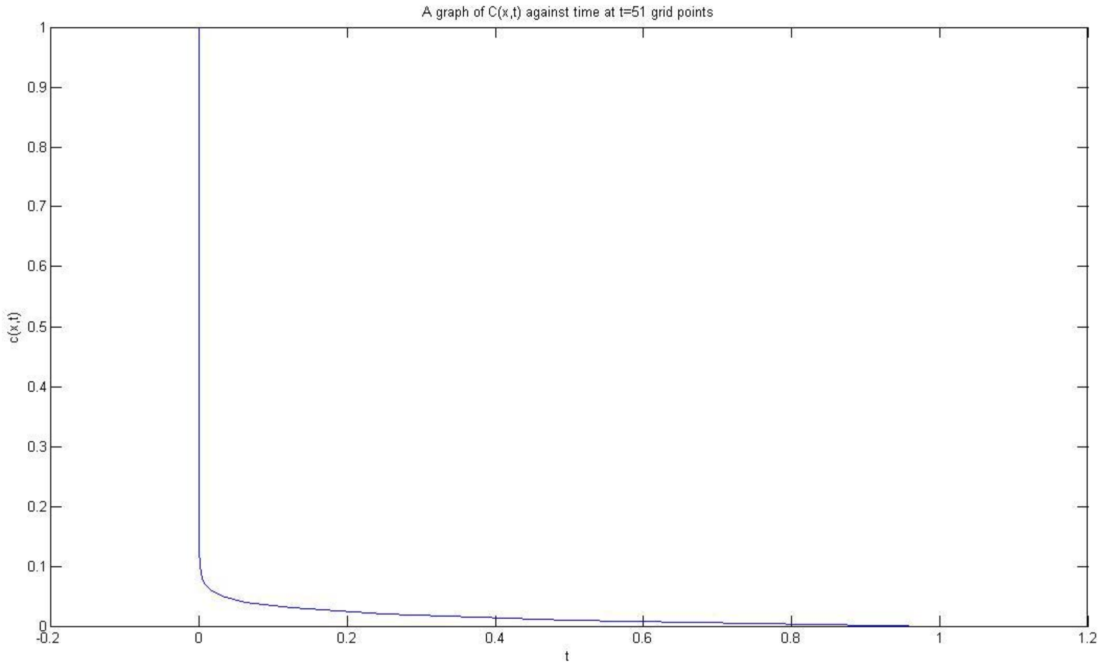

Figure 1: Mass Concentration of Contaminant in Water Bodies.

Consequently, it could be seen that numerical models are powerful tools in environmental impact assessment given that the transport model provides information on contaminant migration including spatial and /or temporal contaminant concentrations and contaminant breakthrough curves at specified terms.

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