# Fuzzy Inference System for Control of Profitability in a Paper Recycling Plant: Mamdani's Design Approach.

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## ABSTRACT

As the demand for materials continues to grow and the supply of natural resources continues to dwindle, recycling of materials has become more important in order to ensure sustainability. Recycling reduces greenhouse gas emissions that may lead to global warming. Recycling also conserves the natural resources on Earth, saves space in the landfills for future generations of people. A sustainable future requires a high degree of recycling. However, Recycling industries face serious economic problems that increase the cost of recycling. This highlights the need of applying fuzzy logic models as one of the best techniques for the effective control of profitability in paper recycling production to ensure profit maximization despite the varying cost of production upon which profits depend.

Fuzzy logic has emerged as a tool to deal with uncertain, imprecise, partial truth, or qualitative decision-making problems to achieve robustness, tractability, and low cost. In order to achieve our objective, a study of a knowledge based system for effective control of profitability in paper recycling is carried out. The Mamdani's Max-Min technique is employed to infer data from the rules developed. This resulted in the establishment of some degrees of influence of input variables on the output. To reinforce the proposed approach, we apply it to a case study performed on the paper recycling industry in Nigeria.

A computer simulation is designed to assist the experimental decision for the best control action. The system is developed using MySQL Database Management System 6.0®, NetBeans 6.5.1®, Java 1.6®, Microsoft Excel 2003®, MatLab® Version 7.6.0.342 (R2007b), Microsoft Windows XP® 2002, and a Pentium IV Personal Computer (PC). The obtained simulation and implementation fuzzy results are investigated, discussed.

(Keywords: industrial efficiency, fuzzy logic, MATLAB, models, profitability)

## INTRODUCTION

Urbanization is one of the most evident global changes worldwide. The rapid and constant growth of urban population has led to a dramatic increase in urban solid waste production, with a crucial socio-economic and environmental impact. However, the growing concern for environmental issues and the need for sustainable development have moved the management of solid waste to the forefront of the public agenda.

Recycling technology has evolved as one of the most useful and environmentally friendly areas of society. All over the world, the need for recycling is heightened by the increasing awareness of product consumers, on the need to maximize the bundle of benefits from the products brought by them.

As the demand for materials continues to grow and the supply of natural resources continues to dwindle, recycling of materials has become more important in order to ensure sustainability. Recycling reduces greenhouse gas emissions that may lead to global warming. Recycling also conserves the natural resources on Earth and saves space in the landfills. A sustainable future requires a high degree of recycling. However, recycling industries face serious economic problems that increase the cost of recycling (Kumaran, 2001).

The profitability of recycling industries is known to be highly dependent on the effective management of resources and management practices, (Craighill and Powell, 1996) (Cunningham, 1969). Taking the paper recycling industry as a case study, the recycling of paper has contributed in no small measure to the conservation of material and consequent low cost of production.

The ultimate resultant effect of the recycling process is high profit generated due to the low cost of production, hence, the link between profitability and recycling industry. Since the ultimate aim of any capitalist industry is to make profit, the concept of profitability is of great significance in science and engineering (http://www.recyclingtoday.com, http://www.recyclinginternational.com).

This highlights the need of applying fuzzy logic models as one of the best techniques for effective control of profitability in paper recycling production to ensure profit maximization despite varying cost of production upon which ultimately profit, in an industry depend.

Fuzzy logic is a powerful technique for solving a wide range of industrial control and information processing applications (Akinyokun, 2002). The fuzzy logic controller has its origin with the E. H. Mamdani (Mamdani, 1977) research, based on theories proposed by L. Zadeh (Zadeh, 1965). It has emerged as a tool to deal with uncertain, imprecise, partial truth or qualitative decision-making problems to achieve robustness, tractability, and low cost.

In order to achieve our objective, a study of a knowledge based system for effective control of profitability in paper recycling is carried out. The Mamdani's Max-Min technique is employed to infer data from the rules developed. This resulted in the establishment of some degrees of influence of input variables on the output. To reinforce the proposed approach, we apply it to a case study performed on paper recycling industry in Nigeria. A computer simulation is designed to assist the experimental decision for the best control action.

The system is developed using MySQL Database Management System 6.0®, NetBeans 6.5.1®, Java 1.6®, Microsoft Excel 2003®, MatLab® Version 7.6.0.342 (R2007b), Microsoft Windows XP® Version 2002, and a Pentium IV Personal Computer (PC). The obtained simulation and implementation fuzzy results are investigated, and discussed.

Wongthatsanekorn (2009) developed a goal programming model for plastic recycling system in Thailand. Kufman (2004) carried out the analysis of technology and infrastructure of paper recycling. Kumar et al. (2008), designed a goal programming model for paper recycling to assist proper management of the paper recycling logistic system, while Udoakpan (2002) carried out the financial implication of establishing a paper recycling plant in Nigeria.

Oke et al. (2006) designed a fuzzy logic model to handle the profitability concept in a plastic industry. Oke et al. (2006) developed a Neuro-Fuzzy linguistic approach in optimizing the flow rate of a Plastic Extruder process. Kumar (2007) designed an optimal blending model for paper manufacturing with competing input materials Misrawati Misman et al. (2008) studied state-of-the-art processes for paper recycling.

This paper outlines the mathematical model of the system. Later sections discuss the research methodology, the model experiment, the results of findings, and finally, some recommendations and our conclusion.

## MATHEMATICAL MODEL

The mathematical model of the proposed work is based on the major components in the concept of profitability using a case in the paper recycling industry. These components include; Selling Price (SP), Cost Price (CP), Quantity Recycled (QR), and Profit (Y). The relationship among these components as used in the concept of profitability is illustrated in Figure 1.

The relationships among components of profitability in Figure 1 show Selling Price (SP) which is the selling price per item of the quantity of recycled product. Cost Price (CP) which is the cost price per item of the quantity of recycled product. The cost price is made up of all expenses incurred directly during the recycling production processes.

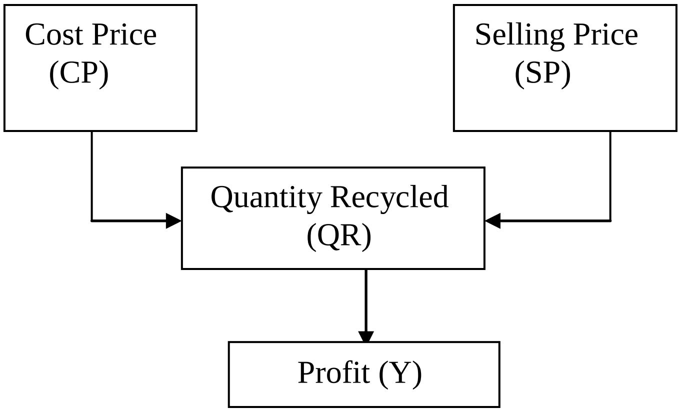


**Figure 1:** Relationships among Components of Profitability (adopted from Oke et al., 2006).

While Quantity Recycled (QR) is the quantity of recycled product which is the output from wastepaper input and it is determined by the conventional control model, based on the following parameters of the paper recycling process:

PM4 = Machine Speed (m/m)

T = Period between the beginning of recycling and the expected completion time.

SUBW = Width of the substance (m)

SUBP = Paper substance or grammage ($g/m^2$)

STRNG = Strange (%) which is the difference between the machine speed and the reelers speed

Thus,

$$QR = PM4\ (m/m) \times T\ (m) \times SUBW\ (m) \times SUBP\ (g/m^2) \times STRNG\ (\%) \quad \textbf{(1)}$$

Profit (Y) is the profit made and is the difference in Selling Price (SP) and Cost Price (CP) multiplied by the quantity of recycled product (QR). This is given as:

$$Y = SP\ (QR) - CP\ (QR) \quad \textbf{(2)}$$

Thus,

$$Y = [(SP - CP)\ (PM4\ (m/m) \times T\ (m) \times SUBW\ (m) \times SUBP\ (g/m^2) \times STRNG\ (\%))] \quad \textbf{(3)}$$

## RESEARCH METHODOLOGY

The fuzzy inference system for effective control of profitability in paper recycling is shown in Figure 2. This system involves three main processes; fuzzification, inference, and defuzzification.

The knowledge base contains the following:

(i) rule-base - that contains knowledge used to characterize Fuzzy Control Rules and Fuzzy Data Manipulation in an FLC, which are defined based on experience and engineering judgment of an expert. In this case, an appropriate choice of the membership functions of a fuzzy set plays a crucial role in the success of an application. The rules are in the form of IF – THEN (production rules).

(ii) data-base: Fuzzy variables are defined by fuzzy sets, which in turn are defined by membership functions. The knowledge base design of profitability control in paper recycling production is made up of both static and dynamic information about the decision variables and about the different factors that influence recycling decision for controlling paper recycling production for profit optimization. There are qualitative and quantitative variables which must be fuzzified, inferred, and defuzzified.

Fuzzification of data is carried out on the transformed data by selecting input parameters into the horizontal axis and projecting vertically to the upper boundary of membership function to determine the degree of membership. This is then used to map the output value specified in the individual rules to an intermediate output measuring fuzzy sets. Parameters used in fuzzy logic model are, Cost Price (CP), Selling Price (SP), and Quantity of Recycled product (QR). These parameters constitute the fuzzy logic input variables used to generate the fuzzy logic model, while the linguistic variable for the model is (SP)(QR) – (CP)(QR) which is the difference between the selling price and cost price of the quantity recycled.

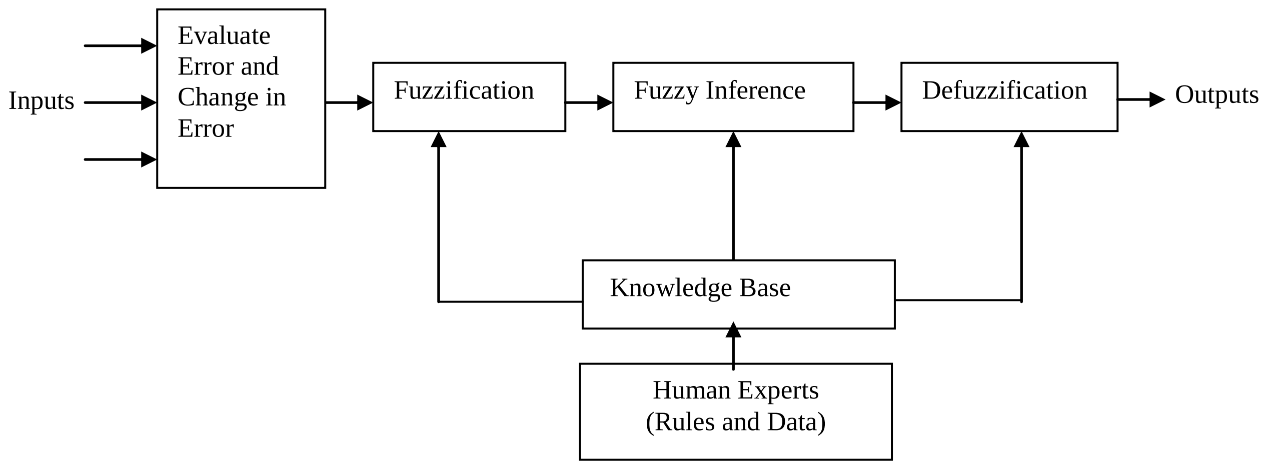


**Figure 2:** Empirical Fuzzy Logic Model for Control of Profitability in Paper Recycling Production.

The following error and change in error terms of the model as defined by (Oke et al., 2006) and are modified and evaluated thus:

Error (E) = (SP – CP)(QR)

Change inn error (CE) = ($\Delta$SP - $\Delta$CP)(QR)
= d/dt Error

Error equals selling price of the quantity recycled minus cost price of the quantity recycled. Change in error equals differentiating selling price of quantity recycled minus cost price of quantity recycled over time.

**Error Terms**

(SP)(QR) – (CP)(QR) = ZE, "Zero-error" term(ZE) (No profit no loss)

($S_P$)($Q_R$) – ($C_P$)($Q_R$) = NE, "Negative-error" term (NE) (Loss)

($S_P$)($Q_R$) – ($C_P$)($Q_R$) = PO, "Positive-error" term (PO) (Profit)

If we consider the model over a period of time, we have:

**Change in Error Terms**

d{(SP)(QR)-(CP)(QR)}/dt = Z, "Zero error-change" (ZE) (No profit no loss over time)

d{(SP)(QR)-(CP)(QR)}/dt = N, "Negative error-change" (NE) (Loss over time)

d{(SP)(QR)-(CP)(QR)}/dt = P, "Positive error-change" (PO) (Profit over time)

We employ the characteristic fuzziness of the model by generating more "error" and "change in error terms by considering the model as changing or varying to large degree over time in order to achieve more effective control of the fuzzy logic model.

**More Error Terms**

(SP)(QR) - (CP)(QR) = << N, "Negative Small error" (NS) (Low Loss)
(SP)(QR) - (CP)(QR) = >>P, "Positive Small error" (PS) (Low Profit)

(SP)(QR) - (CP)(QR) = <<<< N, "Negative Big error" (NB) (High Loss)

(SP)(QR) - (CP)(QR) = >>>> P, " Positive Big error" (PB) (High Profit)

**More Change in Error Terms**

d{(SP)(QR)-(CP)(QP)}/dt = << N, "Negative Small error-change" (NS) (Low loss over time)

d{(SP)(QR)-(CP)(QP)}/dt = >> P, "Positive Small error-change" (PS) (Low profit over time)

d{(SP)(QR)-(CP)(QR)}/dt = <<<< N, "Negative Big error-change" (NB) (High Loss over time) d{(SP)(QR)-(CP)(QR)}/dt = >>>> P, "Positive Big error-change" (PB) (High Profit over time)

In this paper, the universes of discourse for error (*E),change in error (CE)* and *Output* are chosen to be [-100, 100], [-100, 100], and [-100, 100] respectively. Both sets of the linguistic values for the linguistic variables *E* and C*E* are {NB, NS, N, Z, P, PS, PB}, and the set of linguistic values for Output is {HL, LL, L, NPNL, P, LP, HP}, where NB, NS, N, Z, P, PS, and PB represent *negative_big*, *negative_small*, *negative, zero*, *positive*, *positive_small*, and *positive_big*, respectively. The output is defined by fuzzy sets, *high_loss, low_loss, loss, no_profit_no_loss, profit, low_profit and high_profit.*

The linguistic expression E and CE variables and their membership functions are evaluated using triangular membership function as presented in Equations (5) to (11). Triangular curves depend on three parameters $a_1$, $a_2$, and $a_3$ and are given by Equation (4), $a_2$ defines the triangular peak location, while $a_1$ and $a_3$ define the triangular end points. During the process linguistic labels (values) are assigned to the error and change in error indicating the associated degree of influence of membership for each linguistic term that applies to that input variable.

Degrees of membership ($U_x$) are assigned to each linguistic value as expressed in Equations (5) to (11) negative small, negative, zero, positive, positive small and positive big.

$$\mu(x) = \begin{cases} 0 & \text{if } x < a_1 \\ x - a_1/a_2 - a_1 & \text{if } a_1 <= x < a_2 \\ a_3 - x/\ a_3 - a_2 & \text{if } a_2 <= x < a_3 \\ 0 & \text{if } x > a_3 \end{cases} \quad \textbf{(4)}$$

$$\text{Error}(x) = \begin{cases} 0 & & \text{"NB"} \\ (x+63)/34 & \text{if } -67 <= x < -33 & \text{"NS"} \\ -x/33 & \text{if } -33 <= x < 0 & \text{"NE"} \\ (33-x)/66 & \text{if } 0 <= x < 33 & \text{"ZE"} \\ (67-x)/34 & \text{If } 33 <= x < 67 & \text{"PO"} \\ (100-x)/33 & \text{If } 67 <= x < 100 & \text{"PS"} \\ 0 & \text{if } x >= 100 & \text{"PB"} \end{cases} \quad \textbf{(5)}$$

$$\mu_{NS}(x) \begin{cases} 0 & \text{if } x <= -100 \\ (x+100)/33 & \text{if } -100 <= x < -67 \\ (-33+x)/34 & \text{if } -67 <= x < -33 \\ 0 & \text{if } x >= -33 \end{cases} \quad \textbf{(6)}$$

$$\mu_{NE}(x) = \begin{cases} 0 & \text{if } x < -67 \\ (x+67)/34 & \text{if } -67 <= x < -33 \\ -x/33 & \text{if } -33 <= x < 0 \\ 0 & \text{if } x >= 0 \end{cases} \quad \textbf{(7)}$$

$$\mu_{ZE}(x) = \begin{cases} 0 & \text{if } x < -33 \\ (x+33)/33 & \text{if } -33 <= x < 0 \\ (33-x)/33 & \text{if } 0 <= x < 33 \\ 0 & \text{if } x > 33 \end{cases} \quad \textbf{(8)}$$

$$\mu_{PO}(x) = \begin{cases} 0 & \text{if } x <= 0 \\ x/33 & \text{if } 0 <= x < 33 \\ (67-x)/34 & \text{if } 33 <= x < 67 \\ 0 & \text{If } x > 67 \end{cases} \quad \textbf{(9)}$$

$$\mu_{PS}(x) = \begin{cases} 0 & x <= 33 \\ (x-33)/34 & \text{if } 33 <= x < 67 \\ (100-x)/33 & \text{if } 67 <= x < 100 \\ 0 & \text{if } x >= 100 \end{cases} \quad \textbf{(10)}$$

$$\mu_{PB}(x) = \begin{cases} 0 & \text{if } x < 67 \\ (x-67)/33 & \text{if } 67 <= x < 100 \\ 1 & \text{if } x >= 100 \end{cases} \quad \textbf{(11)}$$

$$\mu_{HLoss}(x) = \begin{cases} 0 & \text{if } x <= -100 \\ (x+100)/25 & \text{if } -100 <= x < -75 \\ (-50-x)/25 & \text{if } -75 <= x < -50 \\ 0 & \text{if } x >= -50 \end{cases} \quad \textbf{(12)}$$

$$\mu_{LLoss}(x) = \begin{cases} 0 & \text{if } x < -75 \\ (x+75)/25 & \text{if } -75 <= x < -50 \\ (-25-x)/25 & \text{if } -50 <= x < 25 \\ 0 & \text{if } x >= 25 \end{cases} \quad \textbf{(13)}$$

$$\mu_{Loss}(x) = \begin{cases} 0 & \text{if } x < -50 \\ (x+50)/25 & \text{if } -50 <= x < -25 \\ -x/25 & \text{if } -25 <= x < 0 \\ 0 & \text{if } x > 0 \end{cases} \quad \textbf{(14)}$$

$$\mu_{NPNL}(x) = \begin{cases} 0 & \text{if } x <= -25 \\ (x+25)/25 & \text{if } -25 <= x < 0 \\ (25-x)/25 & \text{if } 0 <= x < 25 \\ 0 & \text{If } x > 25 \end{cases} \quad \textbf{(15)}$$

$$\mu_{Profit}(x) = \begin{cases} 0 & \text{if } x <= 0 \\ x/25 & \text{if } 0 <= x < 25 \\ (50-x)/25 & \text{if } 25 <= x < 50 \\ 0 & \text{if } x >= 50 \end{cases} \quad \textbf{(16)}$$

$$\mu_{LProfit}(x) = \begin{cases} 0 & \text{if } x < 25 \\ (x-25)/25 & \text{if } 25 <= x < 50 \\ (25-x)/25 & \text{if } 50 <= x < 75 \\ 0 & \text{if } x > 75 \end{cases} \quad \textbf{(17)}$$

$$\mu_{HProfit}(x) = \begin{cases} 0 & \text{if } x < 50 \\ (x-50)/25 & \text{if } 50 <= x < 75 \\ (100-x)/25 & \text{if } 75 <= x < 100 \\ 0 & \text{if } x > 100 \end{cases} \quad \textbf{(18)}$$

Linguistic values are assigned to the linguistic variable, error ((SP)(QR) – (CP)(QR)) of profitability as shown in Equation (5). In Equations (6) to (11), each linguistic value is assigned a label emphasizing the degree of the value assigned in (1). For example, Equation (6) evaluates the degree of positive small of the error and change in error, if the value of error is for instance, 55, the degree of influence will evaluate to 0.65 (65%) severity, whereas, 75 evaluates to 0.75 (75%). Fuzzy logic toolbox in Matlab 2007 is employed in this project to model the design. Graphical users interface (GUI) tools are provided by fuzzy logic box (Fuzzy Logic Toolbox Users' guide, 2007). The graphical formats which show the fuzzy membership curves for error, change in error and the output are depicted in Figures 3, 4, and 5 respectively, where triangular membership functions are used to describe the variables.

Each linguistic value of fuzzy output membership function in Figure 5 is assigned a label emphasizing the degree of the value assigned as in Equations 12-.18

Using derivation based on expert experience and control engineering knowledge; the experience of an expert who has been working at the Star Paper Mill located in Aba, Nigeria for over 18 years was used to obtain the rule base. The expert also assisted in defining the fuzzy rules and the fuzzy set. There are 2 inputs in the knowledge base namely; error and change in error, with 7 fuzzy sets each as antecedent parameters and 7 fuzzy sets each as consequent parameters. From the expert knowledge, these are used to generate 49 rules for the rule base defined for the decision-making unit. Some of the rules are presented in Table 1. The Rule matrix for the fuzzy control rules is shown in Table 2.

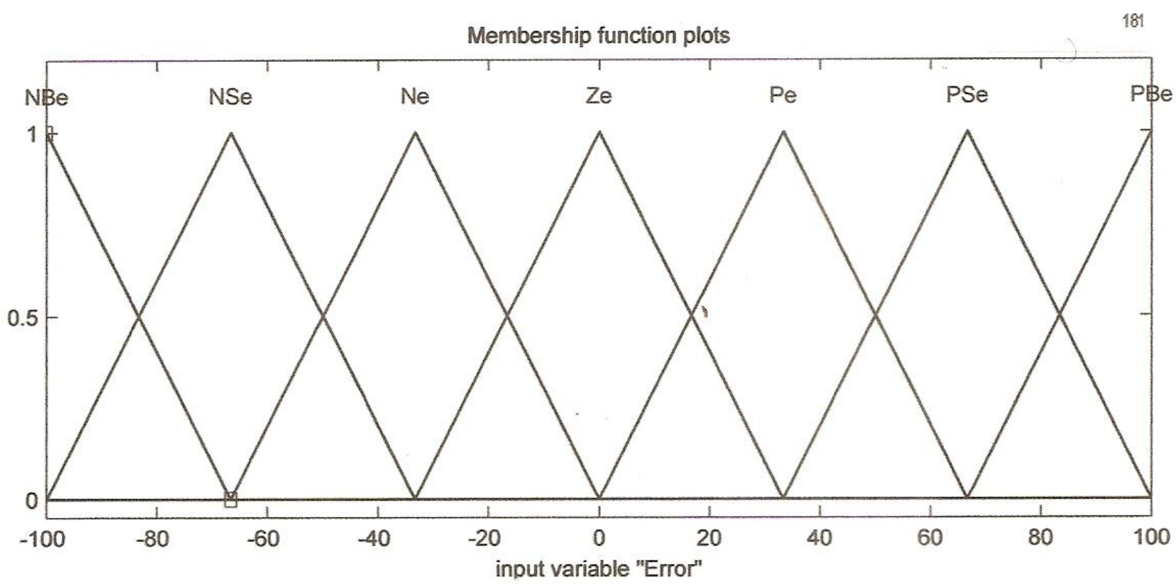


**Figure 3:** Membership Function for Error.

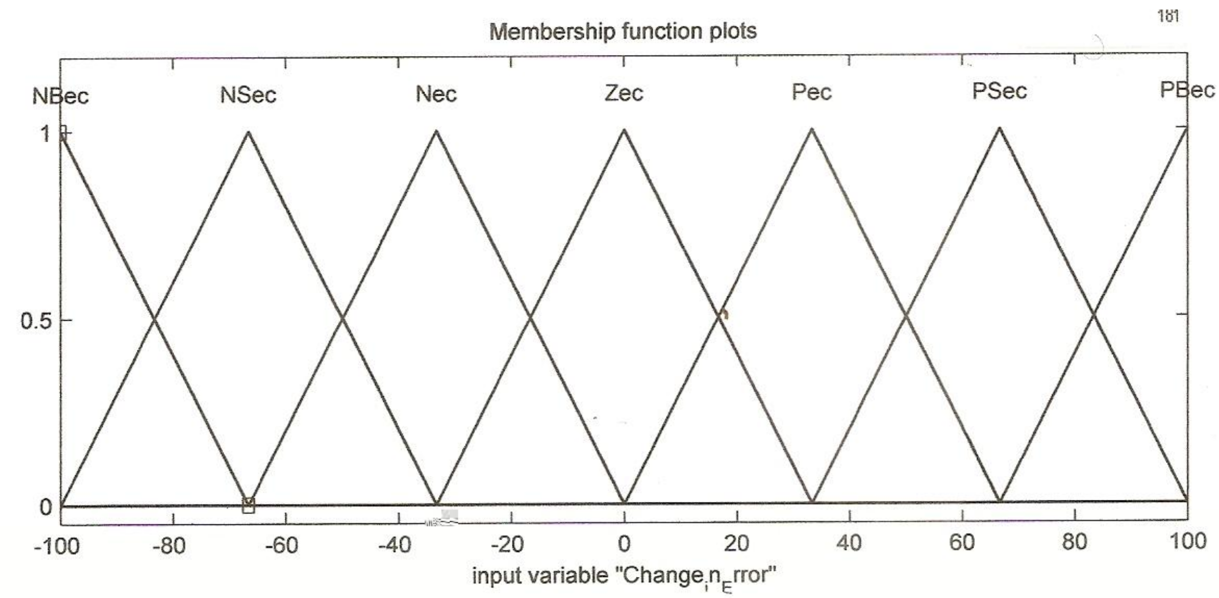


**Figure 4:** Membership Function for Change in Error.

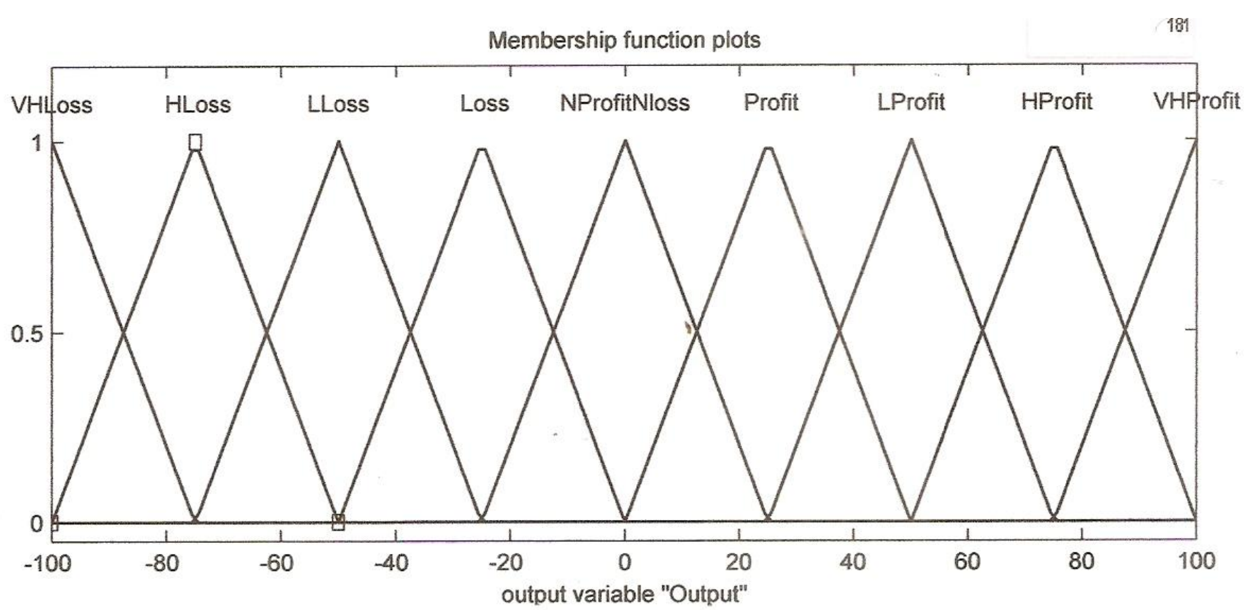


**Figure 5:** Membership Function for Output.

**Table 1:** Fuzzy Rule Base.

| Rule No. | Rules |
|---|---|
| 1. | IF $(S_P)(Q_R)$-$(C_P)(Q_R)$ = NB AND $d[(S_P)(Q_R)$- $(C_P)(Q_R)]/dt$ = NB THEN Output = HighLoss |
| 4. | IF $(S_P)(Q_R)$-$(C_P)(Q_R)$ = NB AND $d[(S_P)(Q_R)$- $(C_P)(Q_R)]/dt$ = ZE THEN Output = NProfitNLoss |
| 8. | IF $(S_P)(Q_R)$-$(C_P)(Q_R)$ = NS AND $d[(S_P)(Q_R)$- $(C_P)(Q_R)]/dt$ = NB THEN Output = HighLoss |
| 14. | IF $(S_P)(Q_R)$-$(C_P)(Q_R)$ = NS AND $d[(S_P)(Q_R)$- $(C_P)(Q_R)]/dt$ = PB THEN Output = Profit |
| 18. | IF $(S_P)(Q_R)$-$(C_P)(Q_R)$ = NE AND $d[(S_P)(Q_R)$- $(C_P)(Q_R)]/dt$ = ZE THEN Output = NProfitNLoss |
| 19. | IF $(S_P)(Q_R)$-$(C_P)(Q_R)$ = NE AND $d[(S_P)(Q_R)$- $(C_P)(Q_R)]/dt$ = PO THEN Output = LowProfit |
| 23. | IF $(S_P)(Q_R)$-$(C_P)(Q_R)$ = ZE AND $d[(S_P)(Q_R)$- $(C_P)(Q_R)]/dt$ = NS THEN Output = LowLoss |
| 24. | IF $(S_P)(Q_R)$-$(C_P)(Q_R)$ = ZE AND $d[(S_P)(Q_R)$- $(C_P)(Q_R)]/dt$ = NE THEN Output = Loss |
| 28. | IF $(S_P)(Q_R)$-$(C_P)(Q_R)$ = ZE AND $d[(S_P)(Q_R)$- $(C_P)(Q_R)]/dt$ = PB THEN Output = HighProfit |
| 32. | IF $(S_P)(Q_R)$-$(C_P)(Q_R)$ = PO AND $d[(S_P)(Q_R)$- $(C_P)(Q_R)]/dt$ = ZE THEN Output = Profit |
| 33. | IF $(S_P)(Q_R)$-$(C_P)(Q_R)$ = ZE AND $d[(S_P)(Q_R)$- $(C_P)(Q_R)]/dt$ = PB THEN Output = HighProfit |
| 41. | IF $(S_P)(Q_R)$-$(C_P)(Q_R)$ = PS AND $d[(S_P)(Q_R)$- $(C_P)(Q_R)]/dt$ = PS THEN Output = LowProfit |
| 42. | IF $(S_P)(Q_R)$-$(C_P)(Q_R)$ = PS AND $d[(S_P)(Q_R)$- $(C_P)(Q_R)]/dt$ = PB THEN Output = HighProfit |
| 45. | IF $(S_P)(Q_R)$-$(C_P)(Q_R)$ = PB AND $d[(S_P)(Q_R)$- $(C_P)(Q_R)]/dt$ = NE THEN Output = LowProfit |
| 48. | IF $(S_P)(Q_R)$-$(C_P)(Q_R)$ = PB AND $d[(S_P)(Q_R)$- $(C_P)(Q_R)]/dt$ = PS THEN Output = Profit |
| 49. | IF $(S_P)(Q_R)$-$(C_P)(Q_R)$ = PB AND $d[(S_P)(Q_R)$- $(C_P)(Q_R)]/dt$ = PB THEN Output = HighProfit |

**Table 2:** Fuzzy Control Rules Matrix.

| E | NB | NS | N | Z | P | PS | PB |
|---|---|---|---|---|---|---|---|
| NB | HL | HL | HL | NPNL | LP | LL | P |
| NS | HL | LL | L | NPNL | LL | NPNL | P |
| N | HL | LL | L | NPNL | LL | P | LP |
| Z | HL | LL | L | NPNL | LP | LP | HP |
| P | LL | L | NPNL | P | P | LP | HP |
| PS | L | LP | P | LP | LP | LP | HP |
| PB | NPNL | P | LP | LP | HP | P | HP |

## Fuzzy Inference

The process of drawing conclusions from existing data is called inference. For each rule, the inference mechanism looks up the membership values in the condition of the rule. Fuzzy inputs are taken to determine the degree to which they belong to each of the appropriate fuzzy sets via membership functions. The aggregation operation is used to calculate the degree of fulfillment or firing strength, $\alpha n$ of the condition of a rule n. A rule, say rule 1, will generate a fuzzy membership value $\mu E1$ coming from the error and a membership value $\mu EC1$ coming from the change in error measurement. $\mu E1$ and $\mu EC1$ are combined by applying fuzzy logical AND to evaluate the composite firing strength of the rule.

The rules use the input membership values as weighting factors to determine their influence on the fuzzy output sets of the final output conclusion. The degrees of truths (R) of the rules are determined for each rule by evaluating the nonzero minimum values using the AND operator. Only the rules that get strength higher than 0, would "fire" the output. The Mamdani max-min inference engine is evaluated to obtain the individual rule outputs as;

$$C'i(w) = (\alpha i1 \wedge Ci1(w)), (\alpha i2 \wedge Ci2(w)), \ldots\ldots(\alpha in \wedge Cin(w)) \qquad \textbf{(19a)}$$

where $Ci'(w)$ is the individual rule's consequence.

The overall system output is computed by aggregating the individual rule outputs from all the rules using OR operator as;

$$C(w) = C'1\ (w) \vee C'2(w) \vee C'2(w) \vee \ldots.. \vee C'n(w) \qquad \textbf{(19b)}$$

## Defuzzification

Defuzzification of data into a crisp output is a process of selecting a representative element from the fuzzy output inferred from the fuzzy control algorithm. A fuzzy inference system maps an input vector to a crisp output value. In order to obtain a crisp output, we need a defuzzification process. The input to the defuzzification process is a fuzzy set (the aggregated output fuzzy set), and the output of the defuzzification process is a single number.

Many defuzzification techniques are proposed and four common defuzzification methods are center-of-area (gravity), center-of-sums, max-criterion and mean of maxima. According to (Obot, 2008), max-criterion produces the point at which the possibility distribution of the action reaches a maximum value and it is the simplest to implement. The center of area (gravity) is the most widely used technique because, when it is used, the defuzzified values tend to move smoothly around the output fuzzy region, thus giving a more accurate representation of fuzzy set of any shape (Cochran and Chen, 2005). The technique is unique, however, and not easy to implement computationally.

Center of gravity (CoG) often uses discretized variables so that CoG, y′ can be approximated to overcome its disadvantage which uses weighted average of the centers of the fuzzy set instead of integration. This approach according to Jadarat and Al-Nimr, (2009) is adopted and modified in this research because it is computationally simple and intuitively plausible. The Crisp is obtained by finding the mathematical mean of all the highest values of the aggregate rules output. This is used to map the fuzzy rules output to a crisp point as shown in Equation 20.

$$\text{Crisp Output} = \frac{\sum_{X \in P} X}{|P|} \qquad \textbf{(20)}$$

where, P is the set of output points with highest values.

## MODEL EXPERIMENT

The study adopts Matlab®/Simulink® and its Fuzzy Logic tool box functions to develop a computer simulation showing the user interface and fuzzy inference to assist the experimental decision for the best control action. Results of evaluation of fuzzy rule base inference for two ranges of inputs, Error ((SP)(QR) – (CP)(QR)) and Change in error (d{(SP)(QR) – (CP)(QR)}/dt) are shown in Tables 3 and Table 4, respectively.

**Table 3:** Rule Base Evaluation for Error and Change in Error at -18 and +25.

| Rule No. | Premise Variables | | Conclusion Part of Rule | Minimum value (non zero) |
|---|---|---|---|---|
| | Error | Change in error | | |
| 19 | 0.5 | 0.75 | LowProfit | 0.5 |
| 18 | 0.5 | 0.25 | NoProfitNoLoss | 0.25 |
| 26 | 0.5 | 0.75 | LowProfit | 0.5 |
| 25 | 0.5 | 0.25 | NoProfitNoLoss | 0.25 |

For example, if Rules 18, 19, 25, and 26 fire from the rule base presented in this paper when error and change in error values are selected to be -18 and +25 and their corresponding degrees of membership are NE = 0.5 ZE = 0.5 PO = 0.0 for error and ZE = 0.25 P0 = 0.75 NE = 0.0 for change in error. The MAX-PRODUCT inference for Low Profit (LP) and NoprofitNoloss (NPNL) membership function is calculated as:

For LP, $\alpha_{19} = 0.5;\ C_{19}(w) = 0.5$

$C'_{19}(w) = (0.5 \wedge 0.5) = 0.5$

$\alpha_{26} = 0.5;\ C_{26}(w) = 0.5$

$C'_{26}(w) = (0.5 \wedge 0.5) = 0.5$ **(21)**

For NPNL, $\alpha_{18} = 0.25;\ C_{18}(w) = 0.25$

$C'_{18}(w) = (0.25 \wedge 0.25) = 0.25$

$\alpha_{25} = 0.25;\ C_{25}(w) = 0.25$

$C'_{25}(w) = (0.25 \wedge 0.25) = 0.25$

$C(w) = C'_{19} \vee C'_{26}(w) \vee C'_{18} \vee C'_{25}(w)$
$= (0.5 \vee 0.5 \vee 0.25 \vee 0.25)$
$= 0.5$ **(22)**

This is then defuzzified to obtain the crisp output for the above range.

Crisp output = 0.35 (35% NPNL) **(23)**

These particular input conditions indicate positive value of 35% (35% NPNL) therefore no profit no loss is expected with 35% possibility and required system response.

**Table 4:** Rule Base Evaluation for Error and Change in Error at +90 and +75.

| Rule No. | Premise Variables: Error | Premise Variables: Change in error | Conclusion Part of rule | Minimum value (non zero) |
|---|---|---|---|---|
| 41 | 0.35 | 0.75 | LowProfit | 0.35 |
| 42 | 0.35 | 0.25 | HighProfit | 0.25 |
| 48 | 0.65 | 0.75 | Profit | 0.65 |
| 49 | 0.65 | 0.25 | HighProfit | 0.25 |

For example, if Rules 41, 42, 48 and 49 fire from the rule base presented in this paper when error and change in error values are selected to be +90 and +75 and their corresponding degrees of membership are PS = 0.35 PB = 0.65 PO = 0.0 for error and PS = 0.75 PB = 0.25 PO = 0.0 for change in error. The MAX-PRODUCT inference for Profit (P), Low Profit (LP) and High Profit (P) membership function is calculated as:

For P, ` $\alpha_{48} = 0.65$; $C_{48}(w) = 0.65$
$C'_{48}(w) = (0.65 \wedge 0.65) = 0.65$

For LP, $\alpha_{41} = 0.35$; $C_{41}(w) = 0.35$
$C'_{41}(w) = (0.35 \wedge 0.35) = 0.35$ **(21)**

For HP, $\alpha_{42} = 0.25$; $C_{42}(w) = 0.25$
$C'_{42}(w) = (0.25 \wedge 0.25) = 0.25$

$\alpha_{49} = 0.25$; $C_{49}(w) = 0.25$
$C'_{49}(w) = (0.25 \wedge 0.25) = 0.25$

$C(w) = C'_{41} \vee C'_{42}(w) \vee C'_{48} \vee C'_{49}(w)$
$= (0.35 \vee 0.25 \vee 0.65 \vee 0.25)$
$= 0.65$ **(22)**

This is then defuzzified to obtain the crisp output for the above range.

Crisp output = 0.43 (43% P) **(23)**

These particular input conditions indicate positive value of 0.43 (43% Profit) therefore profit is expected with 43% possibility and required system response.

The centers of the triangles representing the NB, NS, N, P, PS and PB membership functions for the two inputs are manipulated so as to achieve the desired result.

The values of the errors and change in errors indicated as in Tables 3 and 4 are inserted into the rule base under the view rule editor and the outputs computed for all the cases are recorded. The inference mechanism of fuzzy sets in our example 2 is shown in Figure 6 (generated in the Matlab Fuzzy Logic Toolbox).

**Table 5:** Comparison of MAX-MIN and RSS inference engines

| E | CE | MAX-MIN | | | | | | | ROOS SUM SQUARE | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | HL | LL | L | NPNL | P | LP | HP | HL | LL | L | NPNL | P | LP | HP |
| -100 | -86 | 0.5 | - | - | - | - | - | - | 0.701 | - | - | - | - | - | - |
| -86 | -55 | 0.5 | 0.5 | 0.5 | | | | | 0.591 | 0.5 | 0.3 | - | - | - | - |
| -60 | -30 | - | - | 0.8 | 0.5 | - | - | - | - | - | 0.943 | 0.447 | - | - | - |
| +18 | +18 | - | - | - | 0.5 | 0.5 | 0.5 | - | - | - | - | 0.5 | 0.707 | 0.5 | - |
| +75 | +75 | - | - | - | - | 0.75 | 0.75 | 0.25 | - | - | - | - | 0.75 | 0.75 | 0.35 |

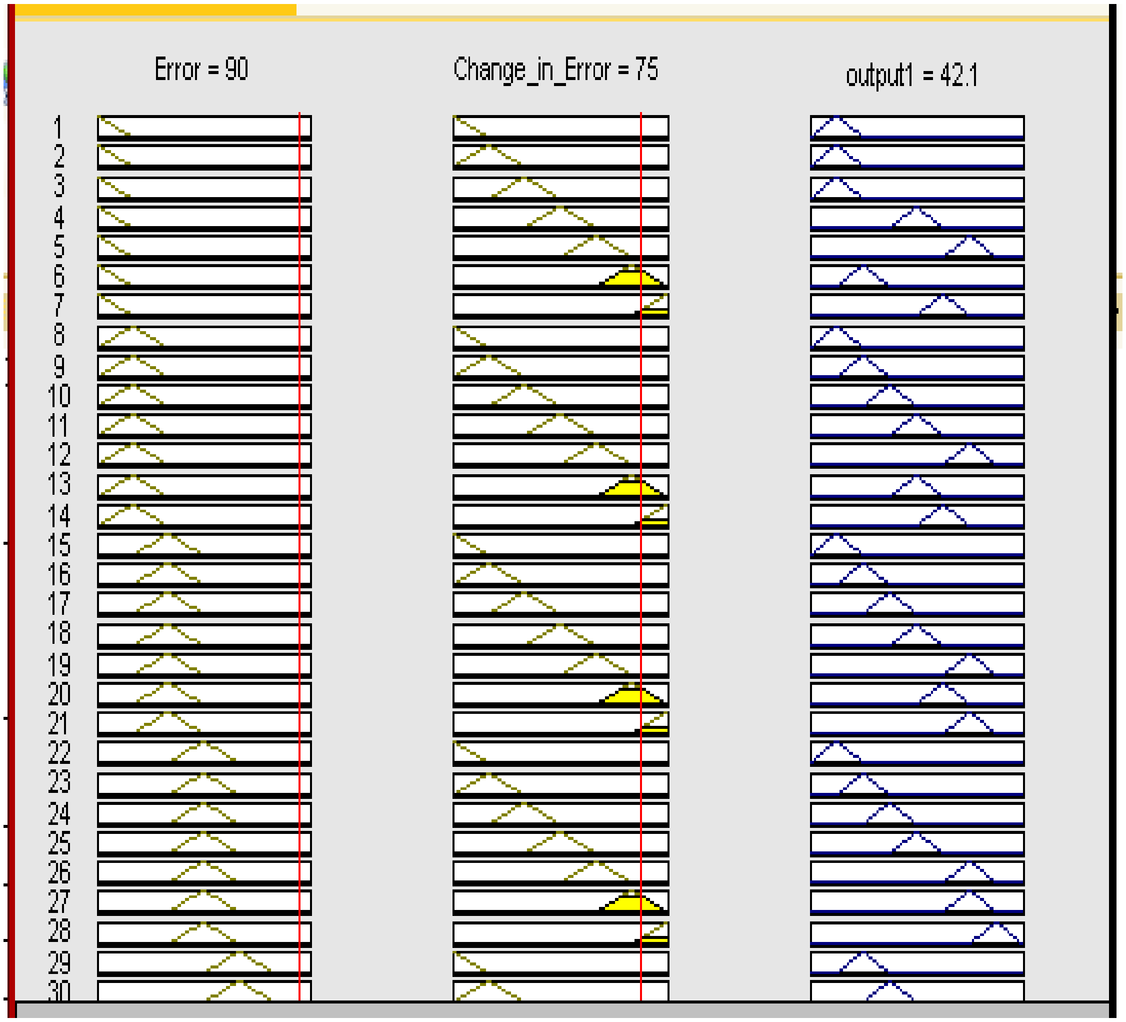


**Figure 6:** Graphical Construction of the Inference Mechanism of Fuzzy Sets in Example 2.

## RESULT AND DISCUSSION

In fuzzy logic implementation, the selection of membership functions and rule base determine the output. Hence, by selecting a triangular membership function, the variables in the system are manipulated and represented judiciously. Also, the rule base is selected from the experience of system expert. Fuzzy logic represents partial "truth" or partial "false" in its modeling. From the study, apart from assigning linguistic variables such as low-loss, no-profit-no-loss, low-profit, etc to the profitability, the degree of influence or severity of each linguistic variable is evaluated.

Tables 3 and 4 show fuzzy logic model of the variables, error (E) and change in error (CE) in order to remove uncertainty, ambiguity and vagueness. The crisp outputs in the two examples cited in this work show the linguistic label and degree of influence on profitability. From the crisp outputs obtained from the graph of fuzzy logic model in figure 6, it is observed that these particular input conditions indicate positive values of 43% (43% Profit). This implies that the selling price of the recycled quantity is more than the cost price of the recycled quantity; therefore profit is expected with 43% possibility.

Considering the degree of relationship between linguistic label and value of fuzzy output membership function, say "Loss", when its value equals 1.0, it indicates that the cost price of the recycled quantity is more than the selling price of the recycled quantity and that industry will run at a loss with 100% possibility. When the fuzzy output value is 0.6, it indicates 60% possibility of loss.

Considering the relationships strength among fuzzy outputs in Figure 5, it indicates that only when "no-profit-no-loss" output value equals 1.0, (100%) that we can conclude that the selling price of the quantity recycled is indeed equal to the cost price and the industry is likely to run at no profit no loss. Relating "no-profit-no-loss' with "profit" for instance, when the value of "no-profit-loss" output is 0.4 showing possibility 40%, it indicates that there is 0.6 (60%) possibility of profit. This implies that it is not likely that the industry will run at no profit no loss altogether when the selling price is only less than or equal to the cost price by 40%. Relating "no-profit-no-loss" with 'loss" with the relationship strength of 0.5 (50%), it shows no profit no loss with 0.5 (50%) possibility and 0.5 (50%) of loss in this case.

Comparing Mamdani's with root sum square (RSS) method of inference, the result indicates that Mamdani's technique is computational simple and is intuitively appealing for many practical problems, especially for fuzzy control. However, a disadvantage of this method is that if at some x Î U the $m_{E_i}^{l}$ $(x_i)$'s are very small the C¢(w) obtained in this method will be very small and this may cause problems in implementation.

From the comparison in Table 5, we have the following observations: (i) If the membership value of the IF part at point x* is small (say, $m_p(x^*_p) < 0.5$), then the Mamdani's inference engines give very small membership values, whereas root sum square gives very large membership values. (ii) RSS inference engine gives largest output membership function, while the minimum inference engine gives a smaller membership values.

Several responses can be observed during the simulation of the system. The system is tuned by modifying the rules and membership functions until the desired system response (output) is achieved. The system can be interfaced to the real world via Java programming language.

## CONCLUSION

It is important to make evident the great potential that fuzzy logic has to offer, such as the need for the mathematical model. Fuzzy Logic Controllers can provide more effective control of non-linear systems than linear controllers, as there is more flexibility in designing the mapping from the input to the output space. Fuzzy logic is capable of resolving conflicts by collaboration, propagation and aggregation and can mimic humanlike reasoning. Another advantage that the fuzzy logic offers is that an autotuning algorithm can be applied to the system, by the means of this reasoning. In this way, the system can learn the control parameters to take.

In our study, we represent the mathematical expression profitability components using linguistic variables. We consider "error" and "change in error" approaches in a vague, ambiguous and uncertain situation. It is shown that fuzzy logic is able to represent common sense knowledge and address the issue of vagueness, ambiguity and uncertainty (Obot, 2008) as it is used to find the exact degree of profit, loss, low-profit, etc in the profitability of an industry. To this end, fuzzy logic can be used to control and ensure the desired output in a model since it can tolerate wide variation in input variables. Fuzzy logic control model shows that profit can be achieved at various levels, but maximum profit is achieved when the selling price is more than the cost price by 100% (1.0). Also loss can be incurred at different levels when the production cost is more than the selling price. The exact level and exact loss or profit has been clearly defined by fuzzy logic control system thereby resolving the conflict of uncertainty and vagueness.

Our case study reveals that production cost depends on waste paper cost, the parameters of paper recycling process and other costs associated with paper recycling. Therefore it is recommended that the parameters affecting production cost, and determine the output of paper recycling be modified so as to reduce production cost and achieve maximum profit. Since the ultimate aim of any capitalist industry is to make profit, the concept of profitability is of great significance and it is evident that the fuzzy logic model developed, if implemented is an effective tool to effectively control profitability in paper recycling to achieve maximum profit. For further optimization of results of our work, hybridization of fuzzy logic and neural network or fuzzy logic and genetic algorithm is recommended for future research.

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