# Optimal Meter Allocation by Consideration of Technical and Economic Problems.

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## ABSTRACT

This paper presents a method for solving the meter placement problem in Power Systems State Estimation. For this purpose, conventional meters (power injection and power flow measurements) are allocated in order to reduce the number (cost) of meters and RTUs, critical measurements, critical sets, and leverage points, and also for improvement in numerical stability of equations. Additionally in this step genetic an algorithm is used for optimization. In the next step, by adding new measurements in best places it is expected that the accuracy of state estimation will be improved. With the intention of this target, conventional measurements and PMUs (separately) are used and compared together.

(Keywords: state estimation, meter placement, network observability, PMU, leverage points)

## INTRODUCTION

The success of state estimation depends on the number, type, and location of the established meters and RTUs on the system. In the design of measurement point locations, first of all it should be considered that the measurement system must satisfy the basic condition of state estimation-observability of the network. In addition to this essential prerequisite, it is also necessary to consider other problems such as accuracy, reliability and economy.

Network observability analysis will determine whether the network is observable or not by the type and placement of the measurements. The topology of network is related to the type and placement of the measurements. Several methods of network observability analysis such as numerical [1, 2] and topological [1, 3] have been introduced to determine whether the network is observable or island observable.
Several research works have been published about algorithms developed with the objective of attaining optimal measurement plans for power system state estimation, taking into account some of the previously described requirements. The cost of installing new meters and reduction number of critical p-set measurements is taken into consideration in the formulations of [4-9]. Accuracy of the weighted least, squares state estimation for the chosen measurement design, is also used as one of the objectives in these studies [10, 14].

Implementation of synchronized phasor measurements presents an opportunity for improvements in power system state estimation. As an addition to standard real and reactive power and voltage and current magnitude measurements, Phasor Measurement Unit (PMU) provides voltage and current phasor measurements ($Ve^{i\theta}, Ie^{j\theta}$). The PMUs are more accurate and also can take measurements synchronously. Thus, the performance of state estimation is improved. Pioneering work in PMU development and utilization is done by Phadke et al. [11]. The PMU placement problem in power systems is mentioned in several papers for example [12-14].

In this paper, the planning of a measurement system implementation is in two steps. In the first step, by using conventional meters (power injection measurement and power flow measurement in pairs unit (P, Q)) a primary outline is achieved. Objective function in this step is observability, reducing cost of meters and RTUs, decreasing critical measurements and critical sets, minimizing the number of leverage points, and improvement in numerical stability. In the next step, by adding PMUs or conventional measurements to primary design done in step 1, the accuracy state estimation is improved and also rate of convergence is speeded up (for PMUs).

It must be stated that detection of bad data in bad leverage points due to tremendous errors or malfunction in meters is not so simple [1] and reducing the number of leverage points is done to avoid this situation. But as we know the existence of good leverage points (free of bad data) causes the accuracy of state estimation to increase [1]. To not lose this positive qualification by adding PMUs is step 2, the accuracy will be increased for compensating the absence of leverage points in pre-designed measurement system in step 1.

## BACKGROUND

### Linear State Estimation

The conventional method for power system state estimation is the weight least squares (WLS) state estimation [1]. The WLS state estimator equations relating to the measurements and the state vector are:

$$z = H.x + e$$

where x and z are the $n \times 1$ state and $m \times 1$ measurement vectors; H is the $m \times n$ Jacobian matrix, e is the $m \times 1$ measurement error vector, m is the number of measurements, and n is the number of buses.

Then, the SE can be formulated as weighted least-squares (WLS) problem:

$$\min J(x) = [z - Hx]R^{-1}[z - H\hat{x}]$$

The state estimate $\hat{x}$ by minimization J(x) in [] can be obtained through the WLS method [1], by satisfying the following Optimality condition:

$$\frac{\partial J(x)}{\partial x} = H^T R^{-1}[z - H\hat{x}] = 0$$

$$\hat{x} = G^{-1}H^T R^{-1}z$$

where $G = H^T R^{-1} H$ is known as gain matrix.

### Observability Analysis

Observability analysis is a search process for portions of a power network for which, given the network and measurement topology, state estimation can be performed.

Usually, the linearized and decoupled state estimator is adopted to perform observability analysis. Hereafter, for the sake of simplicity, the Pɵ (active power-angle) model will be used. A system is said to be observable if the gain matrix is nonsingular, which can be verified during its triangular factorization (no zero pivots, if the reference bus angle is not included) [1-3].

### Condition Number

The condition number of a nonsingular square matrix A is defined as [1, 15, and 16]:

$$K_G = \|A\|.\|A^{-1}\|$$

where $\|...\|$ represents matrix norm. If 2-norm is used, the condition number can be calculated using the following equation:

$$K_G = \frac{\lambda_s}{\lambda_1}$$

where λ denotes the eigenvalues of A, respectively, and subscript s refers to the largest eigenvalues, and subscript 1 refers to the smallest values.

The condition number is equal to unity for identity matrices and tends to infinity for matrices approaching singularity. In state estimation, the sensitivity of the estimate of x to noise is improved (i.e., lessened) when $K_G$ (the condition number of gain matrix) is small, and the sensitivity is worsened (increased) when KG is large. Typical threshold values of KG in state estimation applications, beyond which designers of the state estimator become concerned, are about $10^5$ [15].

### Leverage Points [1, 17]

Some of the measurements of a power system may have a much stronger influence on the state estimate than others due to their location, the local measurement redundancy, the network topology, and parameters. These points are outliers in the space spanned by the row vectors of the Jacobian matrix, meaning that they do not follow the pattern of the point cloud in that space.

Such measurements, referred to as leverage measurements, will distort the solution of the least absolute value estimation, when they carry bad data. There are two cases associated with leverage points. When a measurement is a leverage point and has a wrong metered value, it is a bad leverage point; identification of the bad measurement becomes very difficult by conventional methods. Residual covariance for these measurements will be numerically insignificant. If, however, the measurement is a leverage point and has a good metered value, it is a "good leverage point" and heavily reinforces the M-estimator's performance.

**Projection Statistics**: A robust measure of leveraging effect of a measurement is proposed by Donoho and Gasko [7] and later applied to the power system state estimation by Mili et al. [8]. This measure is called the projection statistics (PSi) and defined for a measurement I as below [16]:

$$PS_i = \max \frac{\left|H_i^t . H_k\right|}{\beta} \qquad for\ k = 1,2,...,m$$

where,

$$\beta = 1.926\ \text{lomed}_{\ i}\left\{\text{lomed}_{\ j\neq i}\left\{\left|H_i^T . H_k + H_j^T . H_k\right|\right\}\right\}$$
$$1 \leq i, j, k \leq m$$

lomedi {x}: low median of the m number in x={x1, x2… xm}

The projection statistics PSi can be shown to approximately behave like chi-square random variable. Furthermore, for measurement i, the related to the sparsity structure of the row Hi. Hence, a measurement i will be identified as leverage point if:

$$PS_i > \chi^2_{k,0.975}$$

where, k is the number of nonzero entries in the row Hi of the measurement Jacobian H.

## Classification of Measurements [1]

**Critical Measurement:** critical measurement is one whose elimination from the measurement se will result in an unobservable system. The residual and standard deviation associated with a critical measurement always equals zero.

$$r(i) = z(i) - \hat{z}(i)$$
$$\sigma_E(i) = \sqrt{E(i,i)} = 0$$

where,

$$E = R - HG^{-1}H^T$$
$$\sigma_E(i) = \sqrt{E(i,i)}$$
$$\hat{z} = H\hat{x}$$

**Redundant Measurement:** A redundant measurement is measurement which is nor critical. Only redundant may have nonzero measurement residuals.

**Critical Set:** A Cset is defined as a group of measurements (noncritical) in which the removal of any of such measurements makes the remaining of the group critical.

Normalized residuals of measurements pertaining to a critical set (Cset) are equal, and their correlation coefficients present maximum values. Suppose that measurements i and j belong to the same critical set. Then, it follows that:

$$\rho_{ij} = \frac{r_N(i)}{r_N(j)} = 1$$

$$\gamma_{ij} = \frac{E(i,j)}{\sqrt{E(i,i)}\sqrt{E(j,j)}} = 1$$

In this paper by a method that is detailed in [18] critical measurements and sets (by above equations) will be detected.

## Genetic Algorithm

GA stems from analogy of the natural selection process. The GA has the following advantage:

i) It is expected that GA is capable of evaluating the global minimum. GA is based on the multi-point search and does not get stuck with local minima.

ii) It is not necessary that the objective function is differentiable. That is, the objective function is arbitrary.

Actually, GA evaluates the optimal solution in maximizing the objective function called the fitness. Using the genetic operators such as crossover, mutation, and reproduction, the optimal solution is searched to maximize the fitness. In this paper, GA is used to determine the optimal solution for redundant measurements for static state estimation. The specified values of the load flow calculation are taken as the basic measurements.

In this paper, that is discussed later, GA will be archived in step 1 for designing primary outline of metering system in this step. The measurement set is assumed to contain only the conventional measurements such as, power injections, and power flows.

## Proposed Method

**Step 1:** As discussed previously, in this step a metering system will be designed, for reducing number (cost) of meters and RTU and abating critical measurement and critical set and leverage points and decrease condition number sake the SE equation converge rapidly and avoiding from ill-conditioned cases type of measurement in this step is power injection measurements and power flow measurements. The random measurement error standard deviation is [8]:

$$\sqrt{R_i} = \frac{(0.02m + 0.005f_s)}{3}$$

where

$$m = \sqrt{P_i^2 + Q_i^2}$$

is the true measurement value, and f s is full-scale value.

**A.1. Fitness Function:** For compliance with above requirements, there a fitness function is proposed:

$$\begin{aligned} FF = {} & Nmeas + k_1 \times NRTU \\ & + k_2 \\ & + k_3 \times Nlepo + k_4 \times Nscr + k_5 \times Npcr \end{aligned}$$

where

FF: fitness function
Nmeas: number of measurements
NRTU: Number of RTUs
Nlepo: Number of leverage points
Nscr: Number of critical measurements
Npcr: Number of critical set
and,
$k_1$, $K_2$, $k_3$, $k_3$, $k_4$, $k_5$: constants

**Step 2:** The weighted least squares method of state estimation is as accurate at the measurements and the model used. SE error is defined as: $\alpha = x^t - x^e$ where: $x^t$ is the true value of system state variables and $x^e$ is the estimated value of system state variables. The covariance matrix of α is defined as [1, 10]:

$$C = E(\alpha\alpha^T) = (H^T R^{-1} H)^{-1}$$

where *E* stands for expectation. The variances of the SE errors stand for the accuracy of SE.

Statistically, they represent the “squared distances” of the estimates from their true values. The smaller the variances are, the better the SE solution is typical. When the system is numerically observable, the state estimation error variances, which are the diagonal elements of matrix C, can be computed and used as indices for the accuracy of the estimated state variables.

In this step by adding new measurements to primary design done in step 1 the accuracy state estimation is improved [1]. The type of measuring device which is added in this step can be PMU or conventional measurements according to opinion and taste of the designer. PMUs have more effect on the enhancement of accuracy, but conventional devices are more economic.

Following that is described separately how to find placement each of these measurements.

## Conventional Measurements

The reduction of error variances is used as an objective in this step to identify the additional measurements that may be needed to improve the accuracy of the SE solutions. The algorithm determines the state variables with low accuracy by the diagonal elements of the SE error covariance matrix C. To be precise, the diagonal elements of covariance matrix are calculated by the sparse inverse method [4]. Then, all the buses are ranked based on estimation variances; $C_{ii}$ A user defined threshold $\mu_0$ may be used to limit the total number of ranked buses on the list. Then, the user will select the buses for improvement from the list.

To improve the quality of the state estimation solutions, the algorithm identifies a list of candidate measurements for each selected low accuracy bus. Any measurement (injection or flow measurement) at the low accuracy buses or at any of their neighboring buses, which is not already available, is considered as a candidate measurement. Addition of the candidate measurements increases the local redundancy and reduces the SE error variances. Meanwhile if the new measurement creates leverage point, it must be omitted from the candidate list. Also the method of identifying leverage points is based on projection statistics [10].

### PMU

The addition of a voltage phase angle measurement to a conventional state estimator could greatly increase the accuracy of the state estimator if implemented correctly. In this step, by adding PMUs to pre-designed metering system in step 1, we will be afforded to increase accuracy.

So this, as to previous parts, denies the system based on regular error variance and has been closed to the designer of the list a few buses that all the variance error are larger (voltage phase angle), installing PMU to be selected. In other words, the "optimal selection" is the placement of PMUs corresponding to the largest $C_{ii}$ values.

### SIMULATION RESULTS

In this section the proposed method is applied to analyze the measurement placement plan of the IEEE-14 bus power system shown in Figure 1.

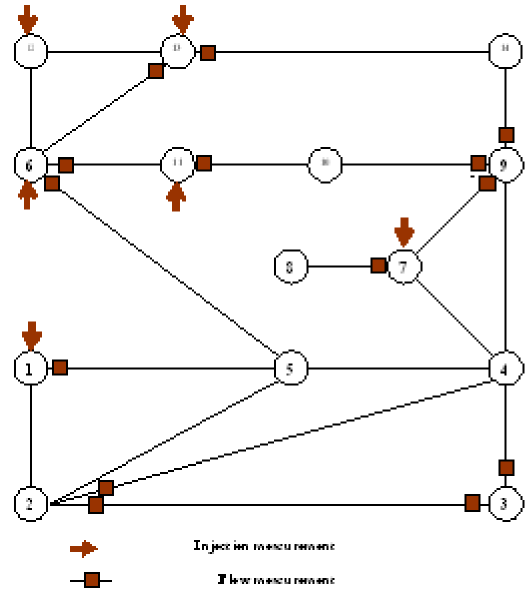


**Figure 1**: IEEE 14-bus system with Measurements.

**Step 1:** The meter placement problem is modeled through GAs considering a binary encoding system in which each individual (chromosome) of a population corresponds to a proposed solution for the problem (metering system). A chromosome is represented by a vector whose elements are associated with meter types and locations. The chromosome dimension then corresponds to the maximum number of meters that can be installed in a given network (twice the number of branches plus the number of buses). The chromosome elements (genes) assume binary values: equal to 1, if a meter is placed and 0, otherwise. It is assumed that all the power measurements are in active-reactive pairs. Therefore, a single gene represents a pair of measurements.

During the search procedure, different values for GA parameters (crossover probability, mutation rate, and population size) have been tested. The search process stopping criterion is based on a previously defined maximum number of generations.

The genetic algorithm parameters used in the first step to run the search for the optimal set of measurements are set as follows:

Maximum generation=200
Population size=100
Crossover probability=0.7

Mutation probability=0.01
Also, constants in FF are:
k1=20, k3=100, k4=104, k5=100

$$k_2 = \begin{cases} 0 & \text{if condition number} \le 1000 \\ 10^2 & \text{if } 1000| < \text{condition number} < 100000 \\ 10^6 & \text{if condition number} \ge 100000 \end{cases}$$

The IEEE 14-bus system example with its measurement configuration shown in Figure 1 is considered to illustrate the proposed method (step 1). There are 5 injections measurements at buses 1, 6, 7, 11, 12, 13 line flow measurements on lines (1-5), (2-3), (3-2), (2-4), (3-4), (6-5), (6-11), (13- 6), (7- 8), (9-7), (9-10), (9-14), (11-10), and (13-14). The evolution of the fitness for the best individual each generation is presented in Figure 2.

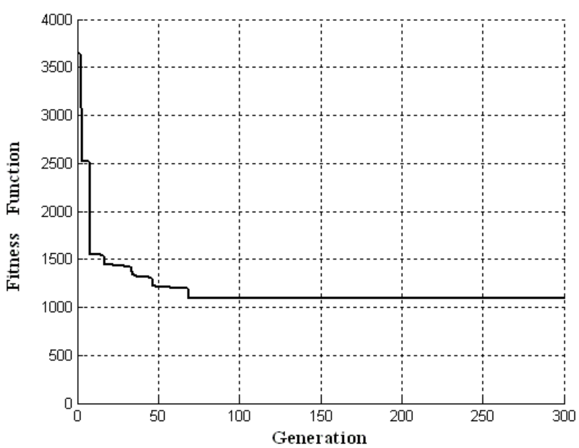


**Figure 2**. Convergence Characteristic of Best Solution (Step 1).

**Step 2:** In this step, the procedure that was explained formerly will be achieved to find the optimal place and number of added measurements (PMUs) or conventional measurement to increase accuracy of state estimation. Also, another assumption is that PMUs are added to be voltage phase angle measurement. The measurement standard deviation of PMUs is assumed is 0.002 (radian.).

Table 1 indicates results of the simulation for normalized errors when PMUs added by the method discussed previously. An improvement in state estimation accuracy was distinguished as PMUs were added to the system.

**Table 1**: Variance of Buses

| Bus Number | Variance |
|---|---|
| 14 | 9.6e-06 |
| 8 | 9.45e-06 |
| 12 | 8.76e-06 |
| 10 | 6.73e-06 |
| 9 | 6.61e-06 |
| 11 | 5.6e-06 |
| 13 | 5.56e-06 |
| 7 | 5.17e-06 |
| 6 | 4.34e-06 |
| 5 | 2.89e-06 |
| 4 | 1.87e-06 |
| 3 | 1.63e-06 |
| 2 | 6.1e-07 |

Table 2 indicates procedure for selection of candidate for adding conventional measurements according to method described above. Tables 3 compares between adding PMUs and conventional measurements and indicates adding PMUs is very better than conventional measurements because of greater improvements in normalized error (accuracy) and condition number if PMUs are placed in buses 14, 8, 12, 10 and conventional measurements installed at buses 14, 8 and on lines (8-7), (12-6).

**Table 2:** Candidate Measurements.

| Bus no. | Candidate | Variance After Adding New Meter | Leverage Point |
|---|---|---|---|
| 14 | (14-13) | 8.4337e-006 | Yes |
| | (14-9) | 8.2473e-006 | no |
| | 14 | 7.2212e-006 | no |
| | 9 | 9.4759e-006 | no |
| 8 | (8-7) | 7.0064e-006 | no |
| | 8 | 7.5624e-006 | no |
| 12 | (12-13) | 8.1682e-006 | no |
| | (12-6) | 7.3338e-006 | no |

$$\text{NE} = \frac{\left\| x_{exact} - \hat{x} \right\|_2}{\left\| x_{exact} \right\|_2}$$

*x:* State Vector

$\hat{x}$ : Estimate of the State Vector

$x_{exact}$ : The "actual" value of $x$

**Table 3:** Comparisons between adding PMUs and Conventional Measurements.

| | Without additional measurements | Adding Conventional meter | Adding PMU |
|---|---|---|---|
| NE | 8.637E-06 | 7.446E-06 | 3.8952E-07 |
| NC | 99032 | 102000 | 3524 |

NE: normalized error
CN: Condition Number

## CONCLUSION

In examining the optimal design principle for arranging measurements presented in this paper, an optimization problem has been formulated where the number of metering systems should be minimized, while some performance requirements should be observed. In step 1, a genetic algorithm has been applied to solve the optimal meter placement problem. Test results with the IEEE 14 bus system show that the proposed methodology is capable of obtaining optimal metering systems. In next step, the metering system is reinforced by adding PMUs or conventional measurements to system that designed in pervious step. The simulation proves that the new model can improve accuracy, the SE equations numerical stability, and convergence speed.

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## SUGGESTED CITATION

Kaihani, R. and A.R. Seifi. 2010. "Optimal Meter Allocation by Consideration of Technical and Economic Problems". *Pacific Journal of Science and Technology*. 11(2):148-155.

Pacific Journal of Science and Technology