General Theory of Games with Virtual Strategies.

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ABSTRACT

Instances of virtual strategies in games encountered in some systems are identified and analyzed based on a mathematical model developed for the systems. Based on the analysis and an analysis in my previous work, two (2) theorems which govern systems behavior are developed.

The theorems are: 1) "A necessary condition for virtual strategies in games and decision systems is such that there must be a primary probability vector/matrix tied to or associated with the game, and the primary probability matrix and the effective primary payoff matrix must change to virtual probability matrices and effective virtual payoff matrices, respectively, in response to changes in virtual strategies such that the utility of the strategist is increased without payoff reduction in the primary payoff matrix, or either decrease or remain the same with payoff reduction in the primary payoff matrix."

2) "Nash equilibrium exists in games with virtual reality strategies". The Mathematical models are subsequently used to prove the theorems.

(Keywords: games, virtual strategies, Nash equilibrium)

INTRODUCTION

Game theory is the study of the ways in which *strategic interactions* among *rational players* produce outcomes with respect to the *preferences* (or *utilities*) of those players, none of which might have been intended by any of them. Since the mathematical theory of games was invented by von Neumann and Morgenstern (von Neumann and Morgenstern, 1947), a lot of work has been done to expound and advance the theory of games.

I had earlier introduced the concept of virtual reality in games in my paper, "Markovian Queue Game with Virtual Reality Strategies", (Nwobi-Okoye, 2009). In that paper, I used the virtual reality concept to model and analyze a unique game which I invented. The virtual strategies as I earlier noted makes the payoff of determining factors to assume a particular situation exists when in reality does not.

In order to throw more light on the concept of virtual strategies, let us consider some typical examples: Have you ever seen scarecrows (human-like figures on farms used to scare away pests)? You would probably have noticed how they help to improve crop yields by scaring away pests. Can you imagine a survival game between a tiger and a tortoise? You can probably envision the tortoise using the virtual strategy to survive the tiger's onslaught. Such strategies abound in the survival game among animals in the animal world. Virtual strategies equally abound in our socioeconomic activities. These range from deceptive labels in products to crowd renting and many other similar strategies.

The Asian economic crisis (Eshan, 1999) could be partly seen from the perspective of improperly applied virtual strategies in n-persons games. Improperly applied virtual strategies leads to zero or negative payoff. Taiwan, to some extent, used some properly applied virtual strategies to help stimulate its economic growth. They did this by labeling their products as being made in the West.

In my previous work, mentioned above, I equally noted that the virtual reality strategy could be extended and applied to game and decision models in many systems ranging from social, political, psychological, management, to economic systems, etc. This paper is therefore an attempt to expound the concept of virtual reality in game theory, to generalize the concept, and to briefly illustrate its application to analyze some game models in specific systems.

This paper is expected to bring to bear a new dimension or direction of thinking to game theorists, and the application of the concept to better understanding and modeling of games.

GAME CHARATERISTICS AND ASSUMPTIONS

The mathematical analysis was based on the following characteristics and assumptions:

1. Could be a zero sum game or non zero sum game.

2. A non-cooperative game.

3. Each competitor aware of the probability matrix/matrices underlying the game introduces dummies or virtual situations to improve his/her utility.

4. Introducing virtual situations or realities changes the payoff matrix, and the nash equilibrium point (NE) point could shift depending on the value of the payoff reduction factor.

5. Virtual strategies are finite and agents adopting such strategies are finite.

6. Playing conditions may be perfectly the same for both parties. i.e. the probability matrix/vector for each competitor is equal, or may be different which makes the game biased in favor of any of the two competitors.

7. The game could be a two persons game or an n persons game, but my analysis is based on two persons game.

8. Virtualization exists only in situations where the payoff determinants assume a particular condition exists, which in reality does not exist.

MATHEMATICAL MODEL OF GAMES WITH VIRTUAL STRATEGIES

Virtual strategies exist only in situations where the payoff determinants assume a particular condition exists, which in reality does not exist.

For virtual reality to exist in a game model these mathematical formulae must be valid:

$$E(k) \cdot G = G_e \quad \textbf{(1)}$$

$$E(v) \cdot G_n = G_{ev} \quad \textbf{(2)}$$

Here:

G = a primary payoff matrix

G_n = a payoff matrix when a virtual strategy is introduced (virtual payoff matrix).

$E(k)$ = a probability vector or matrix associated to the payoff matrix, G

$E(v)$ = a probability vector or matrix associated to the payoff matrix, G_n

G_e = effective matrix obtained by multiplying each payoff by its associated probability of occurrence (effective primary payoff matrix).

G_{ev} = effective matrix obtained by multiplying each payoff by its associated probability of occurrence when a virtual strategy is introduced (effective virtual payoff matrix).

The dot operator in Equations 1 and 2 carries out the operation of multiplying each payoff by its associated probability of occurrence (Nwobi-Okoye, 2009).

The effective primary payoff matrix, G_e and the effective virtual payoff matrix, G_{ev} are shown in Tables 1 and 2.

The grand payoff matrix, GG (secondary virtual payoff matrix), shown in Table 3, was defined by Nwobi-Okoye (2009), and depicts the strategic form representation of the game.

The grand payoff matrix contains the expected monetary value (EMV) for each player for a certain combination of virtual strategies.

Table 1: Effective Primary Payoff Matrix, G_e.

$$
\begin{array}{c|cccc|}
 & 0 & 1 & \ldots & m \\
0 & G_{e\,1X,1X}, G_{e\,1Y,1Y} & G_{e\,1X,1X}, G_{e\,1Y,1Y} & \ldots G_{e\,1X,mX}, G_{e\,1Y,mY} & \\
1 & G_{e\,2X,1X}, G_{e\,2Y,1Y} & G_{e\,2X,2X}, G_{e\,2Y,2Y} & \ldots G_{e\,2X,NX}, G_{e\,2Y,mY} & \\
. & . & . & . & \\
. & . & . & . & \\
. & . & . & . & \\
n & G_{e\,nX,1X}, G_{e\,nY,1Y} & G_{e\,nX,2X}, G_{e\,nY,2Y} & \ldots G_{e\,nX,mX}, G_{e\,nY,mY} &
\end{array}
$$

Here,
$G_{eiX,jX}$ = the payoff for competitor X in the effective primary payoff matrix, G_e
$G_{eiY,jY}$ = the payoff for competitor Y in the effective primary payoff matrix, G_e

Table 2: Effective virtual payoff matrix, $G_{ev.}$

$$
\begin{array}{c|cccc|}
 & 0 & 1 & \ldots & m \\
0 & G_{ev\,1X,1X}, G_{ev\,1Y,1Y} & G_{ev\,1X,1X}, G_{ev\,1Y,1Y} & \ldots G_{ev\,1X,mX}, G_{ev\,1Y,mY} & \\
1 & G_{ev\,2X,1X}, G_{ev\,2Y,1Y} & G_{ev\,2X,2X}, G_{ev\,2Y,2Y} & \ldots G_{ev\,2X,NX}, G_{ev\,2Y,mY} & \\
. & . & . & . & \\
. & . & . & . & \\
. & . & . & . & \\
n & G_{ev\,nX,1X}, G_{ev\,nY,1Y} & G_{ev\,nX,2X}, G_{ev\,nY,2Y} & \ldots G_{ev\,nX,mX}, G_{ev\,nY,mY} &
\end{array}
$$

Here,
$G_{eviX,jX}$ = the payoff for competitor X in the effective virtual payoff matrix, G_e
$G_{eviY,jY}$ = the payoff for competitor Y in the effective virtual payoff matrix, G_e

Table 3: Grand Payoff matrix, GG.

$$
\begin{array}{c|cccc|}
 & 0 & 1 & \ldots & N \\
0 & GG_{1X,1X}, GG_{1Y,1Y} & GG_{1X,1X}, GG_{1Y,1Y} & \multicolumn{2}{c|}{\ldots GG_{1X,NX}, GG_{1Y,NY}} \\
1 & GG_{2X,1X}, GG_{2Y,1Y} & GG_{2X,2X}, GG_{2Y,2Y} & \multicolumn{2}{c|}{\ldots GG_{2X,NX}, GG_{2Y,NY}} \\
. & . & . & \multicolumn{2}{c|}{.} \\
. & . & . & \multicolumn{2}{c|}{.} \\
. & . & . & \multicolumn{2}{c|}{.} \\
N & GG_{NX,1X}, GG_{NY,1Y} & GG_{NX,2X}, GG_{NY,2Y} & \multicolumn{2}{c|}{\ldots GG_{NX,NX}, GG_{NY,NY}}
\end{array}
$$

Here ,
$GG_{iX,jX}$ = the cumulative payoff for player X in the effective payoff matrix, when X uses strategy i-1.
$GG_{iY,jY}$ = the cumulative payoff for player Y in the effective payoff matrix (either primary or virtual effective payoff matrix), when Y uses strategy j-1.
N= maximum value of strategies

In general, the values in the secondary payoff matrix are obtained from the equations below:

$$GG_{iXjX} = \sum_{i=1}^{n} \sum_{j=1}^{m} G_{eiXjX} \cdot \qquad \textbf{(3)}$$

$$GG_{iYjY} = \sum_{i=1}^{n} \sum_{j=1}^{m} G_{eiYjY} \cdots \qquad \textbf{(4)}$$

$$GG_{iXjX} = \sum_{i=1}^{n} \sum_{j=1}^{m} G_{eviXjX} \qquad \textbf{(5)}$$

$$GG_{iYjY} = \sum_{i=1}^{n} \sum_{j=1}^{m} G_{eviYjY} \cdot \qquad \textbf{(6)}$$

Here n denotes the number of rows and m the number of columns in either primary or virtual effective payoff matrix (G_e and G_{ev}).

DEFINITION: A virtual game is therefore defined as a game with a finite set of players i Є J, a grand/secondary payoff matrix, GG, a set of virtual strategies, V = (V_1, V_2, V_3, V_4 … V_n) for each player with each strategy tied to a virtual payoff matrix, G_n, an associated probability matrix/vector, E (v), and an effective virtual payoff matrix, G_{ev}, where G_{ev} = E (v) $\cdot G_n$.

An important generalization or principle could be deduced from the analysis above:

THEOREM 1: "A necessary condition for virtual strategies in games and decision systems is such that there must be a primary probability vector/matrix tied to or associated with the game, and the primary probability matrix and the effective primary payoff matrix must change to virtual probability matrices and effective virtual payoff matrices, respectively, in response to changes in virtual strategies such that the utility of the strategist is increased without payoff reduction in the primary payoff matrix, or either decrease or remain the same with payoff reduction in the primary payoff matrix.".

Players adopting virtual reality strategies must do it in such a way that payoff determining factors (players, people etc) would never find out. Furthermore adopting virtual reality strategies are seen as a fight against nature.

THEOREM 2: Nash equilibrium exists in games with virtual reality strategies.

PROOF OF THE THEOREMS

THEOREM 1:

Just as higher prices lead to an increase in supply or the probability of finding a particular commodity on the market, and lower prices increases demand and the proportion of buyers of a particular commodity, Samuelson (1938), hence, introduction of virtual strategies (virtualization) is driven by potential of increased payoff, hence, increased probability of dominating or winning. A rational agent would always act to maximize his/her utility, Ross and LaCasse (1994 and 1995) and Samuelson (1938). An agent that adopts virtual reality strategy sees the virtual payoff first before his actions changes the probability distribution associated with the payoff matrix.

Consequent to the above law, let virtualization be introduced to the system represented by Equation 1. In response to virtualization, E (k) must change to E (v). Hence, according to Equation 2 we have:

$E(v) \cdot G_n = G_{ev}$

Here,

$G_n = G$ or $G_n \neq G$

E (k), E (v), G_{ev} and G_e, are the variables.

Since $E(k) \neq E(v)$, hence, $G_{ev} \neq G_e$.

Since the players are rational, it follows that E (k) can only change if, and only if, virtualization exists resulting in change to the effective payoff matrix, G_e.

From the proof above, it is important to note that if $G_n \neq G$, there exists a payoff reduction occasioned by the introduction of virtualization.

THEOREM 2: All games with virtual strategies could be represented with the secondary payoff matrix, GG. Since strategies are finite and players are finite, nash equilibrium (Nash, 1950a, 1950b and 1951) must exist in such games.

CASE STUDIES OF VIRTUAL REALITY IN GAMES

Let us now apply the mathematical formulae in the equations above to some game models in some systems.

MANAGEMENT SYSTEM

Here, I assumed the managers use a deceptive strategy such as leaving his vehicle in the organization's premises while out of the premises so that his subordinates/workers would think he is around when actually he is not, and would not relax at their duty posts.

A. No Virtual Reality (Strategy 0)

$$\begin{array}{ccc} & \text{Supervisor} & \text{Supervisor} \\ & \text{Absent} & \text{Present} \\ & 0 & 1 \\ G = | & 1000{,}1000 & 1000{,}1000 | \end{array}$$

G = Maximum possible productivity

Associated probability matrix/vector:

$$E(k) = |\, 0.50{,}0.50 \quad 0.90{,}0.90 \,|$$

The probability value in the probability vector represents the probability of maximum productivity or probability of getting the maximum payoff. In this situation we assume the game situation models the strategic interaction between supervisors from two or more competing firms.

Effective payoff matrix:

$$\begin{array}{ccc} & \text{Supervisor} & \text{Supervisor} \\ & \text{Absent} & \text{Present} \\ & 0 & 1 \\ G_e = 0 | & 500{,}500 & 900{,}900 | \end{array}$$

B. With Virtual Reality (Strategy 1)

$$\begin{array}{ccc} & \text{Supervisor} & \text{Supervisor} \\ & \text{Absent} & \text{Present} \\ & 0 & 1 \\ G_n = & |1000{,}1000 & 1000{,}1000| \end{array}$$

Associated virtual probability matrix/vector:

$$E(v) = |\,0.90{,}0.90 \quad 0.90{,}0.90\,|$$

The probability value in the probability vector represents the probability of maximum productivity or probability of getting the maximum payoff. The virtual strategies are strategies used by the supervisor which makes the workers assume he/she is present when he/she is actually absent.

Effective payoff matrix

$$\begin{array}{ccc} & \text{Supervisor} & \text{Supervisor} \\ & \text{Absent} & \text{Present} \\ & 0 & 1 \\ G_{ev} = 0 & |900{,}900 & 900{,}900| \end{array}$$

The grand payoff matrix, GG, shows what could happen when either or both players (managers) adopts the virtual strategy.

$$GG = \begin{array}{c} \\ 0 \\ 1 \end{array} \begin{array}{cc} 0 & 1 \\ \left| \begin{array}{cc} 1400{,}1400 & 1400{,}1800 \\ 1800{,}1400 & 1800{,}1800 \end{array} \right| \end{array}$$

Here strategy 1 denotes the virtual strategy.

SPORTING/GAMBLING SYSTEM

Here we consider the popular game of “Rock, Paper, and Scissors”. I assumed a player uses a deceptive strategy to know his opponents move/strategy.

A. No Virtual Reality (Strategy 0)

This represents the popular game of Rock, Paper and Scissors.

$$E(k) = \left| \begin{array}{ccc} 1/9{,}1/9 & 1/9{,}1/9 & 1/9{,}1/9 \\ 1/9{,}1/9 & 1/9{,}1/9 & 1/9{,}1/9 \\ 1/9{,}1/9 & 1/9{,}1/9 & 1/9{,}1/9 \end{array} \right|$$

$$G = I \left| \begin{array}{ccc} 0{,}0 & 0{,}-1 & 1{,}0 \\ 1{,}0 & 0{,}0 & 0{,}-1 \\ 0{,}-1 & 1{,}0 & 0{,}0 \end{array} \right|$$

$$G_e = \left| \begin{array}{ccc} 0{,}0 & 0{,}-1/9 & 1/9{,}0 \\ 1/9{,}0 & 0{,}0 & 0{,}-1/9 \\ 0{,}-1/9 & 1/9{,}0 & 0{,}0 \end{array} \right|$$

B. With Virtual Reality (Strategy 1)

If one player (player X) adopts the virtual strategy, we have the following:

$$E(v) = \left| \begin{array}{ccc} 0{,}0 & 0{,}0 & 1/3{,}1/3 \\ 1/3{,}1/3 & 0{,}0 & 0{,}0 \\ 0{,}0 & 1/3{,}1/3 & 0{,}0 \end{array} \right|$$

$$G_n = I \left| \begin{array}{ccc} 0{,}0 & 0{,}-1 & 1{,}0 \\ 1{,}0 & 0{,}0 & 0{,}-1 \\ 0{,}-1 & 1{,}0 & 0{,}0 \end{array} \right|$$

$$G_{ev} = \left| \begin{array}{ccc} 0{,}0 & 0{,}0 & 1/3{,}0 \\ 1/3{,}0 & 0{,}0 & 0{,}0 \\ 0{,}0 & 1/3{,}0 & 0{,}0 \end{array} \right|$$

The grand payoff matrix, GG, shows what happens when either or both players adopts the virtual strategy.

$$GG = \begin{array}{c} \\ 0 \\ 1 \end{array}\begin{array}{c} \begin{matrix} 0 & 1 \end{matrix} \\ \begin{vmatrix} 1/3,-1/3 & 0,-1 \\ 1,0 & 1,-1 \end{vmatrix} \end{array}$$

Here strategy 1 denotes the virtual strategy. Since the game is a zero sum game, the matrix GG could be represented as shown below:

$$GG = \begin{array}{c} \\ 0 \\ 1 \end{array}\begin{array}{c} \begin{matrix} 0 & 1 \end{matrix} \\ \begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix} \end{array}$$

ECONOMIC/MANAGEMENT (COMPETITIVE QUEUING) SYSTEM

A full treatment of this system could be found in the work by Nwobi-Okoye (2009).

A. No Virtual Reality (Strategy 0)

$$E(k) = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array}\begin{array}{c} \begin{matrix} 0 & 1 & 2 \end{matrix} \\ \begin{vmatrix} 0.20,0.20 & 0.60,0.60 & 0.20,0.20 \\ 0.00,0.00 & 0.40,0.40 & 0.60,0.60 \\ 0.00,0.00 & 0.00,0.00 & 1.00,1.00 \end{vmatrix} \end{array}$$

$$G = \begin{vmatrix} 0,0 & 3,3 & 6,6 \\ 0,0 & 0,0 & 3,3 \\ 0,0 & 0,0 & 0,0 \end{vmatrix}$$

$$G_e = \begin{vmatrix} 0.00,0.00 & 1.80,1.80 & 1.20,1.20 \\ 0.00,0.00 & 0.00,0.00 & 1.80,1.80 \\ 0.00,0.00 & 0.00,0.00 & 0.00,0.00 \end{vmatrix}$$

B. With Virtual Reality (Strategy 1)

$$G_n = \begin{vmatrix} -0.25,-0.25 & 2.75,2.75 & 5.75,5.75 \\ 0.00,0.00 & 0.00,0.00 & 3.00,3.00 \\ 0.00,0.00 & 0.00,0.00 & 0.00,0.00 \end{vmatrix}$$

$$E(v) = \begin{vmatrix} 0.10,0.10 & 0.60,0.00 & 0.30,0.00 \\ 0.00,0.60 & 0.40,0.40 & 0.60,0.00 \\ 0.00,0.30 & 0.00,0.60 & 1.00,1.00 \end{vmatrix}$$

$$G_{ev} = \begin{vmatrix} -0.03,-0.03 & 1.65,1.65 & 1.73,1.73 \\ 0.00,0.00 & 0.00,0.00 & 1.80,1.80 \\ 0.00,0.00 & 0.00,0.00 & 0.00,0.00 \end{vmatrix}$$

The grand payoff matrix, GG, shows what could happen when either or both players adopts the virtual strategy. I only showed payoffs for strategy 0 and strategy 1.

$$GG = \begin{vmatrix} 4.80,4.80 & 4.80,5.15 \\ 5.15,4.80 & 5.15,5.15 \end{vmatrix}$$

INDUSTRIAL SYSTEM WITH NO VIRTUAL REALITY

Here I assumed to competitors who wants to introduce products into the market to compete with existing products. They have a choice to either use deceptive labeling or any other virtual strategy as an entry strategy.

A. No Virtual Reality (Strategy 0)

$$G = |10000,10000|$$

G = Market size

Associated probability matrix/vector (market share)

$$E(k) = |\,0.10,0.10\,|$$

The probability value in the probability vector represents the market share for two competing firms.

Effective payoff matrix:

$$G_e = |1000,1000|$$

B. With Virtual Reality (Strategy

$$G_n = |8000{,}8000|$$

G_n = Market size (maximum possible profit)

The reduction in maximum possible profit is a result of the cost of introducing the virtual strategy.

Associated probability matrix/vector (market share)

$$E(v) = |0.20{,}0.20|$$

The probability value in the probability vector represents the market share for two competing firms if both adopt virtual strategies.

Effective payoff matrix:

$$G_{ev} = |1600{,}1600|$$

The grand payoff matrix, GG, shows what could happen when either or both players adopts the virtual strategy.

$$GG = X \begin{matrix} & \begin{matrix} & Y & \\ 0 & & 1 \end{matrix} \\ \begin{matrix} 0 \\ 1 \end{matrix} & \begin{vmatrix} 1000{,}1000 & 1000{,}1600 \\ 1600{,}1000 & 1600{,}1600 \end{vmatrix} \end{matrix}$$

Here strategy 1 denotes the virtual strategy.

MANAGEMENT SYSTEM

Virtual strategies could equally be used to improve productivity in work places by dividing jobs into fragments and not letting the worker know the full extent of job at once. This example models such a situation.

A. No Virtual Reality (Strategy 0)

$$G = |10000{,}10000|$$

G = Maximum productivity

Associated probability matrix/vector

$$E(k) = |0.60{,}0.60|$$

The probability values in the probability vector are for two competing managers, and represent the probability of maximum productivity.

Effective payoff matrix:

$$G_e = |6000{,}6000|$$

B. With Virtual Reality (Strategy 1)

$$G_n = |10000{,}10000|$$

G_n = Maximum Possible Productivity

Associated probability matrix/vector:

$$E(v) = |0.80{,}0.80|$$

The probability values in the probability vector are for two competing managers, and represent the probability of maximum productivity if both adopt virtual strategies.

Effective payoff matrix:

$$G_{ev} = |8000{,}8000|$$

The grand payoff matrix, GG, shows what could happen when either or both players adopts the virtual strategy.

$$GG = X \begin{matrix} & \begin{matrix} & Y & \\ 0 & & 1 \end{matrix} \\ \begin{matrix} 0 \\ 1 \end{matrix} & \begin{vmatrix} 6000{,}6000 & 6000{,}8000 \\ 8000{,}6000 & 8000{,}8000 \end{vmatrix} \end{matrix}$$

Here strategy 1 denotes the virtual strategy.

VIRTUAL STRATEGIES IN A COOPERATIVE GAME

Assuming I am bargaining with a seller to buy a good. Assuming we started bargaining from $1000 to $500 and the seller refused to go lower than $500 but I wanted him to sell at $400. I could pay the $500 but I pretend to be leaving the sellers shop to see if he could call me back to agree to my bargain and lower the price to $400. This is a virtual strategy and has a probability vector associated with it in accordance with the virtual strategy theorem.

If the good sells at $400 my gain is $100 but since my strategy may not work due to the associated probability vector, the effective payoff would be:

$G_{ev} = E(v) \cdot G_n$

If the virtual strategy does not exist, the probability vector is zero (0). If the virtual strategy exists, the probability vector is greater than zero.

A. No Virtual Strategy (Strategy 0)

$$G = |100|$$

G = Maximum Possible Gain

Associated probability matrix/vector:

$$E(k) = |0.0|$$

$$G_e = |0.0|$$

G_e = Gain

B. With Virtual Strategy (Strategy 1)

$$G_n = |100|$$

G_n = Maximum Gain

Associated probability matrix/vector:

$$E(v) = |0.50|$$

$$G_{ev} = |50|$$

G_{ev} = Gain

The grand payoff matrix, GG shows what could happen if the buyer adopts virtual strategy:

$$\begin{matrix} & 0 & 1 \\ GG = & |0.0 & 50| \end{matrix}$$

PSYCHOLOGICAL SYSTEM

Virtual strategies could equally be used to improve emotional stability. Here I assumed two shy individuals adopting strategies to control their emotional problems.

A. No Virtual Reality (Strategy 0)

$$G = |10{,}10|$$

G = Maximum Possible Payoff

Associated probability matrix/vector

$$E(k) = |0.50{,}0.50|$$

The probability values in the probability vector are for the two individuals, and represent the probability of maximum payoff.

Effective payoff matrix:

$$G_e = |5{,}5|$$

B. With Virtual Reality

i. Strategy 1 (Virtual Lens)

$$G_n = |10{,}10|$$

G_n = Possible Payoff

Associated probability matrix/vector:

$$E(v) = |0.70{,}0.70|$$

The probability value in the probability vector represents the probability of maximum payoff if both adopt virtual strategy 1.

Effective payoff matrix:

$$G_{ev} = |7{,}7|$$

ii. Strategy 2 (Alcohol)

$$G_n = |-5{,}-5|$$

$G_n =$ Possible Payoff

Associated probability matrix/vector:

$$E(v) = |\, 0.80, 0.80 \,|$$

The probability value in the probability vector represents the probability of maximum payoff if both adopt virtual strategy 2.

Effective payoff matrix:

$$G_{ev} = |\, 4,4 \,|$$

The grand payoff matrix, GG, shows what could happen when either or both players adopts the virtual strategy.

$$GG = X \begin{array}{c} \\ \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{ccc} & \mathrm{Y} & \\ 0 & 1 & 2 \\ 5,5 & 5,7 & 5,-4 \\ 7,5 & 7,7 & 7,-4 \\ -4,5 & -4,7 & -4,-4 \end{array}$$

Here strategies 1 and 2 denote the virtual strategy.

DISCUSSION

This paper was intended to give a general theoretical background to games with virtual reality strategies and to develop a model for such games hence I did not give an extensive mathematical analysis of any specific type of game in this class. This has been done in my previous work.

The mathematical model developed in this work is idealistic. It is noteworthy that some virtual strategies may not be morally sound; hence, in non-zero sum virtual games, it would be a basis for selecting one equilibrium point from another. Due to repeated and unscientific use of certain virtual strategies, the equilibrium points in some virtual games might shift without the players realizing it in a timely fashion. Furthermore, the mixed strategy equilibrium predicted by the minimax theorem, Loomis (1946) and von Neumann and Morgenstern (1947), may not be the same with that of some virtual games. The analysis of equilibrium points in virtual games would be the subject of my future publication.

It is also noteworthy that new entrants into an existing market may have to adopt the virtual strategies at the early stages of their entry. But such strategies must be scientifically analyzed and carefully applied to avoid failures and its attendant boomeranging effects, as experienced by the Asian tigers in 1997, Eshan (1999).

RECOMMENDATIONS AND CONCLUSIONS

The virtual reality theorem developed in this work is expected to introduce a new dimension to game theory. This totally new concept is expected to spur the development of numerous new sophisticated mathematical models and theorems to advance the concept and push it to new heights in the near future.

Game theorists (management scientists, economists, psychologists, philosophers, political scientists etc) are encouraged to apply the theorems and the concepts expounded in this work to their areas of specialization to model various games based on this promising concept of virtual reality.

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SUGGESTED CITATION

Nwobi-Okoye, C.C. 2010. "General Theory of Games with Virtual Strategies". *Pacific Journal of Science and Technology*. 11(1):317-327.

Pacific Journal of Science and Technology