

Bliss Model for Multiple Players.

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ABSTRACT

The extension of sharing game with asymmetrical information from two players to n players comes with interesting modifications and finds useful application in revenue sharing. Coalitions of players are investigated having in mind the optimality of super-additive domain.

(Keywords: sharing game, asymmetrical information, revenue sharing)

INTRODUCTION

The task of allocating resources from natural wealth in a federation that has not evolved a stable fiscal policy in a popular democracy is often met with agitations, more so when the arrangement is skewed in favor of a/some player(s). "A model of cake division with a blind player" by Christopher Bliss of Nuffield College Oxford describes two players sharing cake under asymmetrical information. Three models of Fixed Payment, Mixed Strategy, and Cake Shrinking solutions are used. Revenues from various organs of Governments are allocated to different interest groups, thus the game is being extended to involve coalitions of players. There would be two broad groups under which each coalition must belong – the advantaged (sighted) and the disadvantaged (blind). The number of coalitions in each group is investigated using the optimality of super additive domain.

The Bliss Model

The problem of implementing an agreed division rule between two players when a player is placed at a more advantageous position is considered with the Bliss Model. A sighted player (player I) is to draw a cake size from a pool of random sizes of cakes. Without a-priori knowledge of the cake size, he enters into agreement with a blind player (player II) who does not see the cake received by

player I but can weigh his own portion to ensure the agreed rule has been obeyed. Three different situations are considered under discrete and continuous random sample spaces of cakes. The improvement of the model is done to ensure maximal utility and avoid waste. A strictly concave utility function 'U' is used in place of actual cake sizes for easy attainment of optimality. The results are:

In fixed payment solution, player I agreed to pay player II a cake size 'b' which is so chosen that,

$$U(b) = \int_0^{s^{max}} U(s - b)p(s)ds$$

Whereby selection of cake is made from $[s_1, s^{max}]$. In cake shrinking solution, it is found that a shrinking function,

$$V(s) = \left(\frac{s}{s^{max}}\right)^{\frac{1-\theta}{\theta}};$$

where $0.5 \leq \theta < 1$ is the agreed division ratio and V is a monotone function with incentive compatibility property, which will induce a true declaration from player I.

In Mixed strategy solution, a weight function 'q' is defined for each cake size such that,

$$q(s) = U(\theta s - b)^{\frac{1-\theta}{\theta}}$$

measures the acceptability of cake size's' by player II. A recursive definition of each q_i brings about an improvement in the model.

The "Multiple Bliss" Model

Let player I be made up of coalitions c_1, c_2, \dots, c_n and player II comprises of coalitions p_1, p_2, \dots, p_r . Three different cases may arise thus:

Case 1: when all the players in the coalitions of each group act in complete agreement, then the game is reduced to Bliss Model with each group acting as player I and player II.

Case 2: when the players in the coalitions of the advantaged group act in unison but the players in the coalitions of the disadvantaged group do not. Then for fixed payment solution, the player I group will present common value while different values will come from the player II. So, for each announced cake size 's' the fixed value 'b' must be so chosen that,

$$U(b) = \sum_s \sum_i U(s - b_i)p(s)$$

where $i = 1, 2, \dots, r$

In cake shrinking solution, each coalition in player II may have different shrinking functions and deploy it independently regardless of other coalitions' opinions. Thus, we have for each coalition p_i in group II we define a shrinking function V_i such that $V = \sum V_i$ and V is as defined in Bliss model. That is V is monotone function such that $0 < V(s) \leq 1 \Rightarrow V(s)s \leq s$ and the shrinking function is so well defined as,

$$V: \mathbb{R} \supset [0, s^{max}] \rightarrow (0, 1]$$

with,

$$\text{group I: } \begin{cases} \sum_j p(s_k)q_j(s_k)U(\theta_{s_k} - b)s_k & \text{accepted at true declaration} \\ \sum_j p(s_k)q_j(s_k)U(\theta_{s_k} - b)s_k V(s_k)s_k & \text{rejected at true declaration} \\ \sum_j p(s_k)q_j(s_k)U(s_k - b - (1 - \theta_{s_k})s_1)s_k & \text{accepted at untrue declaration} \\ \sum_j p(s_k)q_j(s_k)U(s_k - b - (1 - \theta_{s_1})s_1V(s_1)s_k & \text{rejected at untrue declaration} \end{cases}$$

$$\text{group II: } \begin{cases} \sum_j p(s_k)q_j(s_k)U(b + (1 - \theta_{s_k}))s_k & \text{accepted at true declaration} \\ \sum_j p(s_k)q_j(s_k)U(b + (1 - \theta_{s_k}))s_k V(s_k)s_k & \text{rejected at true declaration} \\ \sum_j p(s_k)q_j(s_k)U(b + (1 - \theta_{s_k})s_1)s_k & \text{accepted at untrue declaration} \\ \sum_j p(s_k)q_j(s_k)U(b + (1 - \theta_{s_1})s_1V(s_1)s_k & \text{rejected at untrue declaration} \end{cases}$$

Where $j = 1, 2, \dots, r$ and $s_k < s_1$, $p(s_k)$ is the probability of drawing cake size s_k

$$V(s) = \left(\frac{s}{s^{max}}\right)^{\frac{1-\theta}{\theta}}$$

where $0.5 \leq \theta < 1$ is the agreed division ratio.

Thus, group I's share =

$$\begin{cases} V(s_k)(s_k - (1 - \theta)s_1) & \text{for false declaration} \\ V(s)\theta s & \text{for true declaration} \end{cases}$$

while group II will share $V(s)\theta s$ for any declaration.

Case 3: when the players of the coalitions of both groups do not act in agreement. If each coalition within each group expresses different interest then the number of coalitions equals the number of interests otherwise each group is re-aligned into coalitions of interests. It is however worthy to note that the realignment may occur for every new cake size. So for announcement of cake size s_{nr} there will be n coalitions in group I and r coalitions in group II. The simplest of this arrangement is s_{2r} where there are only two coalitions in group I that is, the coalition that wants to disclose the true value of cake and the one that does not. Hence, the possible outcomes are the elements of the product set $\{c_1, c_2\} \times \{p_1, p_2, \dots, p_r\}$ and each outcome (c_i, p_j) is thus treated as in Bliss model. The result will be:

The weight function q is what each coalition in group II uses to induce true declaration and it is defined as:

$$q(s) = U(\theta s - b) \frac{1-\theta}{\theta}$$

So, if the cakes are drawn from the pool $(0, s_n]$ then on the long run the value of the game becomes:

$$\text{group I: } \left\{ \begin{array}{ll} \sum_{j,k} p(s_k) q_j(s_k) U(\theta s_k - b) s_k & \text{accepted at true declaration} \\ \sum_{j,k} p(s_k) q_j(s_k) U(\theta s_k - b) s_k V(s_k) s_k & \text{rejected at true declaration} \\ \sum_{j,k} p(s_k) q_j(s_k) U(s_k - b - (1 - \theta_{s_k}) s_l) s_k & \text{accepted at untrue declaration} \\ \sum_{j,k} p(s_k) q_j(s_k) U(s_k - b - (1 - \theta_{s_l}) s_l) V(s_l) s_k & \text{rejected at untrue declaration} \end{array} \right.$$

$$\text{group II: } \left\{ \begin{array}{ll} \sum_{j,k} p(s_k) q_j(s_k) U(b + (1 - \theta_{s_k})) s_k & \text{accepted at true declaration} \\ \sum_{j,k} p(s_k) q_j(s_k) U(b + (1 - \theta_{s_k})) s_k V(s_k) s_k & \text{rejected at true declaration} \\ \sum_{j,k} p(s_k) q_j(s_k) U(b + (1 - \theta_{s_k}) s_l) s_k & \text{accepted at untrue declaration} \\ \sum_{j,k} p(s_k) q_j(s_k) U(b + (1 - \theta_{s_l}) s_l) V(s_l) s_k & \text{rejected at untrue declaration} \end{array} \right.$$

Where $k = 1, 2, \dots, n$

We now investigate the number of coalitions that must be in each group for the model to function efficiently. It must be noted that each coalition in the groups seeks to maximize its utility even if it is at conflict with other member(s) of the group(s). However, for a cake size s_k announced by group I stable coalitions are formed which means that all members of the coalition have expressed their contentment with the arrangement. Using the

result of the work of Steven Ketchpel "Forming Coalitions in the face of Uncertain Rewards" we have that stable coalitions are formed in a group partitioned into coalitions $c_1, c_2, c_3, \dots, c_n$ such that each coalition contains distinct members $a_1, a_2, a_3, \dots, a_m$, if the payoffs to each member expressed as utility function is $u(a_j)$, then the partitioning will be stable only if there is no other partitioning such that:

$$\exists \text{ a coalition } \mathbb{C} \text{ belonging to that partition: } \forall a_j^i \in \mathbb{C}, u(a_j^i) > u(a_j)$$

Whereas, $a_j^i = a_j$.

Before the stability of the coalitions is attained, if there are n coalitions, then, the introduction of any new coalition to the system will reorganize the partitioning into $c_1, c_2, c_3, \dots, c_n, c_{n+1}$, this will

bring the system closer to stability only if the members of the new coalitions express better satisfaction than the former arrangement. That is:

$U(C_n) < U(C_{n+1})$ where U is the sum utility of all members of the coalitions C

Likewise it is possible that by reducing the number of coalitions the sum utility is increased which means that $U(C_n) < U(C_{n-1})$ leads to better stability.

Hence, $C_1, C_2, C_3, \dots, C_r$ form stable coalitions if and only if $U(C_{r-1}) < U(C_r) > U(C_{r+1})$.

CONCLUSION AND FURTHER RESEARCH

This paper has described the utility distribution in a skewed system where there is uncertainty on what goes to the coalitions. The scope is limited to only the case where there are only two coalitions in the advantaged group. This may still be extended to a situation where there are many coalitions in the group I and each is presenting different false values and only one is giving the true value of the cake received. The marginal contributions of each member in the coalitions before attainment of stability have been extensively discussed as a stable marriage problem (Gusfield and Irving 1989), the shapely value (Shapely 1953), Banzhaf value (Banzhaf 1965), marginal contribution value, and Average per Capita formula (Dragan and Legaz, 2001).

The attainment of stability in-between a number of coalitions is a variation of super-additive domain (Zlotkin and Rosenschein 1993) in which additional agent or coalition never reduce the sum utility of the entire system. Whatever the marginal contribution of additional agents, once stability is reached the utility cannot be increased. A line of further research is to investigate if another "super-stability" level can be reached by addition and subtraction of coalitions with a fixed number of overall members.

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