Mathematical Foundations of the Relativistic Theory of Quantum Gravity.

Prof. Fran De Aquino

Maranhao State University, Physics Department, S. Luis/MA, Brazil.

*E-mail: deaquino@elo.com.br

ABSTRACT

Starting from the action function we have derived a theoretical background that leads to the quantization of gravity and the deduction of a correlation between the gravitational and inertial masses, which depends on the kinetic momentum of the particle. We show that there is a reaffirmation of the strong equivalence principle and, consequently Einstein's equations are preserved. In fact, such equations are deduced here directly from this new approach to Gravitation. Moreover, we have obtained a generalized equation for inertial forces, which incorporates the Mach's principle into Gravitation. Also, we have deduced the equation of Entropy; the Hamiltonian for a particle in an electromagnetic field and the reciprocal fine structure constant directly from the Gravitation Theory. It is also possible to deduce the expression of the Casimir force and to explain the Inflation Period and the Missing Matter, without assuming the existence of vacuum fluctuations. This new approach to Gravitation will allow us to understand some crucial matters in Cosmology

(Keywords: quantum gravity, quantum cosmology unified field)
(PACs: 04.60.-m; 98.80.Qc; 04.50. +h)

INTRODUCTION

Quantum Gravity was originally studied, by Dirac and others, as the problem of quantizing General Relativity. This approach has many difficulties, detailed by Isham [1]. In the 1970s, physicists tried an even more conventional approach: simplifying the Einstein's equations by assuming that they are *almost linear*, and then applying the standard methods of quantum field theory to the thus oversimplified equations. But this method, too, failed. In the 1980's a very different approach, known as string theory, became popular. Thus far, there are many enthusiasts of string theory. But the mathematical difficulties in string theory are formidable, and it is far from clear that they will be resolved any time soon. At the end of 1997, Isham [2] pointed out several "Structural Problems Facing Quantum Gravity Theory". At the beginning of this new century, the problem of quantizing the gravitational field was still open.

In this work, we propose a new approach to Quantum Gravity. Starting from the generalization of the *action function* we have derived a theoretical background that leads to the quantization of gravity. Einstein's equations of the General Relativity are deduced directly from this theory of Quantum Gravity. Also, this theory leads to a complete description of the Electromagnetic Field, providing a consistent unification of gravity with electromagnetism.

THEORY

We start with the *action* for a free-particle that, as we know, is given by:

$$S = -\alpha \int_a^b ds$$

where α is a quantity which characterizes the particle.

In Relativistic Mechanics, the action can be written in the following form [3]:

$$S = \int_{t_1}^{t_2} L dt = -\int_{t_1}^{t_2} \alpha c \sqrt{1 - V^2/c^2}\, dt$$

where

$$L = -\alpha c \sqrt{1 - V^2/c^2}$$

is the Lagrange's function.

In Classical Mechanics, the Lagrange's function for a free-particle is, as we know, given by:

$L = aV^2$ where V is the speed of the particle and a a quantity *hypothetically* [4] given by:

$$a = m/2$$

where m is the mass of the particle. However, there is no distinction about the kind of mass (if *gravitational mass*, m_g, or *inertial mass* m_i) neither about its sign $(\pm)$.

The correlation between a and α can be established based on the fact that, on the limit $c \to \infty$, the relativistic expression for L must be reduced to the classic expression $L = aV^2$. The result [5] is: $L = \alpha V^2/2c$. Therefore, if $\alpha = 2ac = mc$, we obtain $L = aV^2$. Now, we must decide if $m = m_g$ or $m = m_i$. We will see in this work that the definition of m_g includes m_i. Thus, the right option is m_g, i.e.,

$$a = m_g/2.$$

Consequently, $\alpha = m_g c$ and the generalized expression for the action of a free-particle will have the following form:

$$S = -m_g c \int_a^b ds \qquad (1) \text{ or}$$

$$S = -\int_{t_1}^{t_2} m_g c^2 \sqrt{1 - V^2/c^2}\, dt \qquad (2)$$

where the Lagrange's function is:

$$L = -m_g c^2 \sqrt{1 - V^2/c^2}. \qquad (3)$$

The integral $S = \int_{t_1}^{t_2} m_g c^2 \sqrt{1 - V^2/c^2}\, dt$, preceded by the *plus* sign, cannot have a *minimum*. Thus, the integrand of Equation (2) must be *always positive*. Therefore, if $m_g > 0$, then necessarily $t > 0$; if $m_g < 0$, then $t < 0$. The possibility of $t < 0$ is based on the well-known equation $t = \pm t_0 / \sqrt{1 - V^2/c^2}$ of Einstein's Theory.

Thus if the *gravitational mass* of a particle is *positive,* then t is also *positive* and, therefore, given by $t = +t_0/\sqrt{1 - V^2/c^2}$. This leads to the well-known relativistic prediction that the particle goes to the *future,* if $V \to c$. However, if the *gravitational mass* of the particle is *negative,* then t is *negative* and given by $t = -t_0/\sqrt{1 - V^2/c^2}$. In this case, the prediction is that the particle goes to the *past,* if $V \to c$. Consequently, $m_g < 0$ is the necessary condition for the particle to go to the *past*. Further on, a correlation between *gravitational* and *inertial* masses will be derived, which contains the possibility of $m_g < 0$.

The Lorentz's transforms follow the same rule for $m_g > 0$ and $m_g < 0$, i.e., the sign before $\sqrt{1 - V^2/c^2}$ will be $(+)$ when $m_g > 0$ and $(-)$ if $m_g < 0$.

The *momentum*, as we know, is the vector $\vec{p} = \partial L/\partial \vec{V}$. Thus, from Equation (3) we obtain:

$$\vec{p} = \frac{m_g \vec{V}}{\pm\sqrt{1 - V^2/c^2}} = M_g \vec{V}$$

The sign $(+)$ in the equation above will be used when $m_g > 0$ and the sign $(-)$ if $m_g < 0$. This means that M_g will be always *positive*. Consequently, we will express the momentum $\vec{p}$ in the following form:

$$\vec{p} = \left| \frac{m_g \vec{V}}{\sqrt{1 - V^2/c^2}} \right| = M_g \vec{V} \qquad (4)$$

The derivative $d\vec{p}/dt$ is the *inertial* force F_i which acts on the particle. If the force is perpendicular to the speed, we have:

$$\vec{F}_i = \left| \frac{m_g}{\sqrt{1 - V^2/c^2}} \right| \frac{d\vec{V}}{dt} \qquad (5)$$

However, if the force and the speed have the same direction, we find that:

$$\vec{F}_i = \left|\frac{m_g}{\left(1 - V^2/c^2\right)^{3/2}}\right| \frac{d\vec{V}}{dt} \qquad (6)$$

From Mechanics [6], we know that $\vec{p}\cdot\vec{V} - L$ denotes the *energy* of the particle. Thus, we can write:

$$E_g = \vec{p}\cdot\vec{V} - L = \left|\frac{m_g c^2}{\sqrt{1 - V^2/c^2}}\right| = M_g c^2 \qquad (7)$$

Note that E_g is not null for $V = 0$, but that it has the finite value

$$E_{g0} = m_{g0} c^2 \qquad (8)$$

The Equation (7) can be rewritten in the following form:

$$E_g = m_g c^2 - \frac{m_g c^2}{\sqrt{1 - V^2/c^2}} - m_g c^2 =$$

$$= \frac{m_g}{m_i}\left[m_i c^2 + \underbrace{\left(\frac{m_i c^2}{\sqrt{1 - V^2/c^2}} - m_i c^2 \right)}_{E_{Ki}} \right] =$$

$$= \frac{m_g}{m_i}\left(E_{i0} + E_{Ki}\right) = \frac{m_g}{m_i} E_i \qquad (9)$$

By analogy to the Equation (8), $E_{i0} = m_{i0} c^2$ into the equation above, is the inertial energy *at rest*. Thus, $E_i = E_{i0} + E_{Ki}$ is the *total* inertial energy, where E_{Ki} is the *kinetic inertial energy*. From the Equations (7) and (9) we thus obtain:

$$E_i = \frac{m_{i0} c^2}{\sqrt{1 - V^2/c^2}} = M_i c^2. \qquad (10)$$

For small velocities $(V << c)$, we obtain:

$$E_i \approx m_{i0} c^2 + \tfrac{1}{2} m_i V^2 \qquad (11)$$

where we recognize the classical expression for the kinetic inertial energy of the particle.

The expression for the *kinetic gravitational energy*, E_{Kg}, is easily deduced by comparing Equation (7) with Equation (9). The result is:

$$E_{Kg} = \frac{m_g}{m_i} E_{Ki}. \qquad (12)$$

In the presented picture, we can say that the *gravity*, $\vec{g}$, in a gravitational field produced by a particle of gravitational mass M_g, depends on the particle's gravitational energy, E_g (given by Equation (7)), because we can write:

$$g = -G\frac{E_g}{r^2 c^2} = -G\frac{M_g c^2}{r^2 c^2} = -G\frac{M_g}{r^2} \qquad (13)$$

On the other hand, as we know, the gravitational force is *conservative*. Thus, gravitational energy, in agreement with the energy conservation law, can be expressed by the *decrease* of the inertial energy, i.e.:

$$\Delta E_g = -\Delta E_i \qquad (14)$$

This equation expresses the fact that a decrease of gravitational energy corresponds to an increase of the inertial energy.

Therefore, a variation ΔE_i in E_i yields a variation $\Delta E_g = -\Delta E_i$ in E_g. Thus $E_i = E_{i0} + \Delta E_i$; $E_g = E_{g0} + \Delta E_g = E_{g0} - \Delta E_i$ and

$$E_g + E_i = E_{g0} + E_{i0} \qquad (15)$$

Comparison between (7) and (10) shows that $E_{g0} = E_{i0}$, i.e., $m_{g0} = m_{i0}$. Consequently, we have

$$E_g + E_i = E_{g0} + E_{i0} = 2E_{i0} \qquad (16)$$

However $E_i = E_{i0} + E_{Ki}$. Thus, (16) becomes:

$$E_g = E_{i0} - E_{Ki}. \quad (17)$$

Note the *symmetry* in the equations of E_i and E_g. Substitution of $E_{i0} = E_i - E_{Ki}$ into (17) yields,

$$E_i - E_g = 2E_{Ki} \quad (18)$$

Squaring the Equations (4) and (7) and comparing the result, we find the following correlation between gravitational energy and *momentum*:

$$\frac{E_g^2}{c^2} = p^2 + m_g{}^2c^2. \quad (19)$$

The energy expressed as a function of the *momentum* is, as we know, called *Hamiltonian* or Hamilton's function:

$$H_g = c\sqrt{p^2 + m_g{}^2c^2}. \quad (20)$$

Let us now consider the problem of quantization of gravity. Clearly there is something unsatisfactory about the whole notion of quantization. It is important to bear in mind that the quantization process is a series of rules-of-thumb rather than a well-defined algorithm, and contains many ambiguities. In fact, for electromagnetism we find that there are (at least) two different approaches to quantization and that while they appear to give the same theory they may lead us to very different quantum theories of gravity. Here we will follow a new theoretical strategy: It is known that starting from the Schrödinger equation we may obtain the well-known expression for energy of a particle in periodic motion inside a cubical box of edge length L [7]. The result now is

$$E_n = \frac{n^2h^2}{8m_gL^2} \qquad n = 1,2,3,... \quad (21)$$

Note that the term $h^2/8m_g L^2$ (energy) will be minimum for $L = L_{max}$ where L_{max} is the maximum edge length of a cubical box whose maximum diameter,

$$d_{max} = L_{max}\sqrt{3} \quad (22)$$

is equal to *the maximum* length scale *of the Universe*.

The minimum energy of a particle is obviously its inertial energy at rest $m_gc^2 = m_ic^2$. Therefore we can write:

$$\frac{n^2h^2}{8m_gL_{max}^2} = m_gc^2$$

Then from the equation above follows that,

$$m_g = \pm\frac{nh}{cL_{max}\sqrt{8}} \quad (23)$$

whence we see that there is a *minimum value* for m_g given by:

$$m_{g(min)} = \pm\frac{h}{cL_{max}\sqrt{8}} \quad (24)$$

The *relativistic* gravitational mass $M_g = \left|m_g\left(1 - V^2/c^2\right)^{-\frac{1}{2}}\right|$, defined in the Equation (4), shows that:

$$M_{g(min)} = \left|m_{g(min)}\right| \quad (25)$$

The *box normalization* leads to the conclusion that the *propagation number* $k = \left|\vec{k}\right| = 2\pi/\lambda$ is restricted to the values $k = 2\pi n/L$. This is deduced assuming an *arbitrarily large but finite* cubical box of volume L^3 [8]. Thus, we have:

$$L = n\lambda$$

From this equation, we conclude that

$$n_{max} = \frac{L_{max}}{\lambda_{min}}$$

and

$$L_{min} = n_{min}\lambda_{min} = \lambda_{min}$$

Since $n_{min} = 1$. Therefore we can write that,

$$L_{max} = n_{max} L_{min} \qquad (26)$$

From this equation we thus conclude that:

$$L = nL_{min} \qquad (27)$$

or

$$L = \frac{L_{max}}{n} \qquad (28)$$

Multiplying (27) and (28) by $\sqrt{3}$ and reminding that $d = L\sqrt{3}$, we obtain:

$$d = nd_{min} \qquad or \qquad d = \frac{d_{max}}{n} \qquad (29)$$

Equations above show that the length (and therefore the *space*) is *quantized*.

By analogy to (23) we can also conclude that:

$$M_{g(\boldsymbol{max})} = \frac{n_{\boldsymbol{max}} h}{cL_{\boldsymbol{min}} \sqrt{8}} \qquad (30)$$

since the relativistic gravitational mass, $M_g = \left| m_g \left(1 - V^2/c^2\right)^{-½} \right|$, is just a multiple of m_g .

Equation (26) tells us that $L_{min} = L_{max}/n_{max}$. Thus Equation (30) can be rewritten as follows:

$$M_{g(\boldsymbol{max})} = \frac{n_{\boldsymbol{max}}^2 h}{cL_{\boldsymbol{max}} \sqrt{8}} \qquad (31)$$

Comparison of (31) with (24) shows that

$$M_{g(max)} = n_{max}^2 \left| m_{g(min)} \right| \qquad (32)$$

which leads to following conclusion that:

$$M_g = n^2 \left| m_{g(min)} \right| \qquad (33)$$

This equation shows that *the gravitational mass is quantized*.

Substitution of (33) into (13) leads to quantization of *gravity*, i.e.:

$$g = -\frac{GM_g}{r^2} = n^2 \left(-\frac{G\left| m_{g(min)} \right|}{\left(r_{max}/n\right)^2} \right) =$$
$$= n^4 g_{min} \qquad (34)$$

From the Hubble's law follows that:

$$V_{max} = \tilde{H} l_{max} = \tilde{H}\left(d_{max}/2\right)$$
$$V_{min} = \tilde{H} l_{min} = \tilde{H}\left(d_{min}/2\right)$$

whence

$$\frac{V_{max}}{V_{min}} = \frac{d_{max}}{d_{min}}$$

Equations (29) tell us that $d_{max}/d_{min} = n_{max}$.

Thus the equation above gives:

$$V_{min} = \frac{V_{max}}{n_{max}} \qquad (35)$$

which leads to following conclusion

$$V = \frac{V_{max}}{n} \qquad (36)$$

this equation shows that *velocity* is also quantized.

From this equation one concludes that we can have $V = V_{max}$ or $V = V_{max}/2$, but there is nothing in between.

This shows clearly that V_{max} cannot be equal to c (speed of light in vacuum). Thus, it follows that:

$$
\begin{array}{lll}
n = 1 & V = V_{max} & \\
n = 2 & V = V_{max}/2 & \\
n = 3 & V = V_{max}/3 & Tachyons \\
\dots\dots & \dots\dots\dots\dots & \\
n = n_x - 1 & V = V_{max}/(n_x - 1) & \\
\hline
n = n_x & V = V_{max}/n_x = c \quad \leftarrow & \\
n = n_x + 1 & V = V_{max}/(n_x + 1) & Tardyons \\
n = n_x + 2 & V = V_{max}/(n_x + 2) & \\
\dots\dots & \dots\dots\dots\dots & \\
\end{array}
$$

where n_x *is a big number.*

Then c is the speed *upper limit* of the *Tardyons* and also the speed *lower limit* of the *Tachyons*. Obviously, this limit is *always the same in all inertial frames*. Therefore c can be used as a *reference speed*, to which we may compare any speed, V, as occurs in the relativistic factor $\sqrt{1-V^2/c^2}$. Thus, in this factor, c does not refer to maximum propagation speed of the interactions such as some authors suggest; c is just a speed limit which remains the same in any inertial frame.

The temporal coordinate x^0 of space-time is now $x^0 = V_{max}t$ ($x^0 = ct$ is then obtained when $V_{max} \rightarrow c$).

Substitution of $V_{max} = nV = n(\tilde{H}l)$ into this equation yields $t = x^0/V_{max} = (1/n\tilde{H})(x^0/l)$.

On the other hand, since $V = \tilde{H}l$ and $V = V_{max}/n$ we can write that $l = V_{max}\tilde{H}^{-1}/n$. Thus $(x^0/l) = \tilde{H}(nt) = \tilde{H}t_{max}$.

Therefore we can finally write:

$$
t = (1/n\tilde{H})(x^0/l) = t_{max}/n \qquad (37)
$$

which shows the quantization of *time*.

From Equations (27) and (37) we can easily conclude that the *spacetime is not continuous* it is *quantized*.

Now, let us go back to Equation (20) which will be called the *gravitational* Hamiltonian to distinguish it from the *inertial* Hamiltonian H_i:

$$
H_i = c\sqrt{p^2 + m_{i0}^{\ 2}c^2}. \qquad (38)
$$

Consequently, the Equation (18) can be rewritten in the following form:

$$
H_i - H_g = 2\Delta H_i \qquad (39)
$$

where ΔH_i is the *variation on the inertial Hamiltonian* or *inertial kinetic energy*. A *momentum* variation Δp yields a variation ΔH_i given by:

$$
\Delta H_i = \sqrt{(p+\Delta p)^2 c^2 + m_{i0}^{\ 2}c^4} - \sqrt{p^2c^2 + m_{i0}^{\ 2}c^4} \qquad (40)
$$

By considering that the particle is *initially at rest* $(p = 0)$. Then Equations (20), (38), and (39) give respectively: $H_g = m_g c^2$, $H_i = m_{i0}c^2$ and

$$
\Delta H_i = \left[\sqrt{1 + \left(\frac{\Delta p}{m_{i0}c}\right)^2} - 1\right] m_{i0}c^2
$$

By substituting H_g, H_i, and ΔH_i into Equation (39) we get:

$$
m_g = m_{i0} - 2\left[\sqrt{1 + \left(\frac{\Delta p}{m_{i0}c}\right)^2} - 1\right] m_{i0}. \qquad (41)
$$

This is the *general expression of correlation between the gravitational and inertial mass*. Note that for $\Delta p > m_{i0}c(\sqrt{5}/2)$, the value of m_g becomes *negative*.

Equation (41) shows that m_g decreases of Δm_g for an increase of Δp. Thus, starting from (4) we obtain:

$$p + \Delta p = \frac{\left(m_g - \Delta m_g\right) V}{\sqrt{1 - (V/c)^2}}$$

By considering that the particle is *initially at rest* $(p = 0)$, the equation above gives:

$$\Delta p = \frac{\left(m_g - \Delta m_g\right) V}{\sqrt{1 - (V/c)^2}}$$

From Equation (16) we obtain:

$$E_g = 2E_{i0} - E_i = 2E_{i0} - \left(E_{i0} + \Delta E_i\right) = E_{i0} - \Delta E_i$$

However, Equation (14) tells us that $-\Delta E_i = \Delta E_g$; what leads to $E_g = E_{i0} + \Delta E_g$ or $m_g = m_{i0} + \Delta m_g$. Thus, in the expression of Δp we can replace $\left(m_g - \Delta m_g\right)$ by m_{i0}, i.e.:

$$\Delta p = \frac{m_{i0} V}{\sqrt{1 - (V/c)^2}}$$

We can therefore write:

$$\frac{\Delta p}{m_{i0} c} = \frac{V/c}{\sqrt{1 - (V/c)^2}} \qquad (42)$$

By substitution of the expression above into Equation (41) we thus obtain:

$$m_g = m_{i0} - 2\left[\left(1 - V^2/c^2\right)^{-\frac{1}{2}} - 1\right] m_{i0} \qquad (43)$$

For $V = 0$ we obtain $m_g = m_{i0}$. Then,

$$m_{g(min)} = m_{i0(min)}$$

Substitution of $m_{g(min)}$ into the *quantized* expression of M_g (Equation (33)) gives:

$$M_g = n^2 m_{i0(min)}$$

where $m_{i0(min)}$ is the *elementary quantum of inertial mass* to be determined.

For $V = 0$, the *relativistic* expression $M_g = m_g / \sqrt{1 - V^2/c^2}$ becomes $M_g = M_{g0} = m_{g0}$.

However (43) shows that $m_{g0} = m_{i0}$. Thus, the *quantized* expression of M_g reduces to:

$$m_{i0} = n^2 m_{i0(min)}$$

Then, we can write:

$$\frac{m_{i0}}{\sqrt{1 - V^2/c^2}} = n^2 \frac{m_{i0(min)}}{\sqrt{1 - V^2/c^2}}$$

or

$$M_i = n_i^2 m_{i0(min)} \qquad (44)$$

which shows the quantization of *inertial mass* ; n_i is the *inertial quantum number* .

We will change n in the quantized expression of M_g by n_g in order to define the *gravitational quantum number*. Thus we have:

$$M_g = n_g^2 m_{i0(min)} \qquad (44a)$$

Finally, by substituting m_g given by Equation (43) into the relativistic expression of M_g, we readily obtain:

$$M_g = \left| \frac{m_g}{\sqrt{1 - V^2/c^2}} \right| =$$

$$= \left| M_i - 2\left[\left(1 - V^2/c^2\right)^{-\frac{1}{2}} - 1\right] M_i \right| \qquad (45)$$

The *Lorentz's force* is usually written in the following form:

$$d\vec{p}/dt = q\vec{E} + q\vec{V} \times \vec{B}$$

where $\vec{p} = m_{i0}\vec{V}\Big/\sqrt{1-V^2/c^2}$. However, Equation (4) tells us that $\vec{p} = m_g V\Big/\sqrt{1-V^2/c^2}$.

Therefore, the expressions above must be corrected by multiplying its members by m_g/m_{i0}, i.e.:

$$\vec{p}\,\frac{m_g}{m_{i0}} = \frac{m_g}{m_{i0}}\frac{m_{i0}\vec{V}}{\sqrt{1-V^2/c^2}} = \frac{m_g\vec{V}}{\sqrt{1-V^2/c^2}} = \vec{p}$$

and

$$\frac{d\vec{p}}{dt} = \frac{d}{dt}\left(\vec{p}\,\frac{m_g}{m_{i0}}\right) = \left(q\vec{E} + q\vec{V}\times\vec{B}\right)\frac{m_g}{m_{i0}} \quad (46)$$

That is now the *general expression* for Lorentz's force. Note that it depends on m_g.

When the force is perpendicular to the speed, Equation (5) gives $d\vec{p}\big/dt = m_g\left(d\vec{V}/dt\right)\Big/\sqrt{1-V^2/c^2}$.

By comparing with Equation (46) we thus obtain:

$$\left(m_{i0}\Big/\sqrt{1-V^2/c^2}\right)\left(d\vec{V}/dt\right) = q\vec{E} + q\vec{V}\times\vec{B}$$

Note that this equation is the expression of an *inertial* force.

Starting from this equation, well-known experiments have been carried out in order to verify the relativistic expression: $m_i\Big/\sqrt{1-V^2/c^2}$.

In general, the *momentum* variation Δp is expressed by $\Delta p = F\Delta t$ where F is the applied force during a time interval, Δt. Note that there is no restriction concerning the *nature* of the force F (i.e., it can be mechanical, electromagnetic, etc.).

For example, we can look on the *momentum* variation Δp as due to absorption or emission of *electromagnetic energy* by the particle (by means of *radiation* and/or by means of *Lorentz's force* upon the *charge* of the particle).

In the case of radiation (any type), Δp can be obtained as follows. It is known that the *radiation pressure*, dP, upon an area $dA = dxdy$ of a volume $d\mathcal{V} = dxdydz$ of a particle (the incident radiation normal to the surface dA) is equal to the energy dU absorbed per unit volume $\left(dU/d\mathcal{V}\right)$, i.e.:

$$dP = \frac{dU}{d\mathcal{V}} = \frac{dU}{dxdydz} = \frac{dU}{dAdz} \quad (47)$$

Substitution of $dz = vdt$ (v is the speed of radiation) into the equation above gives:

$$dP = \frac{dU}{d\mathcal{V}} = \frac{\left(dU/dAdt\right)}{v} = \frac{dD}{v} \quad (48)$$

Since $dPdA = dF$ we can write:

$$dFdt = \frac{dU}{v} \quad (49)$$

However we know that $dF = dp/dt$, then,

$$dp = \frac{dU}{v} \quad (50)$$

From Equation (48) it follows that:

$$dU = dPd\mathcal{V} = \frac{d\mathcal{V}dD}{v} \quad (51)$$

Substitution into (50) yields

$$dp = \frac{d\mathcal{V}dD}{v^2} \quad (52)$$

or

$$\int_0^{\Delta p} dp = \frac{1}{v^2}\int_0^D\int_0^{\mathcal{V}} d\mathcal{V}dD$$

whence,

$$\Delta p = \frac{\mathcal{V}D}{v^2} \quad (53)$$

This expression is general for all types of waves including *non-electromagnetic waves* like *sound waves.* In this case, v in Equation (53), will be the speed of sound in the medium and D the *intensity* of the sound radiation.

In the case of *electromagnetic waves*, the Electrodynamics tells us that v will be given by:

$$v = \frac{dz}{dt} = \frac{\omega}{\kappa_r} = \frac{c}{\sqrt{\frac{\varepsilon_r \mu_r}{2}\left(\sqrt{1+(\sigma/\omega\varepsilon)^2}+1\right)}}$$

Where k_r is the real part of the *propagation vector* $\vec{k}$; $k = |\vec{k}| = k_r + ik_i$; ε, μ and σ, are the electromagnetic characteristics of the medium in which the incident (or emitted) radiation is propagating ($\varepsilon = \varepsilon_r \varepsilon_0$ where ε_r is the *relative dielectric permittivity* and $\varepsilon_0 = 8.854\times10^{-12} F/m$; $\mu = \mu_r \mu_0$ where μ_r is the *relative magnetic permeability* and $\mu_0 = 4\pi\times10^{-7} H/m$; σ is the *electrical conductivity*). For an *atom* inside a body, the incident (or emitted) radiation on this atom will be propagating inside the body, and consequently, $\sigma=\sigma_{body}$, $\varepsilon=\varepsilon_{body}$, $\mu=\mu_{body}$.

It is then evident that the *index of refraction* $n_r = c/v$ will be given by:

$$n_r = \frac{c}{v} = \sqrt{\frac{\varepsilon_r \mu_r}{2}\left(\sqrt{1+(\sigma/\omega\varepsilon)^2}+1\right)} \qquad (54)$$

On the other hand, from Equation (50) it follows that:

$$\Delta p = \frac{U}{v}\left(\frac{c}{c}\right) = \frac{U}{c} n_r$$

Substitution into Equation (41) yields,

$$m_g = \left\{1 - 2\left[\sqrt{1+\left(\frac{U}{m_{i0}c^2} n_r\right)^2} - 1\right]\right\} m_{i0} \qquad (55)$$

This equation is general for all types of electromagnetic fields including *gravitoelectromagnetic* fields.

General Relativity tells us that the gravitational field contains components of electromagnetic type called the *gravitoelectromagnetic* field (Figure 1).

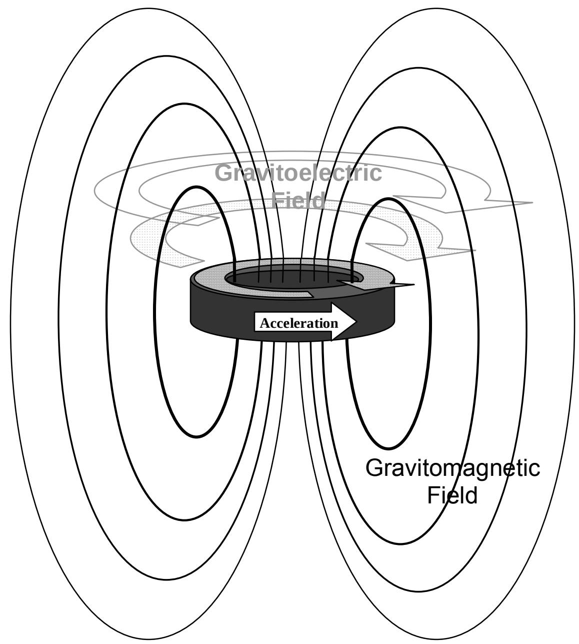


Figure 1: Gravitoelectromagnetic Field.

These components have the property of being produced by moving objects and of acting on objects in movement. It is possible to write the Einstein field equations in weak field approximation in a form very similar to that of Maxwell equations. The Maxwell-like equations for weak gravitational fields are [9]:

$$\nabla . \boldsymbol{D_G} = -\rho$$

$$\nabla \times \boldsymbol{E_G} = -\frac{\partial \boldsymbol{B_G}}{\partial \boldsymbol{t}}$$

$$\nabla . \boldsymbol{B_G} = 0$$

$$\nabla \times \boldsymbol{H_G} = -\boldsymbol{j_G} + \frac{\partial \boldsymbol{D_G}}{\partial \boldsymbol{t}}$$

where $D_G = \varepsilon_{rG}\varepsilon_{0G}E_G$ is the *gravitodisplacement* field (ε_{rG} is the *gravitoelectric relative permittivity* of the medium; ε_{0G} is the *gravitoelectric permittivity* for free space and $\boldsymbol{E_G} = \boldsymbol{g}$ is the *gravitoelectric* field intensity); ρ is the density of local rest mass in the local rest frame of the matter; $B_G = \mu_{rG}\mu_{0G}H_G$ is the *gravitomagnetic* field (μ_{rG} is the *gravitomagnetic relative permeability*, μ_{0G} is the *gravitomagnetic permeability* for free space and $\boldsymbol{H_G}$ is the *gravitomagnetic* field intensity; $\boldsymbol{j_G} = -\sigma_G \boldsymbol{E_G}$ is the local rest-mass current density in this frame (σ_G is the *gravitoelectric conductivity* of the medium).

The *gravitomagnetic permeability* for free space [10,11] is:

$$\mu_{0G} = \frac{16\pi G}{c^2} = 3.73\times10^{-26}\ m/kg$$

Due to the fact that both electromagnetic and gravitational plane waves propagate at the same speed we can write,

$$c = \frac{1}{\sqrt{\varepsilon_0\mu_0}} = \frac{1}{\sqrt{\varepsilon_{0G}\mu_{0G}}}$$

It then follows that the *gravitoelectric permittivity* for free space is:

$$\varepsilon_{0G} = \frac{1}{\mu_{0G}c^2} = \frac{1}{16\pi G}$$

Thus, the impedance for free space is,

$$Z_G = \frac{E_G}{H_G} = \sqrt{\mu_{0G}/\varepsilon_{0G}} = \mu_{0G}c = \frac{16\pi G}{c}$$

In classical electrodynamics the density of energy in an *electromagnetic* field, W_e, has the following expression:

$$W_e = \tfrac{1}{2}\varepsilon_r\varepsilon_0E^2 + \tfrac{1}{2}\mu_r\mu_0H^2$$

In analogy with this expression we define the energy density in a *gravitoelectromagnetic* field, W_G, as follows,

$$W_G = \tfrac{1}{2}\varepsilon_{rG}\varepsilon_{0G}E_G^2 + \tfrac{1}{2}\mu_{rG}\mu_{0G}H_G^2$$

For free space we obtain:

$$\mu_{rG} = \varepsilon_{rG} = 1$$
$$\varepsilon_{0G} = 1/\mu_{0G}\,c^2$$
$$E_G/H_G = \mu_{0G}c$$

and

$$\boldsymbol{B}_G = \mu_{0G}\,\boldsymbol{H}_G$$

Thus, we can rewrite the equation of W_G as follows:

$$W_G = \tfrac{1}{2}\left(\frac{1}{\mu_{0G}c^2}\right)c^2B_G^2 + \tfrac{1}{2}\mu_{0G}\left(\frac{B_G}{\mu_{0G}}\right)^2 = \frac{B_G^2}{\mu_{0G}}$$

Since $U_G = W_G V$, (V is the *volume* of the particle) and $n_r = 1$ for free space we can write (55) in the following form:

$$m_g = \left\{1-2\left[\sqrt{1+\left(\frac{W_G}{\rho\,c^2}\right)^2}-1\right]\right\}m_{i0}$$
$$= \left\{1-2\left[\sqrt{1+\left(\frac{B_G^2}{\mu_{0G}\rho\,c^2}\right)^2}-1\right]\right\}m_{i0} \quad (55a)$$

where $\rho = m_{i0}/\mathrm{V}$.

This equation shows how the gravitational mass of a particle is altered by a *gravitomagnetic* field.

A gravitomagnetic field, according to Einstein's theory of general relativity, arises from moving matter (matter current) just as an ordinary magnetic field arises from moving charges. The rotating Earth is the source of a very weak gravitomagnetic field given by:

$$B_{G,Earth} = -\frac{\mu_{0G}}{16\pi}\left(\frac{M\omega}{r}\right)_{Earth} \approx 10^{-14}\,rad.s^{-1}$$

Perhaps ultra-fast rotating stars can generate very strong gravitomagnetic fields, which can make *negative* the gravitational mass of particles inside and near the star.

According to (55a) this will occur if $B_G > 1.06c\sqrt{\mu_{0G}\rho}$. Usually, however, the gravitomagnetic fields produced by *normal* matter are very weak.

Recently Tajmar, et al., [12] have proposed that in addition to the *London moment*, B_L, ($B_L = -\left(2m^*/e^*\right)\omega \cong 1.1\times10^{-11}\omega$; m^* and e^* are the Cooper-pair mass and charge respectively), a rotating superconductor should exhibit also a large *gravitomagnetic* field, B_G, to explain an apparent mass increase of Niobium Cooper-pairs discovered by Tate et al. [13,14].

According to Tajmar and Matos [15], in the case of *coherent* matter, B_G is given by: $B_G = -2\omega\rho_c\mu_{0G}\lambda_{gr}^2$ where ρ_c is the mass density of *coherent* matter and λ_{gr} is the *graviphoton* wavelength. By choosing λ_{gr} proportional to the local density of *coherent* matter, ρ_c, i.e.,

$$\frac{1}{\lambda_{gr}^2} = \left(\frac{m_{gr}c}{\hbar}\right) = \mu_{0G}\rho_c$$

we obtain,

$$B_G = -2\omega\rho_c\mu_{0G}\lambda_{gr}^2 = -2\omega\rho_c\mu_{0G}\left(\frac{1}{\mu_{0G}\rho_c}\right) =$$

$$= -2\omega$$

and the graviphoton mass, m_{gr}, is:

$$m_{gr} = \mu_{0G}\rho_c\hbar/c$$

Note that if we take the case of *no* local sources of *coherent* matter $(\rho_c = 0)$, the graviphoton mass will be *zero*. However, graviphoton will have non-zero mass inside coherent matter $(\rho_c \neq 0)$. This can be interpreted as a consequence of the graviphoton gaining mass inside the superconductor via the Higgs mechanism due to the breaking of gauge symmetry.

It is important to note that the *minus* sign in the expression for B_G can be understood as due to the change from the normal to the coherent state of matter, (i.e., a switch between real and *imaginary* values for the particles inside the material when going from the normal to the coherent state of matter). Consequently, in this case the variable U in (55) must be replaced by iU_G and not by U_G only. Thus we obtain:

$$m_g = \left\{1 - 2\left[\sqrt{1 - \left(\frac{U_G}{m_{i0}c^2}n_r\right)^2} - 1\right]\right\}m_{i0} \quad (55b)$$

Since $U_G = W_G V$, we can write (55b) for $n_r = 1$, in the following form:

$$m_g = \left\{1 - 2\left[\sqrt{1 - \left(\frac{W_G}{\rho_c c^2}\right)^2} - 1\right]\right\}m_{i0}$$

$$= \left\{1 - 2\left[\sqrt{1 - \left(\frac{B_G^2}{\mu_{0G}\rho_c c^2}\right)^2} - 1\right]\right\}m_{i0} \quad (55c)$$

where $\rho_c = m_{i0}/\mathrm{V}$ is the local density of *coherent* matter. Note the different sign (inside the square root) with respect to (55a).

By means of (55c) it is possible to check the changes in the gravitational mass of the *coherent part* of a given material (e.g. the Cooper-pair fluid). Thus for the *electrons* of the Cooper-pairs we have:

$$m_{ge} = m_{ie} + 2\left[1 - \sqrt{1 - \left(\frac{B_G^2}{\mu_{0G}\rho_e c^2}\right)^2}\right]m_{ie} =$$

$$= m_{ie} + 2\left[1 - \sqrt{1 - \left(\frac{4\omega^2}{\mu_{0G}\rho_e c^2}\right)^2}\right]m_{ie} =$$

$$= m_{ie} + \chi_e m_{ie}$$

where ρ_e is the mass density of the electrons.

In order to check the changes in the gravitational mass of *neutrons* and *protons* (non-coherent part) inside the superconductor, we must use the Equation (55a) and $B_G = -2\omega\rho\mu_{0G}\lambda_{gr}^2$ [Tajmar and Matos, op.cit.]. Due to $\mu_{0G}\rho_c\lambda_{gr}^2 = 1$, that expression of B_G can be rewritten in the following form:

$$B_G = -2\omega\rho\mu_{0G}\lambda_{gr}^2 = -2\omega(\rho/\rho_c)$$

Thus we have:

$$m_{gn} = m_{in} - 2\left[\sqrt{1+\left(\frac{B_G^2}{\mu_{0G}\rho_n c^2}\right)^2} - 1\right]m_{in} =$$

$$= m_{in} - 2\left[\sqrt{1+\left(\frac{4\omega^2(\rho_n/\rho_c)^2}{\mu_{0G}\rho_n c^2}\right)^2} - 1\right]m_{in} =$$

$$= m_{in} - \chi_n m_{in}$$

$$m_{gp} = m_{ip} - 2\left[\sqrt{1+\left(\frac{B_G^2}{\mu_{0G}\rho_p c^2}\right)^2} - 1\right]m_{ip} =$$

$$= m_{ip} - 2\left[\sqrt{1+\left(\frac{4\omega^2(\rho_p/\rho_c)^2}{\mu_{0G}\rho_p c^2}\right)^2} - 1\right]m_{ip} =$$

$$= m_{ip} - \chi_p m_{ip}$$

where ρ_n and ρ_p are the mass density of *neutrons* and *protons,* respectively.

In the Tajmar experiment induced accelerations fields outside the superconductor were observed in the order about $100\mu g$ at angular velocities of about $500 rad.s^{-1}$.

Starting from $g = Gm_{g(initial)}/r$ we can write that $g + \Delta g = G(m_{g(initial)} + \Delta m_g)/r$. Then we get $\Delta g = G\Delta m_g/r$.

For $\Delta g = \eta g = \eta Gm_{g(initial)}/r$ it follows that $\Delta m_g = \eta m_{g(initial)} = \eta m_i$. Therefore a variation of $\Delta g = \eta g$ corresponds to a gravitational mass variation $\Delta m_g = \eta m_{i0}$. Thus $\Delta g \approx 100\mu g = 1\times 10^{-4} g$ corresponds to:

$$\Delta m_g \approx 1\times 10^{-4} m_{i0}$$

On the other hand the total gravitational mass of a particle can be expressed by:

$$\begin{aligned} m_g &= N_n m_{gn} + N_p m_{gp} + N_e m_{ge} + N_p \Delta E/c^2 = \\ &N_n(m_{in} - \chi_n m_{in}) + N_p(m_{ip} - \chi_p m_{ip}) + \\ &+ N_e(m_{ie} - \chi_e m_{ie}) + N_p \Delta E/c^2 = \\ &= (N_n m_{in} + N_p m_{ip} + N_e m_{ie}) + N_p \Delta E/c^2 - \\ &- (N_n\chi_n m_{in} + N_p\chi_p m_{ip} + N_e\chi_e m_{ie}) + N_p \Delta E/c^2 = \\ &= m_i - (N_n\chi_n m_{in} + N_p\chi_p m_{ip} + N_e\chi_e m_{ie}) + N_p \Delta E/c^2 \end{aligned}$$

where ΔE is the interaction energy; N_n, N_p, N_e are the number of neutrons, protons and electrons respectively. Since $m_{in} \cong m_{ip}$ and $\rho_n \cong \rho_p$ it follows that $\chi_n \cong \chi_p$ and consequently the expression of m_g reduces to:

$$m_g \cong m_{i0} - (2N_p\chi_p m_{ip} + N_e\chi_e m_{ie}) + N_p \Delta E/c^2 \quad (55d)$$

Assuming that $N_e\chi_e m_{ie} << 2N_p\chi_p m_{ip}$ and $N_p \Delta E/c^2 << 2N_p\chi_p m_{ip}$ Equation (55d) reduces to:

$$m_g \cong m_{i0} - 2N_p\chi_p m_{ip} = m_i - \chi_p m_i \qquad (55e)$$

or

$$\Delta m_g = m_g - m_{i0} = -\chi_p m_{i0}$$

By comparing this expression with $\Delta m_g \approx 1\times 10^{-4} m_i$ which has been obtained from Tajmar experiment, we conclude that at angular velocities $\omega \approx 500 rad.s^{-1}$ we have:

$$\chi_p \approx 1\times 10^{-4}$$

From the expression of m_{gp} we get:

$$\chi_p = 2\left[\sqrt{1+\left(\frac{B_G^2}{\mu_{0G}\rho_p c^2}\right)^2} - 1\right] =$$
$$= 2\left[\sqrt{1+\left(\frac{4\omega^2\left(\rho_p/\rho_c\right)^2}{\mu_{0G}\rho_p c^2}\right)^2} - 1\right]$$

where $\rho_p = m_p/V_p$ is the mass density of the protons.

In order to calculate V_p we need to know the type of space (metric) inside the proton. It is known that there are just 3 types of space: the space of *positive* curvature, the space of *negative* curvature and the space of *null* curvature. The negative type is obviously excluded since the volume of the proton is *finite*. On the other hand, the space of null curvature is also excluded since the space inside the proton is strongly curved by its enormous mass density. Thus we can conclude that inside the proton the space has *positive* curvature. Consequently, the volume of the proton, V_p, will be expressed by the 3-dimentional space that corresponds to a *hypersphere* in a 4-dimentional space, i.e., V_p will be the space of positive curvature the volume of which is [16]:

$$V_p = \int_0^{2\pi}\int_0^{\pi}\int_0^{\pi} r_p^3 \,\boldsymbol{sin}^2\chi\, \boldsymbol{sin}\,\theta d\chi d\theta d\phi = 2\pi^2 r_p^3$$

In the case of the Earth, for example, $\rho_{Earth} << \rho_p$. Consequently the curvature of the space inside the Earth is approximately *null* (space approximately *flat)*. Then $V_{Earth} \cong \frac{4}{3}\pi r_{Earth}^3$.

For $r_p = 1.4\times10^{-15} m$ we then get,

$$\rho_p = \frac{m_p}{V_p} \cong 3\times10^{16} kg/m^3$$

Starting from the London moment it easy to see that by precisely measuring the magnetic field and the angular velocity of the superconductor, one can calculate the mass of the Cooper-pairs. This has been done for both classical and high-Tc superconductors [17-20].

In the experiment with the highest precision to date, Tate et al., op. cit., reported a disagreement between the theoretically predicted Cooper-pair mass in Niobium of $m^*/2m_e = 0.999992$ and her experimental value of $1.000084(21)$, where m_e is the electron mass. This anomaly was actively discussed in the literature without any apparent solution [21-24].

If we consider that the apparent mass increase from Tate's measurements results from an *increase* in the gravitational mass m_g^* of the Cooper-pairs due to B_G, then we can write:

$$\frac{m_g^*}{2m_e} = \frac{m_g^*}{m_i^*} = 1.000084$$
$$\Delta m_g^* = m_g^* - m_{g(initial)}^* = m_g^* - m_i^* =$$
$$= 1.000084 m_i^* - m_i^* =$$
$$= +0.84\times10^{-4} m_i^* = \chi^* m_i^*$$

where $\chi^* = 0.84\times10^{-4}$.

From (55c) we can write that,

$$m_g^* = m_i^* + 2\left[1-\sqrt{1-\left(\frac{4\omega^2}{\mu_{0G}\rho^* c^2}\right)^2}\right] m_i^* =$$
$$= m_i^* + \chi^* m_i^*$$

where ρ^* is the Cooper-pair mass density.

Consequently we can write:

$$\chi^* = 2\left[1-\sqrt{1-\left(\frac{4\omega^2}{\mu_{0G}\rho^* c^2}\right)^2}\right] = 0.84\times10^{-4}$$

From this equation we then obtain,

$$\rho^* \cong 3\times10^{16}\, kg/m^3$$

Note that $\rho_p \cong \rho^*$.

Now we can calculate the graviphoton mass, m_{gr}, inside the Cooper-pairs fluid (coherent part of the superconductor) as:

$$m_{gr} = \mu_{0G}\rho^* \hbar/c \cong 4\times10^{-52}\, kg$$

Outside the coherent matter $(\rho_c = 0)$ the graviphoton mass will be *zero* $(m_{gr} = \mu_{0G}\rho_c\hbar/c = 0)$.

Substitution of ρ_p, $\rho_c = \rho^*$ and $\omega \approx 500 rad.s^{-1}$ into the expression of χ_p gives:

$$\chi_p \approx 1\times10^{-4}$$

Compare this value with that one obtained from the Tajmar experiment.

Therefore the decrease in the gravitational mass of the superconductor, expressed by (55e), is:

$$m_{g,SC} \cong m_{i,SC} - \chi_p m_{i,SC}$$
$$\cong m_{i,SC} - 10^{-4} m_{i,SC}$$

This corresponds to a decrease of the order of $10^{-2}\%$ in respect to the initial gravitational mass of the superconductor. However, we must also consider the *gravitational shielding effect,* produced by this decrease of $\approx 10^{-2}\%$ in the gravitational mass of the particles inside the superconductor (see Figure 2). Therefore, the *total* weight decrease in the superconductor will be much greater than $10^{-2}\%$. According to Podkletnov experiment [25] it can reach up to 1% of the total weight of the superconductor at $523.6 rad.s^{-1}$ $(5000 rpm)$. In this experiment a slight decrease (up to $\approx 1\%$) in the weight of samples hung above the disk (rotating at 5000rpm) was observed. A smaller effect on the order of 0.1% has been observed when the disk is not rotating. The percentage of weight decrease is the same for samples of different masses and chemical compounds. The effect does not seem to diminish with increases in elevation above the disk. There appears to be a "shielding cylinder" over the disk that extends upwards for at least 3 meters. No weight reduction has been observed under the disk.

It is easy to see that the decrease in the weight of samples hung above the disk (inside the "shielding cylinder" over the disk) in the Podkletnov experiment is also a consequence of the *Gravitational Shielding Effect* showed in Figure 2.

In order to explain the *Gravitational Shielding Effect*, we start with the gravitational field, $\vec{g} = -\frac{GM_g}{R^2}\hat{\mu}$, produced by a particle with gravitational mass, M_g. The gravitational flux, ϕ_g, through a spherical surface, with area S and radius R, concentric with the mass M_g, is given by:

$$\phi_g = \oint_S \vec{g}d\vec{S} = g\oint_S dS = gS =$$
$$= \frac{GM_g}{R^2}\left(4\pi R^2\right) = 4\pi GM_g$$

Note that the flux ϕ_g does not depend on the radius R of the surface S, i.e., it is the *same* through any surface concentric with the mass M_g.

Now consider a particle with gravitational mass, m'_g, placed into the gravitational field produced by M_g. According to Equation (41), we can have $m'_g/m'_{i0} = -1$, $m'_g/m'_{i0} \cong 0$ [1], $m'_g/m'_{i0} = 1$, etc. In the first case, the gravity acceleration, g', upon the particle m'_g, is $\vec{g}' = -g = +\frac{GM_g}{R^2}\hat{\mu}$.

[1] The quantization of the gravitational mass (Equation (33)) shows that for *n* = 1 the gravitational mass is not *zero* but equal to $m_{g(min)}$. In spite of the gravitational mass of a particle never to be null, the Equation (41) shows that it can be turned very close to zero.

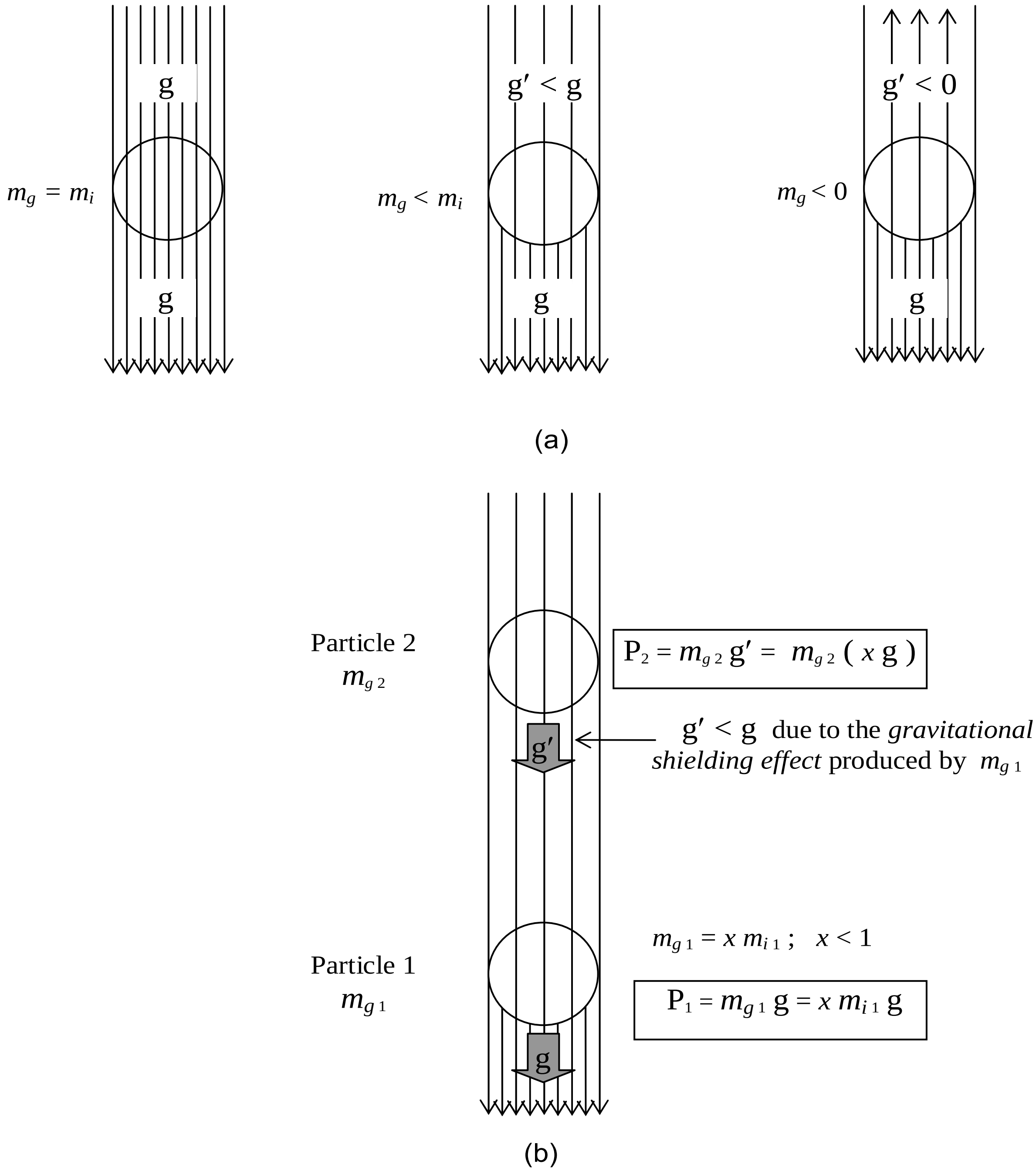


Figure 2: The Gravitational Shielding Effect.

This means that in this case, the gravitational flux, ϕ'_g, through the particle m'_g will be given by $\phi'_g = g'S = -gS = -\phi_g$ (i.e., it will be *symmetric* in respect to the flux when $m'_g = m'_{i0}$ (third case)). In the second case $\left(m'_g \cong 0\right)$, the intensity of the gravitational force between m'_g and M_g will be very close to zero. This is equivalent to say that the gravity acceleration upon the particle with mass m'_g will be $g' \cong 0$. Consequently we can write that

$\phi'_g = g'S \cong 0$. It is easy to see that there is a correlation between m'_g / m'_{i0} and ϕ'_g / ϕ_g, i.e.,

_ If $m'_g / m'_{i0} = -1 \quad \Rightarrow \quad \phi'_g / \phi_g = -1$

_ If $m'_g / m'_{i0} = 1 \quad \Rightarrow \quad \phi'_g / \phi_g = 1$

_ If $m'_g / m'_{i0} \cong 0 \quad \Rightarrow \quad \phi'_g / \phi_g \cong 0$

Just a simple algebraic form contains the requisites mentioned above, the correlation:

$$\frac{\phi'_g}{\phi_g} = \frac{m'_g}{m'_{i0}}$$

By making $m'_g / m'_{i0} = \chi$ we get:

$$\phi'_g = \chi \ \phi_g$$

This is the expression of the gravitational flux through m'_g. It explains the *Gravitational Shielding Effect* presented in Figure 2.

As $\phi_g = gS$ and $\phi'_g = g'S$ we obtain:

$$g' = \chi \ g$$

This is the gravity acceleration inside m'_g.

Figure 2(b) shows the gravitational shielding effect produced by two particles at the same direction. In this case, the gravity acceleration inside and above the second particle will be $\chi^2 g$ if $m_{g2} = m_{i1}$.

These particles are representative of any material particles or material *substance* (solid, liquid, gas, plasma, electrons flux, etc.), whose gravitational mass have been reduced by the factor χ. Thus, *above* the substance, the gravity acceleration g' is reduced at the same proportion $\chi = m_g / m_{i0}$, and, consequently, $g' = \chi \ g$, where g is the gravity acceleration *under* the substance.

Figure 3 shows an experimental set-up in order to check the factor χ above *a high-speed electrons flux*. As we have shown (Equation 43), the *gravitational mass* of a particle decreases with the increase of the velocity V of the particle.

Since the theory says that the factor χ is given by the correlation m_g / m_{i0} then, in the case of an electrons flux, we will have that $\chi = m_{ge} / m_{ie}$ where m_{ge} as function of the velocity V is given by Equation (43). Thus, we can write that:

$$\chi = \frac{m_{ge}}{m_{ie}} = \left\{ 1 - 2\left[\frac{1}{\sqrt{1 - V^2/c^2}} - 1 \right] \right\}$$

Therefore, if we know the velocity V of the electrons we can calculate χ. (m_{ie} is the electron mass at rest).

When an electron penetrates the electric field E_y (Figure 3) an electric force, $\vec{F}_E = -e\vec{E}_y$, will acts upon the electron. The direction of $\vec{F}_E$ will be contrary to the direction of $\vec{E}_y$. The magnetic force $\vec{F}_B$ which acts upon the electron, due to the magnetic field $\vec{B}$, is $\vec{F}_B = eVB\hat{\mu}$ and will be opposite to $\vec{F}_E$ because the electron charge is negative.

By adjusting conveniently B we can make $\left|\vec{F}_B\right| = \left|\vec{F}_E\right|$. Under these circumstances in which the total force is zero, the spot produced by the electrons flux on the surface α returns from O' to O and is detected by the galvanometer G. That is, there is no deflection for the cathodic rays. Then it follows that $eVB = eE_y$ since $\left|\vec{F}_B\right| = \left|\vec{F}_E\right|$. Then, we get:

$$V = \frac{E_y}{B}$$

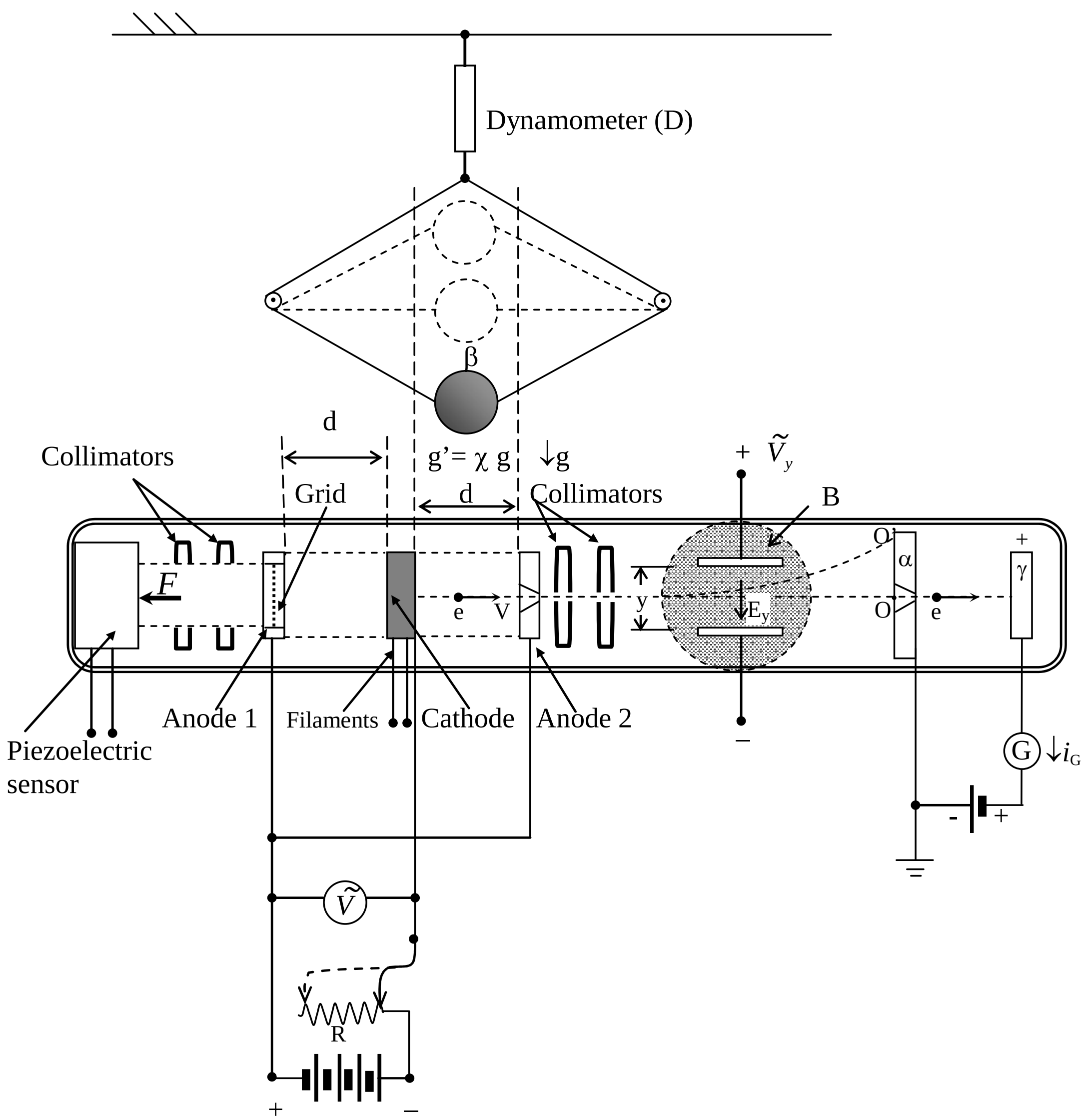


Figure 3: Experimental Set-Up in Order to Check the Factor χ above a High-Speed Electrons Flux. (The set-up may also check the velocities and the gravitational masses of the electrons).

This gives a measure of the velocity of the electrons.

Thus, by means of the experimental set-up, shown in Figure 3, we can easily obtain the velocity V of the electrons below the body β, in order to calculate the *theoretical* value of χ. The *experimental* value of can be obtained by dividing the weight, $P'_\beta = m_{g\beta} g'$ of the body β for a voltage drop $\widetilde{V}$ across the anode and cathode, by its weight, $P_\beta = m_{g\beta} g$, when the voltage $\widetilde{V}$ is *zero*, i.e.:

$$\chi = \frac{P'_\beta}{P_\beta} = \frac{g'}{g}$$

According to Equation (4), the gravitational mass, M_g, is defined by:

$$M_g = \left| \frac{m_g}{\sqrt{1 - V^2/c^2}} \right|$$

While Equation (43) define m_g by means of the following expression:

$$m_g = \left\{ 1 - 2\left[\frac{1}{\sqrt{1 - V^2/c^2}} - 1 \right] \right\} m_{i0}$$

In order to check the gravitational mass of the electrons it is necessary to known the pressure P produced by the electrons flux. Thus, we have put a piezoelectric sensor in the bottom of the glass tube as shown in Figure 3. The electrons flux radiated from the cathode is accelerated by the anode1 and strikes on the piezoelectric sensor yielding a pressure P which is measured by means of the sensor.

Let us now deduce the correlation between P and M_{ge}. When the electrons flux strikes on the sensor, the electrons transfer to it a *momentum* $Q = n_e q_e = n_e M_{ge} V$.

Since $Q = F\Delta t = 2Fd/V$, we conclude that:

$$M_{ge} = \frac{2d}{V^2}\left(\frac{F}{n_e} \right)$$

The amount of electrons, n_e, is given by $n_e = \rho S d$ where ρ is the amount of electrons per unit of volume (electrons/m^3); S is the cross-section of the electrons flux and d the distance between cathode and anode.

In order to calculate n_e we will start from the *Langmuir-Child law* and the *Ohm vectorial law*, respectively given by:

$$J = \alpha \frac{\tilde{V}^{\frac{3}{2}}}{d} \text{ and } J = \rho_c V \text{ , } \quad (\rho_c = \rho/e)$$

where J is the thermoionic current density; $\alpha = 2.33 \times 10^{-6} A.m^{-1}.V^{-\frac{3}{2}}$ is the called *Child's constant*; $\tilde{V}$ is the voltage drop across the anode and cathode electrodes, and V is the velocity of the electrons.

By comparing the Langmuir-Child law with the Ohm vectorial law we obtain:

$$\rho = \frac{\alpha \tilde{V}^{\frac{3}{2}}}{ed^2 V}$$

Thus, we can write that:

$$n_e = \frac{\alpha \tilde{V}^{\frac{3}{2}} S}{edV}$$

and

$$M_{ge} = \left(\frac{2ed^2}{\alpha V \tilde{V}^{\frac{3}{2}}} \right) P$$

where $P = F/S$, is the pressure to be measured by the piezoelectric sensor.

In the experimental set-up the total force F acting on the piezoelectric sensor is the resultant of all the forces F_ϕ produced by each electrons flux that passes through each hole of area S_ϕ in the grid of the anode 1, and is given by:

$$F = nF_\phi = n(PS_\phi) = \left(\frac{\alpha n S_\phi}{2ed^2} \right) M_{ge} V \tilde{V}^{\frac{3}{2}}$$

where n is the number of holes in the grid. By means of the piezoelectric sensor we can measure F and consequently obtain M_{ge}.

We can use the equation above to evaluate the magnitude of the force F to be measured by the piezoelectric sensor. First, we will find the expression of V as a function of $\tilde{V}$ since the electrons speed V depends on the voltage, $\tilde{V}$.

We will start from Equations (46) which is the general expression for Lorentz's force, i.e.,

$$\frac{d\vec{p}}{dt} = \left(q\vec{E} + q\vec{V} \times \vec{B} \right) \frac{m_g}{m_{i0}}$$

When the force and the speed have the same direction Equation (6) gives:

$$\frac{d\vec{p}}{dt} = \left| \frac{m_g}{\left(1 - V^2/c^2\right)^{\frac{3}{2}}} \right| \frac{d\vec{V}}{dt}$$

By comparing these expressions we obtain:

$$\frac{m_{i0}}{\left(1 - V^2/c^2\right)^{\frac{3}{2}}} \frac{d\vec{V}}{dt} = q\vec{E} + q\vec{V} \times \vec{B}$$

In the case of electrons accelerated by an electric field only $(B = 0)$, the equation above gives:

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{e\vec{E}}{m_{ie}} \left(1 - V^2/c^2\right) \sqrt{\frac{2e\widetilde{V}}{m_{ie}}}$$

Therefore the velocity V of the electrons in the experimental set-up is:

$$V = \sqrt{2ad} = \left(1 - V^2/c^2\right)^{\frac{3}{4}} \sqrt{\frac{2e\widetilde{V}}{m_{ie}}}$$

From Equation (43) we conclude that $m_{ge} \cong 0$ when $V \cong 0.745c$. Substitution of this value of V into equation above gives $\widetilde{V} \cong 479.1KV$. This is the voltage drop necessary to be applied across the anode and cathode electrodes in order to obtain $m_{ge} \cong 0$.

Since the equation above can be used to evaluate the velocity V of the electrons flux for a given $\widetilde{V}$, then we can use the obtained value of V to evaluate the intensity of $\vec{B}$ in order to produce $eVB = eE_y$ in the experimental set-up. Then by adjusting B we can check when the electrons flux is detected by the galvanometer G. In this case, as we have already seen, $eVB = eE_y$, and the velocity of the electrons flux is calculated by means of the expression $V = E_y/B$. Substitution of V into the expressions of m_{ge} and M_{ge}, respectively given by:

$$m_{ge} = \left\{1 - 2\left[\frac{1}{\sqrt{1 - V^2/c^2}} - 1\right]\right\} m_{ie}$$

and

$$M_{ge} = \left| \frac{m_{ge}}{\sqrt{1 - V^2/c^2}} \right|$$

yields the corresponding values of m_{ge} and M_{ge} which can be compared with the values obtained in the experimental set-up:

$$m_{ge} = \chi m_{ie} = \left(P'_\beta / P_\beta\right) m_{ie}$$

$$M_{ge} = \frac{F}{V\widetilde{V}^{\frac{3}{2}}} \left(\frac{2ed^2}{\alpha n S_\phi}\right)$$

where P'_β and P_β are measured by the dynamometer D and F is measured by the piezoelectric sensor.

If we have $nS_\phi \cong 0.16m^2$ and $d = 0.08m$ in the experimental set-up then it follows that:

$$F = 1.82 \times 10^{14} M_{ge} V \widetilde{V}^{\frac{3}{2}}$$

By varying $\widetilde{V}$ from 10KV up to 500KV we note that the maximum value for F occurs when $\widetilde{V} \cong 344.7KV$. Under these circumstances, $V \cong 0.7c$ and $M_{ge} \cong 0.28 m_{ie}$. Thus the maximum value for F is:

$$F_{max} \cong 1.9N \cong 190gf$$

Consequently, for $\widetilde{V}_{max} = 500KV$, the piezoelectric sensor must satisfy the following characteristics:

– Capacity 200gf
– Readability 0.001gf

Let us now return to the explanation for the findings of Podkletnov's experiment. Next, we will explain the decrease of 0.1% in the weight of the superconductor when the disk is only levitating but not rotating.

Equation (55) shows how the gravitational mass is altered by *electromagnetic* fields.

The expression of n_r for $\sigma >> \omega\varepsilon$ can be obtained from (54), in the form:

$$n_r = \frac{c}{v} = \sqrt{\frac{\mu\sigma c^2}{4\pi f}} \qquad (56)$$

Substitution of (56) into (55) leads to,

$$m_g = \left\{1 - 2\left[\sqrt{1 + \frac{\mu\sigma}{4\pi f}\left(\frac{U}{m_i c}\right)^2} - 1\right]\right\} m_{i0}$$

This equation shows that *atoms of ferromagnetic materials with very-high* μ can have gravitational masses strongly reduced by means of *Extremely Low Frequency* (ELF) electromagnetic radiation. It also shows that atoms of *superconducting materials* (due to *very-high* σ) can also have its gravitational masses strongly reduced by means of ELF electromagnetic radiation.

Alternatively, we may put Equation (55) as a function of the *power density* (or intensity), D, of the radiation. The integration of (51) gives $U = VD/v$. Thus, we can write (55) in the following form:

$$m_g = \left\{1 - 2\left[\sqrt{1 + \left(\frac{n_r^2 D}{\rho c^3}\right)^2} - 1\right]\right\} m_{i0} \qquad (57)$$

where $\rho = m_{i0}/V$.

For $\sigma >> \omega\varepsilon$, n_r will be given by (56) and consequently (57) becomes:

$$m_g = \left\{1 - 2\left[\sqrt{1 + \left(\frac{\mu\sigma D}{4\pi f \rho c}\right)^2} - 1\right]\right\} m_{i0} \qquad (58)$$

In the case of *Thermal radiation*, it is common to relate the energy of photons to *temperature*, *T*, through the relation,

$$\langle hf \rangle \approx \kappa T$$

where $\kappa = 1.38 \times 10^{-23} J/°K$ is the *Boltzmann's constant*. On the other hand it is known that:

$$D = \sigma_B T^4$$

where $\sigma_B = 5.67 \times 10^{-8} watts/m^2 °K^4$ is the *Stefan-Boltzmann's constant*. Thus we can rewrite (58) in the following form:

$$m_g = \left\{1 - 2\left[\sqrt{1 + \left(\frac{\mu\sigma\sigma_B h T^3}{4\pi\kappa\rho c}\right)^2} - 1\right]\right\} m_{i0} \qquad (58a)$$

Starting from this equation we can evaluate the effect of the *thermal radiation* upon the gravitational mass of the Copper-pair fluid, $m_{g,CPfluid}$. Below the *transition temperature,* T_c, $(T/T_c < 0.5)$ the conductivity of the superconducting materials is usually greater than $10^{22} S/m$ [26]. On the other hand the *transition temperature*, for high critical temperature (HTC) superconducting materials, is in the order of $10^2 K$. Thus (58a) gives:

$$m_{g,CPfluid} = \left\{1 - 2\left[\sqrt{1 + \frac{\sim 10^{-9}}{\rho_{CPfluid}^2}} - 1\right]\right\} m_{i,CPfluid} \qquad (58a)$$

Assuming that the number of Copper-pairs per unit volume is $N \approx 10^{26} m^{-3}$ [27] we can write:

$$\rho_{CPfluid} = Nm^* \approx 10^{-4} kg/m^3$$

Substitution of this value into (58a) yields:

$$m_{g,CPfluid} = m_{i,CPfluid} - 0.1\ m_{i,CPfluid} \qquad (58a)$$

This means that the gravitational masses of the electrons are decreased of ~10%. This corresponds to a decrease in the gravitational mass of the superconductor given by:

$$\frac{m_{g,SC}}{m_{i,SC}}=\frac{N\left(m_{ge}+m_{gp}+m_{gn}+\Delta E/c^2\right)}{N\left(m_{ie}+m_{ip}+m_{in}+\Delta E/c^2\right)}=$$
$$=\left(\frac{m_{ge}+m_{gp}+m_{gn}+\Delta E/c^2}{m_{ie}+m_{ip}+m_{in}+\Delta E/c^2}\right)=$$
$$=\left(\frac{0.9m_{ie}+m_{ip}+m_{in}+\Delta E/c^2}{m_{ie}+m_{ip}+m_{in}+\Delta E/c^2}\right)=$$
$$=0.999976$$

where ΔE is the interaction energy. Therefore, a decrease of $(1-0.999976)\approx 10^{-5}$, (i.e., approximately $10^{-3}\%$ in respect to the initial gravitational mass of the superconductor) due to the local *thermal radiation* only. However, here we must also consider the *gravitational shielding effect* produced, in this case, by the decrease of $\approx 10^{-3}\%$ in the gravitational mass of the particles inside the superconductor (Figure 2). Therefore the *total* weight decrease in the superconductor will be much greater than $\approx 10^{-3}\%$. This can explain the smaller effect on the order of 0.1% observed in the Podkletnov measurements when the disk is not rotating.

Let us now consider an electric current I through a conductor subjected to electromagnetic radiation with power density D and frequency f.

Under these circumstances the *gravitational mass* m_{ge} of the *electrons* of the conductor, according to Equation (58), is given by:

$$m_{ge}=\left\{1-2\left[\sqrt{1+\left(\frac{\mu\sigma D}{4\pi f\rho c}\right)^2}-1\right]\right\}m_e$$

where $m_e=9.11\times10^{-31}kg$. Note that if the radiation upon the conductor has extremely-low frequency (ELF radiation) then m_{ge} can be strongly reduced. For example, if $f\approx 10^{-6}Hz$, $D\approx 10^5 W/m^2$ and the conductor is made of *copper:*

($\mu\cong\mu_0$; $\sigma=5.8\times10^7 S/m$ and $\rho=8900kg/m^3$)

then

$$\left(\frac{\mu\sigma D}{4\pi f\rho c}\right)\approx 1$$

and consequently $m_{ge}\approx 0.1m_e$.

According to Equation (6) the force upon each *free electron* is given by:

$$\vec{F}_e=\left|\frac{m_{ge}}{\left(1-V^2/c^2\right)^{3/2}}\right|\frac{d\vec{V}}{dt}=e\vec{E}$$

where E is the applied electric field. Therefore the decrease of m_{ge} produces an increase in the velocity V of the free electrons and consequently the *drift velocity* V_d is also increased. It is known that the density of electric current J through a conductor [28] is given by:

$$\vec{J}=\Delta_e\vec{V}_d$$

where Δ_e is the density of the free electric charges (For cooper conductors $\Delta_e=1.3\times10^{10}C/m^3$). Therefore increasing V_d produces an increase in the electric current I. Thus if m_{ge} is reduced 10 times $(m_{ge}\approx 0.1m_e)$ the drift velocity V_d is increased 10 times as well as the electric current. Thus we conclude that strong fluxes of ELF radiation upon electric/electronic circuits can suddenly increase the electric currents and consequently damage these circuits.

Since the *orbital electrons* moment of inertia is given by $I_i=\Sigma(m_i)_j r_j^2$, where m_i refers to *inertial mass* and not to gravitational mass, then the *momentum* $L=I_i\omega$ of the conductor *orbital electrons* are not affected by the ELF radiation. Consequently this radiation just affects the conductor *free electron* velocities. Similarly, in the case of superconducting materials, the *momentum*, $L = I_i \omega$, of the *orbital electrons* are not affected by the gravitomagnetic fields.

The vector $\vec{D}=(U/V)\vec{v}$, which we may define from (48), has the same direction of the propagation vector $\vec{k}$ and evidently corresponds to the *Poynting vector*. Then $\vec{D}$ can be replaced by $\vec{E}\times\vec{H}$. Thus we can write $D=EH=E(B/\mu)=E[(E/v)/\mu]=(1/v\mu)E^2$.

For $\sigma >> \omega\varepsilon$ the Equation (54) tells us that $v=\sqrt{4\pi f/\mu\sigma}$. Consequently, we obtain:

$$D = E^2\sqrt{\frac{\sigma}{4\pi f\mu}}$$

This expression refers to the instantaneous values of D and E. The average value for E^2 is equal to $\frac{1}{2}E_m^2$ because E varies sinusoidaly (E_m is the maximum value for E). Consequently the equation above tells us that the average density $\overline{D}$ is:

$$\overline{D} = \tfrac{1}{2}E_m^2\sqrt{\frac{\sigma}{4\pi f\mu}}$$

Substitution of this expression into (58) yields the expression for $\overline{m}_g$. Substitution of the expression of D into (58) gives:

$$m_g = \left\{1-2\left[\sqrt{1+\frac{\mu}{c^2}\left(\frac{\sigma}{4\pi f}\right)^3\frac{E^4}{\rho^2}}-1\right]\right\}m_{i0} \quad (59a)$$

Note that for *extremely-low frequencies* the value of f^{-3} in this equation becomes highly expressive. Since $E = vB$ the equation (59a) can also be put as a function of B, i.e.,

$$m_g = \left\{1-2\left[\sqrt{1+\left(\frac{\sigma}{4\pi f\mu c^2}\right)\frac{B^4}{\rho^2}}-1\right]\right\}m_{i0} \quad (59b)$$

For *conducting* materials with $\sigma \approx 10^7 S/m$; $\mu_r = 1$; $\rho \approx 10^3 kg/m^3$ the expression (59b) gives:

$$m_g = \left\{1-2\left[\sqrt{1+\left(\frac{\approx 10^{-12}}{f}\right)B^4}-1\right]\right\}m_{i0}$$

This equation shows that the decreasing in the *gravitational mass* of these conductors can become experimentally detectable for example, starting from 100Teslas at 10mHz.

One can then conclude that an interesting situation arises when a body penetrates a magnetic field in the direction of its center. The *gravitational mass* of the body decreases progressively. This is due to the intensity increase of the magnetic field upon the body while it penetrates the field. In order to understand this phenomenon we might, based on (43), think of the inertial mass as being formed by two parts: one *positive* and another *negative*. Thus, when the body penetrates the magnetic field its negative inertial mass increase, but its total inertial mass decreases, i.e., although there is an increase of inertial mass, the total inertial mass (which is equivalent to *gravitational mass*) will be reduced.

On the other hand, Equation (4) shows that the *velocity of the body must increase* as consequence of the gravitational mass decreasing since the *momentum* is conserved. Consider for example a spacecraft with velocity V_s and gravitational mass M_g. If M_g is reduced to m_g then the velocity becomes:

$$V_s' = (M_g/m_g)V_s$$

In addition, Equations 5 and 6 tell us that the *inertial forces* depend on m_g. Only in the particular case of $m_g = m_{i0}$ the expressions (5) and (6) reduce to the well-known Newtonian expression $F = m_{i0}a$. Consequently, one can conclude that the *inertial effects* on the spacecraft will also be reduced due to the *decreasing of its gravitational mass*. Obviously this leads to a new concept of aerospace flight.

Now consider an electric current $i = i_0 sin 2\pi ft$ through a conductor. Since the current density, $\vec{J}$, is expressed by $\vec{J} = di/d\vec{S} = \sigma\vec{E}$, then we can write that $E = i/\sigma S = (i_0/\sigma S) sin 2\pi ft$. Substitution of this equation into (59a) gives:

$$m_g = \left\{1 - 2\left[\sqrt{1 + \frac{i_0^4 \mu}{64\pi^3 c^2 \rho^2 S^4 f^3 \sigma} sin^4 2\pi ft} - 1\right]\right\} m_{i0} \quad (59c)$$

If the conductor is a *supermalloy* rod $(1 \times 1 \times 400 mm)$ then $\mu_r = 100{,}000$ (initial); $\rho = 8770 kg/m^3$; $\sigma = 1.6 \times 10^6 S/m$ and $S = 1 \times 10^{-6} m^2$. Substitution of these values into equation above yields the following expression for the *gravitational mass* of the supermalloy rod:

$$m_{g(sm)} = \left\{1 - 2\left[\sqrt{1 + \left(5.71 \times 10^{-12} i_0^4 / f^3\right) sin^4 2\pi ft} - 1\right]\right\} m_{i(sm)}$$

Some oscillators like the HP3325A (Op.002 High Voltage Output) can generate sinusoidal voltages with *extremely-low* frequencies down to $f = 1 \times 10^{-6} Hz$ and amplitude up to 20V (into 50Ω load). The maximum output current is $0.08A_{pp}$.

Thus, for $i_0 = 0.04A$ $(0.08A_{pp})$ and $f < 2.25 \times 10^{-6} Hz$ the equation above shows that the *gravitational mass* of the rod becomes *negative* at $2\pi ft = \pi/2$; for $f \cong 1.7 \times 10^{-6} Hz$ at $t = 1/4f = 1.47 \times 10^5 s \cong 40.8h$ it shows that $m_{g(sm)} \cong -m_{i(sm)}$.

This leads to the idea of the *Gravitational Motor*. Figure 4 shows a type of gravitational motor (Rotational Gravitational Motor) based on the possibility of gravity control on a ferromagnetic wire.

It is important to realize that this is not the unique way of decreasing the gravitational mass of a body. It was noted earlier that the expression (53) is general for all types of waves including non-electromagnetic waves like sound waves for example. In this case, the velocity v in (53) will be the *speed of sound in the body* and D the *intensity* of the sound radiation. Thus from (53) we can write that:

$$\frac{\Delta p}{m_i c} = \frac{VD}{m_i c} = \frac{D}{\rho c v^2}$$

It can easily be shown that $D = 2\pi^2 \rho f^2 A^2 v$ where $A = \lambda P / 2\pi\rho v^2$; A and P are respectively the amplitude and maximum pressure variation of the sound wave. Therefore we readily obtain:

$$\frac{\Delta p}{m_{i0} c} = \frac{P^2}{2\rho^2 c v^3}$$

Substitution of this expression into (41) gives:

$$m_g = \left\{1 - 2\left[\sqrt{1 + \left(\frac{P^2}{2\rho^2 c v^3}\right)^2} - 1\right]\right\} m_{i0}$$

This expression shows that in the case of sound waves the decreasing of gravitational mass is relevant for *very strong pressures* only.

It is known that in the nucleus of the Earth the pressure can reach values greater than $10^{13} N/m^2$. The equation above tells us that sound waves produced by pressure variations of this magnitude can cause strong decreasing of the *gravitational mass* at the surroundings of the point where the sound waves were generated. This obviously must cause an abrupt decreasing of the pressure at this place since pressure = weight /area = $m_g g$/area). Consequently a local instability will be produced due to the opposite internal pressure. The conclusion is that this effect may cause Earthquakes.

Consider a sphere of radius r around the point where the sound waves were generated (at $\approx 1{,}000 km$ depth; the Earth's radius is $6{,}378 km$).

If the *maximum* pressure, at the explosion place (sphere of radius r_0), is $P_{max} \approx 10^{13} N/m^2$ and the pressure at the distance $r = 10 km$ is $P_{min} = (r_0/r)^2 P_{max} \approx 10^9 N/m^2$ then we can consider that in the sphere $P = \sqrt{P_{max} P_{min}} \approx 10^{11} N/m^2$.

$$\alpha = \left\{1 - 2\left[\sqrt{1 + \frac{i^4 \mu}{64\pi^3 c^2 \rho^2 S^4 f^3 \sigma}} - 1\right]\right\}$$

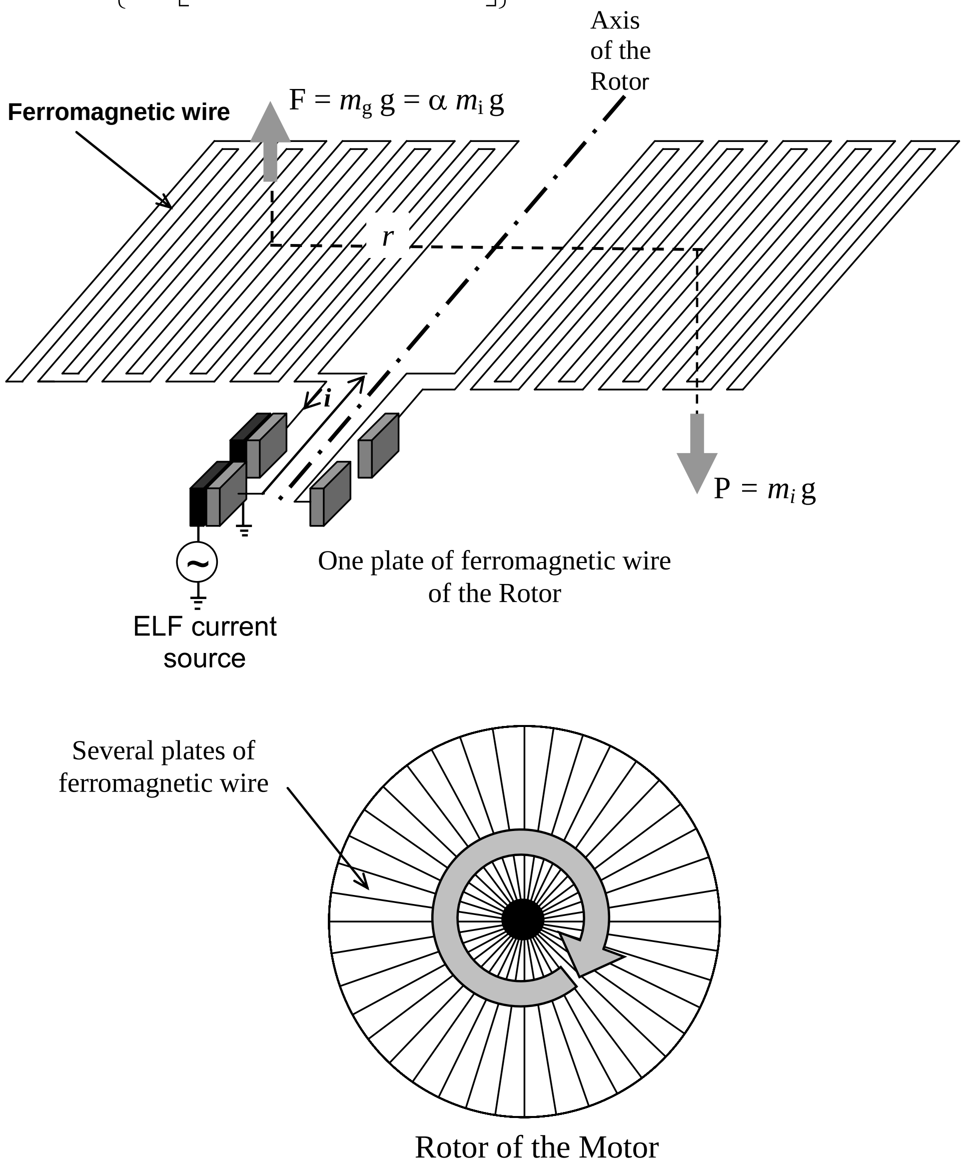


Figure 4: Rotational Gravitational Motor.

Thus assuming $v \approx 10^3 m/s$ and $\rho \approx 10^3 kg/m^3$ we can calculate the variation of gravitational mass in the sphere by means of the equation of m_g ,i.e.,

$$\Delta m_g = m_{g(initial)} - m_g =$$

$$= m_{i0} - \left\{1 - 2\left[\sqrt{1 + \left(\frac{P^2}{2\rho^2 c v^3}\right)^2} - 1\right]\right\} m_{i0} =$$

$$= 2\left[\sqrt{1 + \left(\frac{P^2}{2\rho^2 c v^3}\right)^2} - 1\right]\rho V \approx 10^{11} kg$$

The *transitory* loss of this great amount of gravitational mass may produce evidently a strong pressure variation and consequently a strong Earthquake.

Finally, we can evaluate the energy necessary to generate those sound waves. From (48) we can write $D_{max} = P_{max}v \approx 10^{16} W / m^2$. Thus, the released power is $P_0 = D_{max}(4\pi r_0^2) \approx 10^{21} W$ and the energy ΔE released at the time interval Δt must be $\Delta E = P_0 \Delta t$. Assuming $\Delta t \approx 10^{-3} s$ we readily obtain:

$$\Delta E = P_0 \Delta t \approx 10^{18}\ joules \approx 10^4\ Megatons$$

This is the amount of energy released by a magnitude 9 earthquake $(M_s = 9)$, (i.e., $E = 1.74 \times 10^{(5+1.44M_s)} \cong 10^{18}\ joules$). The maximum magnitude in the *Richter* scale is 12. Note that the sole releasing of this energy at 1000km depth (without the effect of gravitational mass decreasing) cannot produce an earthquake, since the sound waves reach 1km depth with pressures less than 10N/cm^2.

Let us now return to the Theory. The equivalence between frames of non-inertial reference and gravitational fields assumed $m_g \equiv m_i$ because the inertial forces were given by $\vec{F}_i = m_i \vec{a}$, while the equivalent gravitational forces, by $\vec{F}_g = m_g \vec{g}$. Thus, to satisfy the equivalence ($\vec{a} \equiv \vec{g}$ and $\vec{F}_i \equiv \vec{F}_g$) it was *necessary that* $m_g \equiv m_i$. Now, the inertial force, $\vec{F}_i$, is given by Equation (6), and from the Equation (13) we can obtain the gravitational force, $\vec{F}_g$. Thus, $\vec{F}_i \equiv \vec{F}_g$ leads to:

$$\frac{m_g}{\left(1 - V^2/c^2\right)^{3/2}}\vec{a} \equiv G\frac{m'_g}{\left(r'\sqrt{1 - V^2/c^2}\right)^2}\frac{m_g}{\sqrt{1 - V^2/c^2}} \equiv$$

$$\equiv \left(G\frac{m'_g}{r'^2}\right)\frac{m_g}{\left(1 - V^2/c^2\right)^{3/2}} \equiv \vec{g}\frac{m_g}{\left(1 - V^2/c^2\right)^{3/2}}$$

hence:

$$\vec{a} \equiv \vec{g}$$

Consequently, the equivalence is evident, and therefore the Einstein's equations from the General Relativity continue obviously valid.

The new expression for F_i (Equations (5) and (6)) shows that the inertial forces are proportional to the *gravitational mass*, m_g. This means that these forces result from the gravitational interaction between the particle and the other gravitational masses of the Universe, just as Mach's *principle* predicts. Therefore the new expression for the inertial forces incorporates the Mach's principle into Gravitation Theory, and furthermore reveals that the inertial effects upon a particle can be reduced because, as we have seen, the gravitational mass may be reduced.

When $m_g = m_{i0}$ the *nonrelativistic* equation for inertial forces, $\vec{F}_i = m_g \vec{a}$, reduces to $\vec{F}_i = m_{i0}\vec{a}$. This is the well-known *Newton's second law* for motion.

In Einstein's Special Relativity Theory the motion of a free-particle is described by means of $\delta S = 0$ [29]. Now based on Equation (1), $\delta S = 0$ will be given by the following expression:

$$\delta S = -m_g c \delta \int ds = 0.$$

which also describes the motion of the particle inside the gravitational field. Thus, the Einstein's equations from the General Relativity can be derived starting from $\delta(S_m + S_g) = 0$, where S_g and S_m refer to *action* of the gravitational field and the action of the matter, respectively [30].

The variations δS_g and δS_m can be written as follows [31]:

$$\delta S_g = \frac{c^3}{16\pi G}\int \left(R_{ik} - \tfrac{1}{2} g_{ik} R\right)\delta g^{ik}\sqrt{-g}\,d\Omega \qquad (60)$$

$$\delta S_m = -\frac{1}{2c}\int T_{ik}\delta g^{ik}\sqrt{-g}\,d\Omega \qquad (61)$$

where R_{ik} is the Ricci's tensor; g_{ik} the metric tensor and T_{ik} the matter's energy-momentum tensor:

$$T_{ik} = \left(P + \varepsilon_g\right)\mu_i \mu_k + P g_{ik} \qquad (62)$$

where P is the pressure and $\varepsilon_g = \rho_g c^2$ is now, the density of gravitational energy, E_g, of the particle; ρ_g is then the density of gravitational mass of the particle (i.e., M_g at the volume unit).

Substitution of (60) and (61) into $\delta S_m + \delta S_g = 0$ yields:

$$\frac{c^3}{16\pi G}\int \left(R_{ik} - \tfrac{1}{2} g_{ik} R - \tfrac{8\pi G}{c^4} T_{ik}\right)\delta g^{ik}\sqrt{-g}\, d\Omega = 0$$

whence,

$$\left(R_{ik} - \tfrac{1}{2} g_{ik} R - \tfrac{8\pi G}{c^4} T_{ik}\right) = 0 \qquad (63)$$

because the δg_{ik} are arbitrary.

Equations (63) in the following form:

$$R_{ik} - \tfrac{1}{2} g_{ik} R = \tfrac{8\pi G}{c^4} T_{ik} \qquad (64)$$

or

$$R_i^k - \tfrac{1}{2} g \delta_i^k R = \tfrac{8\pi G}{c^4} T_i^k . \qquad (65)$$

are the Einstein's equations from the General Relativity.

It is known that these equations are *only valid* if the spacetime is *continuous*. We have shown at the beginning of this work that the spacetime *is not continuous* it is *quantized*. However, the spacetime can be considered approximately "*continuous*" when the *quantum number* n is very large (Classical limit). Therefore, just under these circumstances the Einstein's equations from the General Relativity can be used in order to "*classicalize*" the quantum theory by means of approximated description of the spacetime.

Forward, we will show that the length $d_{\min}$ of Equation (29) is given by:

$$d_{\min} = \tilde{k} l_{planck} = \tilde{k}\left(G\hbar/c^3\right)^{\frac{1}{2}} \approx 10^{-34} m$$

(See Equation (100)).

On the other hand, we will find in the Equation (129) the length scale of the *initial Universe* (i.e., $d_{initial} \approx 10^{-14} m$). Thus, from the Equation (29) we get: $n = d_{initial}/d_{\min} = 10^{-14}/10^{-34} \approx 10^{20}$ this is the quantum number of the spacetime at *initial instant*. That quantum number is sufficiently large for the spacetime to be considered approximately "*continuous*" starting from the beginning of the Universe. Therefore the Einstein's equations can be used even at the Initial Universe.

Now it is easy to conclude because the attempt of quantization of gravity starting from the General Relativity was a bad theoretical strategy.

Making on the obtained Einstein's equations for the gravitational field, the transition to the Classical Mechanics, we obtain:

$$\Delta\Phi = 4\pi G\left(\frac{\varepsilon_g}{c^2}\right) = 4\pi G \rho_g \qquad (66)$$

This is the nonrelativistic equation for the gravitational field, the general solution of which is

$$\Phi = -G\int \frac{\varepsilon_g dV}{rc^2} \qquad (67)$$

This equation expresses the nonrelativistic potential of the gravitational field for any distribution of mass. In particular, for only one particle with gravitational energy $E_g = m_g c^2$, the result is:

$$\Phi = -GE_g / rc^2 \qquad (68)$$

Thus, the gravity $\vec{g}$ into the gravitational field created by the particle is:

$$\vec{g} = -\frac{\partial\Phi}{\partial r} = -G\frac{E_g}{\vec{r}^2 c^2} = -G\frac{m_g}{\vec{r}^2} . \qquad (69)$$

Therefore, the gravitational force $\vec{F}_g$ which acts on that field, upon another particle of gravitational mass m'_g is then given by:

$$\vec{F}_g = m'_g \vec{g} = -G\frac{m_g m'_g}{\vec{r}^2} \qquad (70)$$

If $m_g > 0$ and $m'_g < 0$, or $m_g < 0$ and $m'_g > 0$ the force will be *repulsive*; *the force will never be null* due to the existence of a *minimum value* for m_g (see Equation (24)). However, if $m_g < 0$ and $m'_g < 0$, or $m_g > 0$ and $m'_g > 0$ the force will be *attractive*. Just for $m_g = m_i$ and $m'_g = m'_i$ we obtain the *Newton's attraction law*.

Since the gravitational interaction can be repulsive, besides attractive, such as the electromagnetic interaction, then the *graviton* must have spin 1 (called *graviphoton*) and not 2. Consequently, the gravitational forces are also *gauge* forces because they are yield by the exchange of so-called "virtual" *quanta* of spin 1, such as the electromagnetic forces and the weak and strong nuclear forces.

Let us now deduce the *Entropy Differential Equation* starting from the Equation (55). Comparison of Equations (55) and (41) shows that $Un_r = \Delta pc$. For small velocities $(V << c)$, so that $Un_r << m_{i0}c^2$. Under these circumstances, the development of the (55) in power of $\left(Un_r / m_{i0}c^2\right)$, gives:

$$m_g = m_{i0} - \left(\frac{Un_r}{m_{i0}c^2}\right)^2 m_{i0} \qquad (71)$$

In the particular case of *thermal radiation,* it is usual to relate the energy of the photons to temperature, through the relationship $\langle h\nu \rangle \approx kT$ where $k = 1.38\times10^{-23} J/K$ is the Boltzmann's constant. Thus, in that case, the energy absorbed by the particle will be $U = \eta\langle h\nu\rangle \approx \eta kT$, where η is a particle-dependent absorption/emission coefficient. Therefore, Equation (71) may be rewritten in the following form:

$$m_g = m_{i0} - \left[\left(\frac{n_r \eta k}{c^2}\right)^2 \frac{T^2}{m_{i0}{}^2}\right] m_{i0} \qquad (72)$$

For electrons at T=300K, we have:

$$\left(\frac{n_r \eta k}{c^2}\right)^2 \frac{T^2}{m_e{}^2} \approx 10^{-17}$$

Comparing (72) with (18), we obtain:

$$E_{Ki} = \frac{1}{2}\left(\frac{n_r \eta k}{c}\right)^2 \frac{T^2}{m_{i0}}. \qquad (73)$$

The derivative of E_{Ki} with respect to temperature T is:

$$\frac{\partial E_{Ki}}{\partial T} = \left(n_r \eta k/c\right)^2 \left(T/m_{i0}\right) \qquad (74)$$

Thus,

$$T\frac{\partial E_{Ki}}{\partial T} = \frac{\left(n_r \eta kT\right)^2}{m_{i0}c^2} \qquad (75)$$

Substitution of $E_{Ki} = E_i - E_{i0}$ into (75) gives:

$$T\left(\frac{\partial E_i}{\partial T} + \frac{\partial E_{i0}}{\partial T}\right) = \frac{\left(n_r \eta kT\right)^2}{m_{i0}c^2} \qquad (76)$$

By comparing the Equations (76) and (73) and considering that $\partial E_{i0}/\partial T = 0$ because E_{i0} does not depend on T, the Equation (76) reduces to:

$$T\left(\partial E_i/\partial T\right) = 2E_{Ki} \qquad (77)$$

However, Equation (18) shows that $2E_{Ki} = E_i - E_g$. Therefore Equation (77) becomes:

$$E_g = E_i - T\left(\partial E_i/\partial T\right) \qquad (78)$$

Here, we can identify the energy E_i with the *free-energy* of the system-F and E_g with the *internal energy* of the system-U, thus we can write the Equation (78) in the following form:

$$U = F - T\left(\partial F/\partial T\right) \qquad (79)$$

This is the well-known equation of Thermodynamics. On the other hand, remembering $\partial Q = \partial\tau + \partial U$ (1st principle of Thermodynamics) and;

$$F = U - TS \qquad (80)$$

(Helmholtz's function), we can easily obtain from (79), the following equation:

$$\partial Q = \partial\tau + T\partial S. \qquad (81)$$

For *isolated systems*, $\partial\tau = 0$, we have

$$\partial Q = T\partial S \qquad (82)$$

which is the well-know *Entropy Differential Equation*.

Let us now consider the Equation (55) in the *ultra-relativistic case* where the inertial energy of the particle $E_i = M_i c^2$ is much greater than its inertial energy at rest, $m_{i0}c^2$.

Comparison between (4) and (10) leads to $\Delta p = E_i V/c^2$ which, in the ultra-relativistic case, gives $\Delta p = E_i V/c^2 \cong E_i/c \cong M_i c$. On the other hand, comparison between (55) and (41) shows that $Un_r = \Delta pc$. Thus, $Un_r = \Delta pc \cong M_i c^2 >> m_{i0}c^2$. Consequently, Equation (55) reduces to:

$$m_g = m_{i0} - 2\,Un_r/c^2 \qquad (83)$$

Therefore, the *action* for such particle, in agreement with the Equation (2), is:

$$\begin{aligned} S &= -\int_{t_1}^{t_2} m_g c^2 \sqrt{1 - V^2/c^2}\,dt = \\ &= \int_{t_1}^{t_2} \left(-m_i + 2Un_r/c^2\right)c^2 \sqrt{1 - V^2/c^2}\,dt = \\ &= \int_{t_1}^{t_2} \left[-m_i c^2 \sqrt{1 - V^2/c^2} + 2Un_r\sqrt{1 - V^2/c^2}\right]dt. \end{aligned} \qquad (84)$$

The integrant function is the *Lagrangean*, i.e.,

$$L = -m_{i0}c^2\sqrt{1 - V^2/c^2} + 2Un_r\sqrt{1 - V^2/c^2} \qquad (85)$$

Starting from the Lagrangean we can find the Hamiltonian of the particle, by means of the well-known general formula:

$$H = V(\partial L/\partial V) - L.$$

The result is:

$$H = \frac{m_{i0}c^2}{\sqrt{1 - V^2/c^2}} + Un_r\left[\frac{\left(4V^2/c^2 - 2\right)}{\sqrt{1 - V^2/c^2}}\right]. \qquad (86)$$

The second term on the right hand side of Equation (86) results from the particle's interaction with the *electromagnetic field*. Note the similarity between the obtained Hamiltonian and the well-known Hamiltonian for the particle in an electromagnetic field [32]:

$$H = m_{i0}c^2\Big/\sqrt{1 - V^2/c^2} + Q\varphi. \qquad (87)$$

in which Q is the electric charge and φ, the field's *scalar* potential. The quantity $Q\varphi$ expresses, as we know, the particle's interaction with the electromagnetic field. Such as the second term on the right hand side of the Equation (86).

It is therefore evident that it is the same quantity, expressed by different variables. Thus, we can conclude that, in ultra-high energy conditions $\left(Un_r \cong M_i c^2 > m_{i0}c^2\right)$, the gravitational and electromagnetic fields can be described by the *same* Hamiltonian, (i.e., in these circumstances they are *unified)* !

It is known that starting from that Hamiltonian we may obtain a complete description of the electromagnetic field. This means that from the present theory for gravity we can also derive *the equations of the electromagnetic field*.

Due to $Un_r = \Delta pc \cong M_i c^2$ the second term on the right hand side of Equation (86) can be written as follows:

$$\begin{aligned} \Delta pc\left[\frac{\left(4V^2/c^2 - 2\right)}{\sqrt{1 - V^2/c^2}}\right] &= \\ &= \left[\frac{\left(4V^2/c^2 - 2\right)}{\sqrt{1 - V^2/c^2}}\right]M_i c^2 = \\ &= Q\varphi = \frac{QQ'}{4\pi\varepsilon_0 R} = \frac{QQ'}{4\pi\varepsilon_0 r\sqrt{1 - V^2/c^2}} \end{aligned}$$

hence,

$$\left(4V^2/c^2-2\right)M_ic^2=\frac{QQ'}{4\pi\varepsilon_0 r}$$

The factor $\left(4V^2/c^2-2\right)$ becomes equal to 2 in the ultra-relativistic case, then it follows that:

$$2M_ic^2=\frac{QQ'}{4\pi\varepsilon_0 r} \qquad (88)$$

From (44) we know that there is a minimum value for M_i given by $M_{i(min)}=m_{i(min)}$. Equation (43) shows that $m_{g(min)}=m_{i0(min)}$ and Equation (23) gives $m_{g(min)}=\pm h/cL_{max}\sqrt{8}=\pm h\sqrt{3/8}/cd_{max}$. Thus we can write:

$$M_{i(min)}=m_{i0(min)}=\pm h\sqrt{3/8}/cd_{max} \qquad (89)$$

According to (88) the value $2M_{i(min)}c^2$ is correlated to $\left(QQ'/4\pi\varepsilon_0 r\right)_{min}=Q^2_{min}/4\pi\varepsilon_0 r_{max}$, i.e.,

$$\frac{Q^2_{min}}{4\pi\varepsilon_0 r_{max}}=2M_{i(min)}c^2 \qquad (90)$$

where Q_{min} is the *minimum electric charge* in the Universe (therefore equal to minimum electric charge of the quarks, i.e., $\frac{1}{3}e$); r_{max} is the *maximum distance* between Q and Q', which should be equal to the so-called "diameter", d_c, of the *visible* Universe ($d_c=2l_c$ where l_c is obtained from the Hubble's law for $V=c$, i.e., $l_c=c\tilde{H}^{-1}$). Thus from (90) we readily obtain

$$\begin{aligned}Q_{min}&=\sqrt{\pi\varepsilon_0 hc\sqrt{24}\left(d_c/d_{max}\right)}=\\&=\sqrt{\left(\pi\varepsilon_0 hc^2\sqrt{96}\tilde{H}^{-1}/d_{max}\right)}=\\&=\tfrac{1}{3}e \qquad (91)\end{aligned}$$

hence we find:

$$d_{max}=3.4\times10^{30}\,m$$

This will be the maximum "diameter" that the Universe will reach. Consequently, Equation (89) tells us that the *elementary quantum* of matter is:

$$m_{i0(min)}=\pm h\sqrt{3/8}/cd_{max}=\pm3.9\times10^{-73}kg$$

Now, by combination of gravity and the *uncertainty principle* we will derive the expression for the *Casimir force*.

An uncertainty Δm_i in m_i produces a uncertainty Δp in p and therefore an uncertainty Δm_g in m_g, which according to Equation (41), is given by:

$$\Delta m_g=\Delta m_i-2\left[\sqrt{1+\left(\frac{\Delta p}{\Delta m_i c}\right)^2}-1\right]\Delta m_i \qquad (92)$$

From the uncertainty principle for position and momentum, we know that the product of the uncertainties of the simultaneously measurable values of the corresponding position and momentum components is at least of the order of magnitude of $\hbar$, i.e.,

$$\Delta p\Delta r\sim\hbar$$

Substitution of $\Delta p\sim\hbar/\Delta r$ into (92) yields:

$$\Delta m_g=\Delta m_i-2\left[\sqrt{1+\left(\frac{\hbar/\Delta m_i c}{\Delta r}\right)^2}-1\right]\Delta m_i \qquad (93)$$

Therefore if:

$$\Delta r<<\frac{\hbar}{\Delta m_i c} \qquad (94)$$

then the expression (93) reduces to:

$$\Delta m_g\cong-\frac{2\hbar}{\Delta rc} \qquad (95)$$

Note that Δm_g does not depend on m_g.

Consequently, the uncertainty ΔF in the gravitational force $F = -Gm_g m'_g / r^2$, will be given by:

$$\Delta F = -G\frac{\Delta m_g \Delta m'_g}{(\Delta r)^2} = = -\left[\frac{2}{\pi(\Delta r)^2}\right]\frac{hc}{(\Delta r)^2}\left(\frac{G\hbar}{c^3}\right) \qquad (96)$$

The amount $\left(G\hbar/c^3\right)^{1/2} = 1.61\times 10^{-35}m$ is called the *Planck length*, l_{planck}, (the length scale on which quantum fluctuations of the metric of the space time are expected to be of order unity). Thus, we can write the expression of ΔF as follows:

$$\Delta F = -\left(\frac{2}{\pi}\right)\frac{hc}{(\Delta r)^4}l^2_{planck} = = -\left(\frac{\pi}{480}\right)\frac{hc}{(\Delta r)^4}\left[\left(\frac{960}{\pi^2}\right)l^2_{planck}\right] = = -\left(\frac{\pi A_0}{480}\right)\frac{hc}{(\Delta r)^4} \qquad (97)$$

or

$$F_0 = -\left(\frac{\pi A_0}{480}\right)\frac{hc}{r^4} \qquad (98)$$

which is the expression of the *Casimir force* for $A = A_0 = \left(960/\pi^2\right)l^2_{planck}$.

This suggests that A_0 is an *elementary area* related to existence of a *minimum length* $d_{min} = \tilde{k}l_{planck}$. What is in accordance with the *quantization of space* (29) and which point out to the existence of d_{min}.

It can be easily shown that the *minimum area* related to d_{min} is the area of an *equilateral triangle* of side length d_{min}, i.e.,

$$A_{min} = \left(\tfrac{\sqrt{3}}{4}\right)d^2_{min} = \left(\tfrac{\sqrt{3}}{4}\right)\tilde{k}^2 l^2_{planck}$$

On the other hand, the *maximum area* related to d_{min} is the area of a *sphere* of radius d_{min}, i.e.,

$$A_{max} = \pi d^2_{min} = \pi\tilde{k}^2 l^2_{planck}$$

Thus, the elementary area:

$$A_0 = \delta_A d^2_{min} = \delta_A \tilde{k}^2 l^2_{planck} \qquad (99)$$

must have a value between A_{min} and A_{max}, i.e.,

$$\tfrac{\sqrt{3}}{4} < \delta_A < \pi$$

The previous assumption that $A_0 = \left(960/\pi^2\right)l^2_{planck}$ shows that $\delta_A \tilde{k}^2 = 960/\pi^2$ what means that:

$$5.6 < \tilde{k} < 14.9$$

Therefore we conclude that:

$$d_{min} = \tilde{k}l_{planck} \approx 10^{-34}m. \qquad (100)$$

The $n-esimal$ area after A_0 is:

$$A = \delta_A (nd_{min})^2 = n^2 A_0 \qquad (101)$$

It can also be easily shown that the *minimum volume* related to d_{min} is the volume of a *regular tetrahedron* of edge length, d_{min}, i.e.,

$$\Omega_{min} = \left(\tfrac{\sqrt{2}}{12}\right)d^3_{min} = \left(\tfrac{\sqrt{2}}{12}\right)\tilde{k}^3 l^3_{planck}$$

The *maximum volume* is the volume of a *sphere* of radius, d_{min}, i.e.,

$$\Omega_{max} = \left(\tfrac{4\pi}{3}\right)d^3_{min} = \left(\tfrac{4\pi}{3}\right)\tilde{k}^3 l^3_{planck}$$

Thus, the elementary volume $\Omega_0 = \delta_V d^3_{min} = \delta_V \tilde{k}^3 l^3_{planck}$ must have a value between Ω_{min} and Ω_{max}, i.e.,

$$\left(\tfrac{\sqrt{2}}{12}\right) < \delta_V < \tfrac{4\pi}{3}$$

On the other hand, the $n-esimal$ volume after Ω_0 is:

$$\Omega = \delta_V \left(nd_{min}\right)^3 = n^3 \Omega_0 \qquad n = 1,2,3,\ldots,n_{max}.$$

The existence of n_{max} given by (26), i.e.,

$$n_{max} = L_{max}/L_{min} = d_{max}/d_{min} = \\ = \left(3.4\times10^{30}\right)/\tilde{k}l_{planck} \approx 10^{64}$$

shows that the Universe must have a *finite volume* whose value at the present stage is:

$$\Omega_{Up} = n_{Up}^3 \Omega_0 = \left(d_p/d_{min}\right)^3 \delta_V d_{min}^3 = \delta_V d_p^3$$

where d_p is the present length scale of the Universe. In addition as $\left(\frac{\sqrt{2}}{12}\right) < \delta_V < \frac{4\pi}{3}$ we conclude that the Universe must have a *polyhedral* space topology with volume between the volume of a *regular tetrahedron* of edge length, d_p, and the volume of the *sphere of* diameter, d_p.

A recent analysis of astronomical data suggests not only that the Universe is *finite*, but also that it has a *dodecahedral* space topology [33, 34], what is in strong accordance with the theoretical predictions above.

From (22) and (26) we have that $L_{max} = d_{max}/\sqrt{3} = n_{max} d_{min}/\sqrt{3}$. Since (100) gives $d_{min} \cong 10^{-34} m$ and $n_{max} \cong 10^{64}$ we conclude that $L_{max} \cong 10^{30} m$. From the *Hubble's law* and (22) we have:

$$V_{max} = \tilde{H} l_{max} = \tilde{H}\left(d_{max}/2\right) = \left(\sqrt{3}/2\right)\tilde{H} L_{max}$$

where $\tilde{H} = 1.7\times10^{-18} s^{-1}$. Therefore we obtain:

$$V_{max} \cong 10^{12} m/s$$

This is the speed upper limit imposed by the *quantization of velocity* (Equation 36). It is known that the speed upper limit for *real* particles is equal to c. However, also it is known that *imaginary* particles can have velocities greater than c (*Tachyons*).

Thus, we conclude that V_{max} is the speed upper limit for *imaginary particles in our ordinary space-time*. Forwards, we will see that there also exists a speed upper limit to the *imaginary* particles in the *imaginary* space-time.

Now, multiplying Equation (98) (the expression of F_0) by n^2 we obtain:

$$F = n^2 F_0 = -\left(\frac{\pi n^2 A_0}{480}\right)\frac{hc}{r^4} = -\left(\frac{\pi A}{480}\right)\frac{hc}{r^4} \qquad (102)$$

This is the general expression of the *Casimir force*. Thus we conclude that *the Casimir effect* is just a gravitational effect related to the *uncertainty principle*.

Note that the Equation (102) arises only when Δm_i and $\Delta m_i'$ satisfy Equation (94). If only Δm_i satisfies Equation (94), (i.e., $\Delta m_i << \hbar/\Delta rc$ but $\Delta m_i' >> \hbar/\Delta rc$) then Δm_g and $\Delta m_g'$ will be respectively given by:

$$\Delta m_g \cong -2\hbar/\Delta rc \quad and \quad \Delta m_g' \cong \Delta m_i$$

Consequently, the expression (96) becomes:

$$\Delta F = \frac{hc}{(\Delta r)^3}\left(\frac{G\Delta m_i'}{\pi c^2}\right) = \frac{hc}{(\Delta r)^3}\left(\frac{G\Delta m_i' c^2}{\pi c^4}\right) = \\ = \frac{hc}{(\Delta r)^3}\left(\frac{G\Delta E'}{\pi c^4}\right) \qquad (103)$$

However, from the uncertainty principle for *energy* and *time* we know that:

$$\Delta E \sim \hbar/\Delta t \qquad (104)$$

Therefore we can write the expression (103) in the following form:

$$\Delta F = \frac{hc}{(\Delta r)^3}\left(\frac{G\hbar}{c^3}\right)\left(\frac{1}{\pi \Delta t' c}\right) =$$

$$= \frac{hc}{(\Delta r)^3} l_{planck}^2 \left(\frac{1}{\pi \Delta t' c}\right) \qquad (105)$$

From the General Relativity Theory we know that $dr = cdt/\sqrt{-g_{00}}$. If the field is *weak* then $g_{00} = -1 - 2\phi/c^2$ and
$dr = cdt/\left(1+\phi/c^2\right) = cdt/\left(1 - Gm/r^2c^2\right)$.

For $Gm/r^2 c^2 << 1$ we obtain $dr \cong cdt$. Thus, if $dr = dr'$ then $dt = dt'$. This means that we may change $(\Delta t' c)$ by (Δr) into (105). The result is:

$$\Delta F = \frac{hc}{(\Delta r)^4}\left(\frac{1}{\pi} l_{planck}^2\right) =$$

$$= \left(\frac{\pi}{480}\right)\frac{hc}{(\Delta r)^4}\underbrace{\left(\frac{480}{\pi^2} l_{planck}^2\right)}_{\frac{1}{2}A_0} =$$

$$= \left(\frac{\pi A_0}{960}\right)\frac{hc}{(\Delta r)^4}$$

or

$$F_0 = \left(\frac{\pi A_0}{960}\right)\frac{hc}{r^4}$$

whence

$$F = \left(\frac{\pi A}{960}\right)\frac{hc}{r^4} \qquad (106)$$

Now the Casimir force is *repulsive*, and its intensity is half of the intensity previously obtained (102).

Consider the case when both Δm_i and $\Delta m_i'$ do not satisfy Equation (94), and,

$$\Delta m_i >> \hbar/\Delta rc$$
$$\Delta m_i' >> \hbar/\Delta rc$$

In this case, $\Delta m_g \cong \Delta m_i$ and $\Delta m_g' \cong \Delta m_i'$. Thus,

$$\Delta F = -G\frac{\Delta m_i \Delta m_i'}{(\Delta r)^2} = -G\frac{\left(\Delta E/c^2\right)\left(\Delta E'/c^2\right)}{(\Delta r)^2} =$$

$$= -\left(\frac{G}{c^4}\right)\frac{(\hbar/\Delta t)^2}{(\Delta r)^2} = -\left(\frac{G\hbar}{c^3}\right)\frac{hc}{(\Delta r)^2}\left(\frac{1}{c^2\Delta t^2}\right) =$$

$$= -\left(\frac{1}{2\pi}\right)\frac{hc}{(\Delta r)^4} l_{planck}^2 =$$

$$= -\left(\frac{\pi}{1920}\right)\frac{hc}{(\Delta r)^4}\left(\frac{960}{\pi^2} l_{planck}^2\right) = -\left(\frac{\pi A_0}{1920}\right)\frac{hc}{(\Delta r)^4}$$

whence

$$F = -\left(\frac{\pi A}{1920}\right)\frac{hc}{r^4} \qquad (107)$$

The force will be *attractive* and its intensity will be the *fourth part* of the intensity given by the first expression (102) for the Casimir force.

We can also use this theory to explain some relevant cosmological phenomena. For example, the recent discovery that the cosmic expansion of the Universe may be *accelerating*, and not decelerating as many cosmologists had anticipated [35].

We start from the Equation (6) which shows that the *inertial forces*, $\vec{F}_i$, whose acts on a particle, in the case of the force and speed have the *same direction*, is given by:

$$\vec{F}_i = \left|\frac{m_g}{\left(1 - V^2/c^2\right)^{3/2}}\right|\vec{a}$$

Substitution of m_g given by (43) into the expression above gives:

$$\vec{F}_i = \left|\frac{3}{\left(1 - V^2/c^2\right)^{3/2}} - \frac{2}{\left(1 - V^2/c^2\right)^2}\right| m_{i0}\vec{a}$$

whence we conclude that a particle with rest inertial mass, m_{i0}, subjected to a force, $\vec{F}_i$, acquires an acceleration $\vec{a}$ given by:

$$\vec{a} = \frac{\vec{F}_i}{\left| \frac{3}{\left(1 - V^2/c^2\right)^{3/2}} - \frac{2}{\left(1 - V^2/c^2\right)^2} \right| m_{i0}}$$

By substituting the well-known expression of Hubble's law for velocity, $V = \tilde{H}l$, ($\tilde{H} = 1.7 \times 10^{-18} s^{-1}$ is the Hubble constant) into the expression of $\vec{a}$, we get *the acceleration for any particle in the expanding Universe*, i.e.,

$$\vec{a} = \frac{\vec{F}_i}{\left| \frac{3}{\left(1 - \tilde{H}^2 l^2/c^2\right)^{3/2}} - \frac{2}{\left(1 - \tilde{H}^2 l^2/c^2\right)^2} \right| m_{i0}}$$

Obviously the distance l increases with the expansion of the Universe. Under these circumstances it is easy to see that the term:

$$\left| \frac{3}{\left(1 - \tilde{H}^2 l^2/c^2\right)^{3/2}} - \frac{2}{\left(1 - \tilde{H}^2 l^2/c^2\right)^2} \right|$$

decreases, *increasing the acceleration* of the expanding Universe.

Let us now consider the phenomenon of gravitational deflection of light. A light ray, from a distant star, under the Sun's gravitational force field describes the usual central force hyperbolic orbit. The deflection of the light ray is illustrated in Figure 5, with the bending greatly exaggerated for a better view of the angle of deflection.

The distance CS is the distance d of closest approach. The angle of deflection of the light ray, δ, is shown in the Figure 5 and is:

$$\delta = \pi - 2\beta.$$

where β is the angle of the asymptote to the hyperbole. It then follows that:

$$\tan \delta = \tan(\pi - 2\beta) = -\tan 2\beta$$

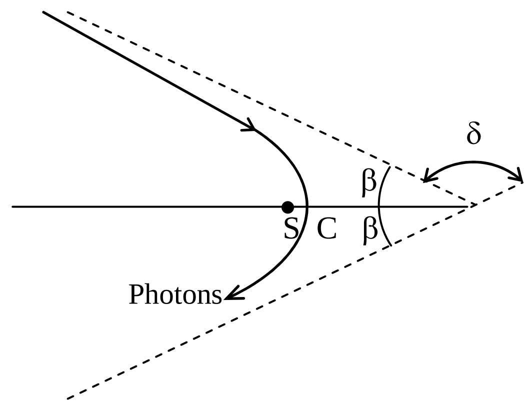


Figure 5: Gravitational Deflection about the Sun.

From the Figure 5 we obtain:

$$\tan \beta = \frac{V_y}{c}.$$

Since δ and β are too small we can write that:

$$\delta = 2\beta \quad \text{and} \quad \beta = \frac{V_y}{c}$$

then

$$\delta = \frac{2V_y}{c}$$

Consider the motion of the photons at some time t after it has passed the point of closest approach. We impose Cartesian Co-ordinates with the origin at the point of closest approach, the x axis pointing along its path and the y axis towards the Sun. The gravitational pull of the Sun is:

$$P = G\frac{M_{gS} M_{gp}}{r^2}$$

where M_{gp} is the relativistic *gravitational* mass of the photon and M_{gS} the relativistic gravitational mass of the Sun. Thus, the component in a perpendicular direction is:

$$F_y = G\frac{M_{gS}M_{gp}}{r^2}\boldsymbol{sin}\,\beta =$$

$$= G\frac{M_{gS}M_{gp}}{d^2+c^2t^2}\frac{d}{\sqrt{d^2+c^2t^2}}$$

According to Equation (6) the expression of the force F_y is:

$$F_y = \left|\frac{m_{gp}}{\left(1-V_y^2/c^2\right)^{\frac{3}{2}}}\right|\frac{dV_y}{dt}$$

By substituting Equation (43) into this expression we get:

$$F_y = \left|\frac{3}{\left(1-V_y^2/c^2\right)} - \frac{2}{\left(1-V_y^2/c^2\right)^{\frac{3}{2}}}\right|M_{ip}\frac{dV_y}{dt}$$

For $V_y << c$, we can write this expression in the following form $F_y = M_{ip}\left(dV_y/dt\right)$. This force acts on the photons for a time t causing an increase in the transverse velocity:

$$dV_y = \frac{F_y}{M_{ip}}dt$$

Thus the component of transverse velocity acquired after passing the point of closest approach is:

$$V_y = \frac{M_{gp}}{M_{ip}}\int\frac{dGM_{gS}}{\left(d^2+c^2t^2\right)^{\frac{3}{2}}}dt =$$

$$= \frac{GM_{gS}}{dc}\left(\frac{M_{gp}}{M_{ip}}\right)$$

Since the angle of deflection δ is given by:

$$\delta = 2\beta = \frac{2V_y}{c}$$

We readily obtain:

$$\delta = \frac{2V_y}{c} = \frac{2GM_{gS}}{c^2d}\left(\frac{M_{gp}}{M_{ip}}\right)$$

If $M_{gp}/M_{ip} = 2$ the expression above gives:

$$\delta = \frac{4GM_{gS}}{c^2d}$$

As we know, this is the correct formula indicated by the experimental results.

Equation (41) shows that:

$$\frac{m_{gp}}{m_{ip}} = \left\{1-2\left[\sqrt{1+\left(\frac{\Delta p}{m_{ip}c}\right)^2}-1\right]\right\}$$

or

$$\frac{M_{gp}}{M_{ip}} = \left\{1-2\left[\sqrt{1+\left(\frac{\Delta p}{M_{ip}c}\right)^2}-1\right]\right\}$$

By making, $M_{gp}/M_{ip} = m_{gp}/m_{ip} = 2$ and $\Delta p = h/\lambda$ into these expressions, we obtain:

$$M_{ip} = m_{ip} = \frac{2}{\sqrt{3}}\left(\frac{hf}{c^2}\right)i$$

Therefore:

$$M_{gp} = m_{gp} = \frac{4}{\sqrt{3}}\left(\frac{hf}{c^2}\right)i$$

This means that the gravitational and inertial masses of the photon are *imaginaries*, and *invariants* with respect to speed of photon. On the other hand, we can write that:

$$m_{ip} = m_{ip(real)} + m_{ip(imaginary)} = \frac{2}{\sqrt{3}}\left(\frac{hf}{c^2}\right)i$$

and

$$m_{gp} = m_{gp(real)} + m_{gp(imaginary)} = \frac{4}{\sqrt{3}}\left(\frac{hf}{c^2}\right)i$$

This means that we must have:

$$m_{ip(real)} = m_{gp(real)} = 0$$

The Equation (41) shows that if the *inertial mass* of a particle is *null* then its *gravitational mass* is given by:

$$m_g = \pm \frac{2\Delta p}{c}$$

where Δp is the *momentum* variation due to the energy absorbed by the particle. If the energy of the particle is *invariant* then $\Delta p = 0$ and, consequently its *gravitational mass* will also be null. This is the case of the photons, i.e., they have an invariant energy hf and *momentum* h/λ. As they cannot absorb additional energy, the variation in the *momentum* will be null $(\Delta p = 0)$ and, therefore, their *gravitational masses* will also be null. However, if the energy of the particle is not invariant (it is able to absorb energy) then the absorbed energy will transfer the *amount of motion* (*momentum*) to the particle, and consequently its *gravitational mass* will be increased. This means that the *motion* generates gravitational mass. On the other hand, if the *gravitational mass* of a particle is null then its *inertial mass*, according to Equation (41), will be given by:

$$m_i = \pm \frac{2}{\sqrt{5}} \frac{\Delta p}{c}$$

From the Equations (4) and (7) we get:

$$\Delta p = \left(\frac{E_g}{c^2}\right)\Delta V = \left(\frac{p_0}{c}\right)\Delta V$$

Thus we have:

$$m_g = \pm\left(\frac{2p_0}{c^2}\right)\Delta V \text{ and } m_i = \pm\frac{2}{\sqrt{5}}\left(\frac{p_0}{c^2}\right)\Delta V$$

Note that, such as the gravitational mass, the inertial mass is also directly related to the motion (i.e., it also generated by the motion).

Thus, we can conclude that is the motion, or rather, the *velocity* is what makes the two types of mass.

In this picture, the fundamental particles can be considered as *immaterial vortex of velocity*; it is the velocity of these vortexes that causes the fundamental particles to have masses. That is, there is not matter in the usual sense; just *motion*. The difference between matter and energy just consists of the diversity of the motion direction; *rotating*, closed in itself, in the matter; *ondulatory*, with open cycle, in the energy (Figure 6).

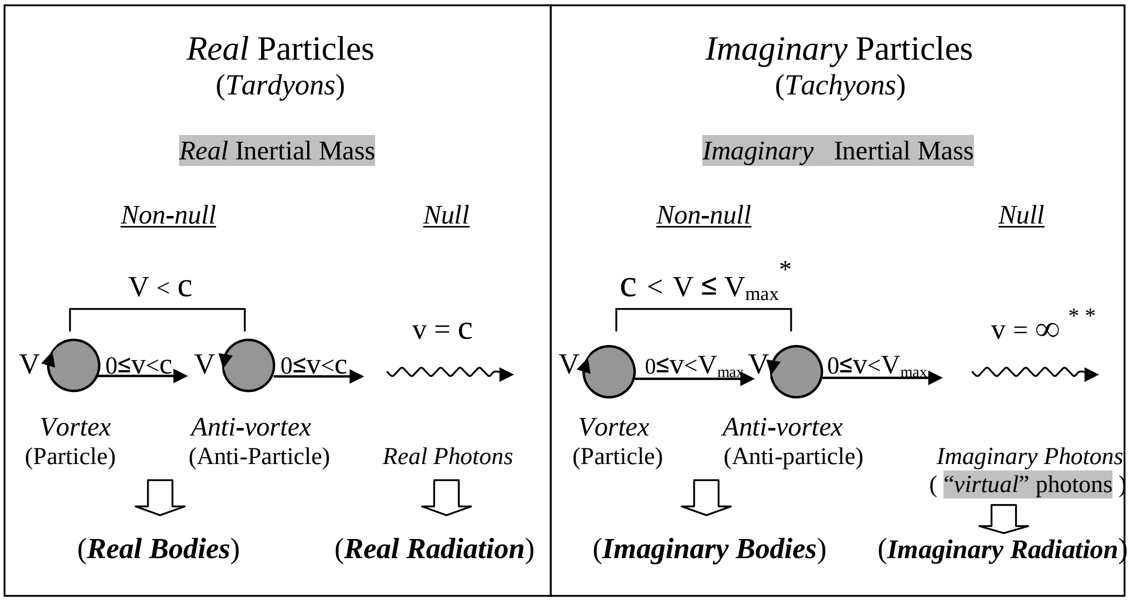
Under this context, the *Higgs mechanism*† appears as a process, by which the velocity of an immaterial vortex can be increased or decreased by making the vortex (particle) *gain* or *lose mass*. If is the *real motion* is what makes the *real mass* then, by analogy, we can say that the *imaginary mass* is made by the *imaginary motion*. This is not only a simple generalization of the process based on the theory of the *imaginary functions*, but also a fundamental conclusion related to the concept of *imaginary mass* that, as it will be show, provides a coherent explanation for the *materialization* of the fundamental particles, in the beginning of the Universe.

It is known that the simultaneous disappearance of a pair (electron/positron) liberates an amount of energy, $2m_{ie(real)}c^2$, under the form of two photons with frequency f, in such a way that:

$$2m_{ie(real)}c^2 = 2hf$$

Since the photon has *imaginary* masses associated to it, the phenomenon of transformation of the energy $2m_{ie(real)}c^2$ into $2hf$ suggests that the imaginary energy of the photon, $m_{gp(imaginary)}c^2$, comes from the transformation of imaginary energy of the electron, $m_{ge(imaginary)}c^2$, just as the real energy of the photon, hf, results from the transformation of real energy of the electron.

† The Standard Model is the name given to the current theory of fundamental particles and how they interact. This theory includes: *Strong interaction* and a combined theory of weak and electromagnetic interaction, known as *electroweak* theory. One part of the Standard Model is not yet well established. *What causes the fundamental particles to have masses*? The simplest idea is called the *Higgs mechanism*. This mechanism involves one additional particle, called the Higgs boson, and one additional force type, mediated by exchanges of this boson.



* V_{max} is the *speed upper limit* for Tachyons with *non-null* imaginary inertial mass. It has been previously obtained starting from the *Hubble's law* and Eq.(22). The result is: $V_{max} = \left(\sqrt{3}/2\right)\tilde{H}L_{max} \cong 10^{12}\, m.s^{-1}$.

** In order to communicate instantaneously the *interactions* at infinite distance the velocity of the *quanta* ("virtual" photons) must be *infinity* and consequently their imaginary masses must be *null* .

Figure 6: Real and Imaginary Particles.

Then, it follows that:

$$m_{ge(imaginary)}c^2 = m_{gp(imaginary)}c^2$$

As $m_{gp(imaginary)} = \frac{4}{\sqrt{3}}\left(hf/c^2\right) i$, we then conclude that:

$$m_{ge(imaginary)} = m_{gp(imaginary)} = $$
$$= \frac{4}{\sqrt{3}}\left(hf_e/c^2\right) i = $$
$$= \frac{4}{\sqrt{3}}\left(h/\lambda_e c\right) i = \frac{4}{\sqrt{3}} m_{ie(real)}\, i$$

where $\lambda_e = h/m_{ie(real)}c$ is the *Broglies' wavelength* for the electron.

In addition to the imaginary masses $m_{ie(imaginary)}$ and $m_{ge(imaginary)}$ the electron has *real* inertial and gravitational masses, respectively, given by

$$m_{ie(real)} = 9.11\times10^{-31} kg$$

$$m_{ge(real)} = \left\{1 - 2\left[\frac{1}{\sqrt{1 - V^2/c^2}} - 1\right]\right\} m_{ie(real)}$$

The existence of *imaginary* mass associated to a *real* particle suggests the possible existence of *imaginary particles* with imaginary masses in nature.

In this case, the concept of *wave associated* to a particle (De Broglie's waves) would also be applied to the imaginary particles. Then, by analogy, the imaginary wave associated to an imaginary particle with imaginary masses $m_{i\psi}$

and $m_{g\psi}$ would be described by the following expressions:

$$\vec{p}_{\psi} = \hbar \vec{k}_{\psi}$$
$$E_{\psi} = \hbar \omega_{\psi}$$

Henceforth, for simplicity's sake, we will use the Greek letter ψ to replace the word *imaginary*; $\vec{p}_{\psi}$ is the *momentum* carried by the ψ *wave* and E_{ψ} its energy; $\left|\vec{k}_{\psi}\right| = 2\pi/\lambda_{\psi}$ is the propagation number and λ_{ψ} the wavelength of the ψ *wave;* $\omega_{\psi} = 2\pi f_{\psi}$ is the cyclical frequency.

According to Equation (4) the *momentum* $\vec{p}_{\psi}$ is:

$$\vec{p}_{\psi} = M_{g\psi}\vec{V}$$

where V is the velocity of the ψ particle.

By comparing the expressions of $\vec{p}_{\psi}$ we get:

$$\lambda_{\psi} = \frac{h}{M_{g\psi}V}$$

It is known that the variable quantity which characterizes the De Broglie's waves is called *wave function,* usually indicated by symbol Ψ. The wave function associated with a material particle describes the dynamic state of the particle: its value at a particular point x, y, z, t is related to the probability of finding the particle in that place and instant. Although Ψ does not have a physical interpretation, its square Ψ^2 (or $\Psi\ \Psi^*$) calculated for a particular point x, y, z, t *is proportional to the probability of finding the particle in that place and instant*.

Since Ψ^2 is proportional to the probability P of finding the particle described by Ψ, the integral of Ψ^2 on the *whole space* must be finite – inasmuch as the particle is somewhere.

On the other hand, if,

$$\int_{-\infty}^{+\infty} \Psi^2 dV = 0$$

the interpretation is that the particle will not exist. However, if,

$$\int_{-\infty}^{+\infty} \Psi^2 dV = \infty \qquad (108)$$

The particle will be everywhere simultaneously.

In Quantum Mechanics, the wave function Ψ corresponds, as we know, to the displacement y of the undulatory motion of a rope. However, Ψ, as opposed to y, is not a measurable quantity and can, hence, be a complex quantity. For this reason, it is assumed that Ψ is described in the $x-direction$ by:

$$\Psi = \Psi_0 e^{-(2\pi\, i/h)(Et-px)}$$

This is the expression of the wave function for a *free* particle, with total energy E and *momentum* $\vec{p}$, moving in direction $+x$.

As to the imaginary particle, the *imaginary particle wave function* will be denoted by Ψ_{ψ} and, by analogy with the expression of Ψ, will be expressed by:

$$\Psi_{\psi} = \Psi_{0\psi} e^{-(2\pi\, i/h)(E_{\psi}t-p_{\psi}x)}$$

Therefore, the *general expression* of the wave function for a *free* particle can be written in the following form:

$$\Psi = \Psi_{0(real)} e^{-(2\pi\, i/h)(E_{(real)}t-p_{(real)}x)} + \\ + \Psi_{0\psi} e^{-(2\pi\, i/h)(E_{\psi}t-p_{\psi}x)}$$

It is known that the *uncertainty principle* can also be written as a function of ΔE (uncertainty in the energy) and Δt (uncertainty in the time), i.e.,

$$\Delta E.\Delta t \geq \hbar$$

This expression shows that a variation of energy ΔE, during a time interval Δt, can only be detected if $\Delta t \geq \hbar/\Delta E$.

Consequently, a variation of energy ΔE , during a time interval $\Delta t < \hbar/\Delta E$, cannot be experimentally detected. This is a limitation imposed by Nature and not by our equipments.

Thus, a *quantum* of energy $\Delta E = hf$ that varies during a time interval $\Delta t = 1/f = \lambda/c < \hbar/\Delta E$ (wave period) cannot be experimentally detected. This is an *imaginary* photon or a "*virtual*" photon.

Now, consider a particle with energy $M_g c^2$. The DeBroglie's gravitational and inertial wavelengths are respectively, $\lambda_g = h/M_g c$ and $\lambda_i = h/M_i c$. In Quantum Mechanics, particles of matter and quanta of radiation are described by means of *wave packet* (DeBroglie's waves) with average wavelength λ_i . Therefore, we can say that during a time interval $\Delta t = \lambda_i/c$, a *quantum* of energy $\Delta E = M_g c^2$ varies.

According to the uncertainty principle, the particle will be detected if $\Delta t \ge \hbar/\Delta E$, i.e., if $\lambda_i/c \ge \hbar/M_g c^2$ or $\lambda_i \ge \lambda_g/2\pi$. This condition is usually satisfied when $M_g = M_i$. In this case, $\lambda_g = \lambda_i$ and obviously, $\lambda_i > \lambda_i/2\pi$. However, when M_g decreases λ_g increases and $\lambda_g/2\pi$ can become greater than λ_i , making the particle *non-detectable* or *imaginary*.

According to Equations (7) and (41) we can write M_g in the following form:

$$M_g = \left|\frac{m_g}{\sqrt{1-V^2/c^2}}\right| = \left|\frac{\chi\ m_i}{\sqrt{1-V^2/c^2}}\right| = |\chi| M_i$$

where

$$\chi = \left\{1 - 2\left[\sqrt{1+\left(\Delta p/m_{i0}c\right)^2} - 1\right]\right\}$$

Since the condition to make the particle *imaginary* is:

$$\lambda_i < \frac{\lambda_g}{2\pi}$$

and

$$\frac{\lambda_g}{2\pi} = \frac{\hbar}{M_g c} = \frac{\hbar}{|\chi| M_i c} = \frac{\lambda_i}{2\pi|\chi|}$$

Then we get:

$$|\chi| < \frac{1}{2\pi} = 0.159$$

This means that when:

$$-0.159 < \chi < +0.159$$

The particle becomes *imaginary*. Under these circumstances, we can say that the particle made a transition to the *imaginary space-time*.

Note that, when a particle becomes imaginary, its gravitational and inertial masses also become imaginary. However, the factor $\chi = M_{g(imaginary)}/M_{i(imaginary)}$ remains *real* because:

$$\chi = \frac{M_{g(imaginary)}}{M_{i(imaginary)}} = \frac{M_g i}{M_i i} = \frac{M_g}{M_i} = real$$

Thus, if the gravitational mass of the particle is reduced by means of the absorption of an amount of electromagnetic energy U , for example, we have:

$$\chi = \frac{M_g}{M_i} = \left\{1 - 2\left[\sqrt{1+\left(U/m_{i0}c^2\right)^2} - 1\right]\right\}$$

This shows that the energy U of the electromagnetic field *remains acting on* the imaginary particle. In practice, this means that *electromagnetic fields act on imaginary particles*.

The gravity acceleration on a *imaginary* particle (due to the rest of the imaginary Universe) are given by:

$$g'_j = \chi \; g_j \qquad j = 1,2,3,\ldots,n.$$

where $\chi = M_{g(imaginary)} / M_{i(imaginary)}$ and $g_j = -Gm_{gj(imaginary)} / r_j^2$.

Thus, the gravitational forces acting on the particle are given by:

$$F_{gj} = M_{g(imaginary)} g'_j =$$
$$= M_{g(imaginary)}\left(-\chi G m_{gj(imaginary)} / r_j^2\right) =$$
$$= M_g i\left(-\chi G m_{gj} i / r_j^2\right) = +\chi G M_g m_{gj} / r_j^2 \;.$$

Note that these forces are *real*. Remind that, the Mach's principle says that the *inertial effects* upon a particle are consequence of the gravitational interaction of the particle with the rest of the Universe. Then we can conclude that the *inertial forces* upon an *imaginary* particle are also real.

Equation (7) shows that, in the case of imaginary particles, the relativistic mass is:

$$M_{g(imaginary)} = \left|\frac{m_{g(imaginary)}}{\sqrt{1 - V^2/c^2}}\right| =$$
$$= \left|\frac{m_g i}{i\sqrt{V^2/c^2 - 1}}\right| = \left|\frac{m_g}{\sqrt{V^2/c^2 - 1}}\right|$$

This expression shows that *imaginary* particles can have velocities V greater than c in our ordinary space-time (Tachyons). The *quantization of velocity* (Equation 36) shows that there is a speed upper limit $V_{max} > c$. As we have already calculated previously, $V_{max} \approx 10^{12} m.s^{-1}$, (Equation 102).

Note that this is the speed upper limit for *imaginary* particles *in our ordinary space-time* not in the *imaginary* space-time (Figure 7) because the *infinite* speed of the "virtual" *quanta* of the interactions shows that *imaginary* particles can have *infinite speed* in the *imaginary* space-time.

While the speed upper limit for imaginary particles in the ordinary space-time is $V_{max} \approx 10^{12} m.s^{-1}$, the speed upper limit for *real* particles in the *imaginary* space-time is c, because the relativistic expression of the mass shows that the velocity of *real* particles cannot be greater than c in *any space-time*.

The uncertainty principle permits that particles make "*virtual*" transitions, during a time interval Δt, if $\Delta t < \hbar / \Delta E$. The "*virtual*" transition of *mesons* emitted from nucleons that does not change of mass, during a time interval $\Delta t < \hbar / m_\pi c^2$, is a well-known example of "*virtual*" transition of particles. During a "virtual" transition of a *real* particle, the speed upper limit in the *imaginary* space-time is c, while the speed upper limit for an *imaginary* particle in the our ordinary space-time is $V_{max} \approx 10^{12} m s^{-1}$ (Figure 8).

There is a crucial cosmological problem to be solved: the problem of the *hidden mass*. Most theories predict that the amount of known matter, detectable and available in the universe, is only about 1/10 to 1/100 of the amount needed to close the universe. That is, to achieve the density sufficient to close-up the universe by maintaining the gravitational curvature (escape velocity equal to the speed of light) at the outer boundary.

Equation (43) may solve this problem. We will start by substituting the expression of *Hubble's* law for velocity, $V = \tilde{H} l$, into Equation (43). The expression obtained shows that particles which are at distances $l = l_0 = \left(\sqrt{5}/3\right)\left(c/\tilde{H}\right) = 1.3 \times 10^{26} m$ have *quasi null* gravitational mass $m_g = m_{g(min)}$; beyond this distance, the particles have *negative* gravitational mass. Therefore, there are two well-defined regions in the Universe; the region of the bodies with *positive* gravitational masses and the region of the bodies with *negative* gravitational mass. The total gravitational mass of the first region, in accordance with Equation (45), will be given by:

$$M_{g1} \cong M_{i1} = \frac{m_{i1}}{\sqrt{1 - \bar{V}_1^2/c^2}} \cong m_{i1}$$

where m_{i1} is the total *inertial mass* of the bodies of the mentioned region; $\bar{V}_1 << c$ is the average velocity of the bodies at region 1. The total gravitational mass of the second region is:

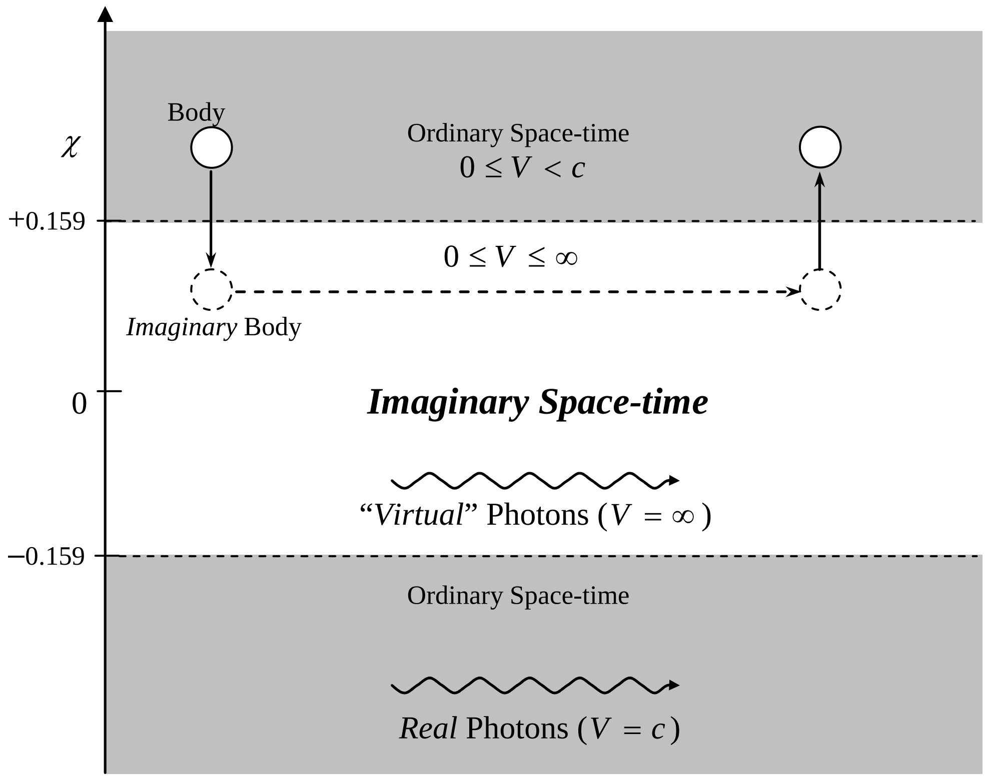


Figure 7: Travel in the Imaginary Space-Time.
(Similarly to the "virtual" photons, imaginary bodies can have infinite speed in the imaginary space-time).

$$M_{g2} = \left|1 - 2\left(\frac{1}{\sqrt{1-\overline{V}_2^2/c^2}} - 1\right)\right| M_{i2}$$

where $\overline{V}_2$ is the average velocity of the bodies ; $M_{i2} = m_{i2} \Big/ \sqrt{1-\overline{V}_2^2/c^2}$ and m_{i2} is the total *inertial mass* of the bodies of region 2.

Now consider that from Equation (7), we can write:

$$\xi = \frac{E_g}{V} = \frac{M_g c^2}{V} = \rho_g c^2$$

where ξ is the *energy density* of matter.

Note that the expression of ξ only reduces to the well-known expression ρc^2, where ρ is the sum of the inertial masses per volume unit, when $m_g = m_i$. Therefore, in the derivation of the well-known difference:

$$\frac{8\pi G \rho_U}{3} - \tilde{H}^2$$

which gives the *sign of the curvature* of the Universe [36], we must use $\xi = \rho_{gU} c^2$ instead of $\xi = \rho_U c^2$. The result obviously is:

$$\frac{8\pi G \rho_{gU}}{3} - \tilde{H}^2 \qquad (109)$$

where

$$\rho_{gU} = \frac{M_{gU}}{V_U} = \frac{M_{g1} + M_{g2}}{V_U} \qquad (110)$$

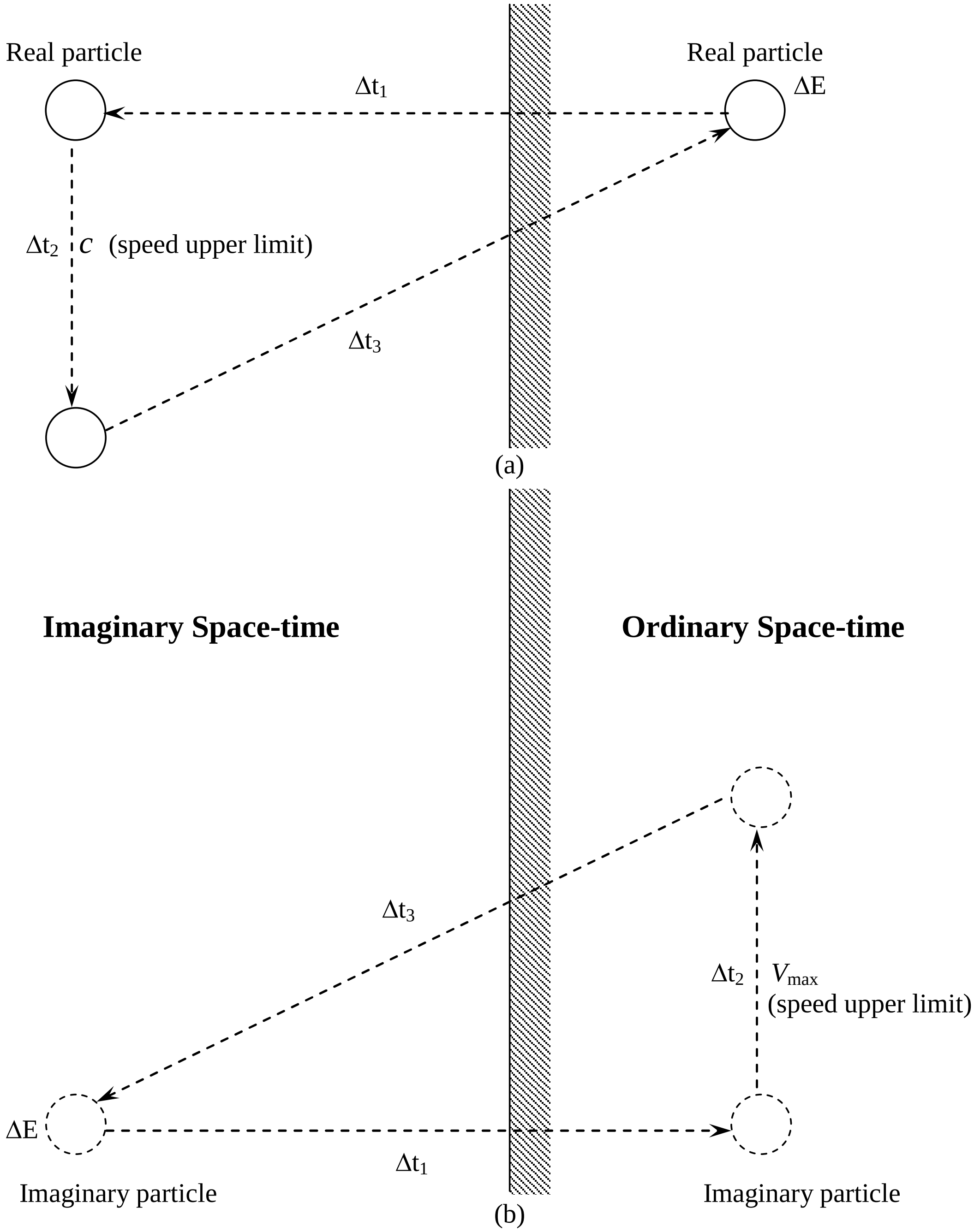


Figure 8: "Virtual" Transitions.

(a) "Virtual" Transitions of a Real Particle to the Imaginary Space-Time.
(The speed upper limit for real particle in the imaginary space-time is c).

(b) "Virtual" Transitions of an Imaginary particle to the Ordinary Space Time.
(The speed upper limit for imaginary particle in the ordinary space time $V_{max} \approx 10^{12} m.s^{-1}$).

(Note that to occur a "virtual" transition it is necessary that Note that to occur a "virtual" transition it is necessary that $\Delta t=\Delta t_1+\Delta t_2+\Delta t_3<\hbar/\Delta E$. Thus, even at principle, it will be impossible to determine any variation of energy in the particle (uncertainty principle).

M_{gU} and V_U are respectively, the total gravitational mass and the volume of the Universe.

Substitution of M_{g1} and M_{g2} into expression (110) gives:

$$\rho_{gU}=\frac{m_{iU}+\left[\left|\frac{3}{\sqrt{1-\overline{V}_2^2/c^2}}-\frac{2}{1-\overline{V}_2^2/c^2}\right|m_{i2}-m_{i2}\right]}{V_U}$$

where $m_{iU}=m_{i1}+m_{i2}$ is the total inertial mass of the Universe.

The volume V_1 of the region 1 and the volume V_2 of the region 2, are respectively given by:

$$V_1=2\pi^2 l_0^3 \qquad and \qquad V_2=2\pi^2 l_c^3-V_1$$

where $l_c=c/\widetilde{H}=1.8\times10^{26}m$ is the so-called "radius" of the visible Universe. Moreover, $\rho_{i1}=m_{i1}/V_1$ and $\rho_{i2}=m_{i2}/V_2$. Due to the *hypothesis of the uniform distribution of matter in the space*, it follows that $\rho_{i1}=\rho_{i2}$. Thus, we can write:

$$\frac{m_{i1}}{m_{i2}}=\frac{V_1}{V_2}=\left(\frac{l_0}{l_c}\right)^3=0.38$$

Similarly,

$$\frac{m_{iU}}{V_U}=\frac{m_{i2}}{V_2}=\frac{m_{i1}}{V_1}$$

Therefore,

$$m_{i2}=\frac{V_2}{V_U}m_{iU}=\left[1-\left(\frac{l_0}{l_c}\right)^3\right]m_{iU}=0.62m_{iU}$$

and $m_{i1}=0.38m_{iU}$.

Substitution of m_{i2} into the expression of ρ_{gU} yields:

$$\rho_{gU}=\frac{m_{iU}+\left|\frac{1.86}{\sqrt{1-\overline{V}_2^2/c^2}}-\frac{1.24}{1-\overline{V}_2^2/c^2}-0.62\right|m_{iU}}{V_U}$$

Due to $\overline{V}_2\cong c$, we conclude that the term between bracket (hidden mass) *is much greater than* $10m_{iU}$. The amount m_{iU} is the mass of known matter in the universe (1/10 to 1/100 of the amount needed to close the Universe).

Consequently, the total mass:

$$m_{iU}+\left|\frac{1.86}{\sqrt{1-\overline{V}_2^2/c^2}}-\frac{1.24}{1-\overline{V}_2^2/c^2}-0.62\right|m_{iU}$$

must be sufficient to close the Universe. This solves therefore the problem of the *hidden mass*.

There is another cosmological problem to be solved: the problem of the *anomalies* in the spectral red-shift of certain galaxies and stars.

Several observers have noticed red-shift values that cannot be explained by the Doppler-Fizeau effect or by the Einstein effect (the gravitational spectrum shift, supplied by Einstein's theory).

This is the case of the so-called *Stefan's quintet* (a set of five galaxies which were discovered in 1877), whose galaxies are located at approximately the same distance from the Earth, according to very reliable and precise measuring methods. But, when the velocities of the galaxies are measured by its red-shifts, the velocity of one of them is much greater than the velocity of the others.

Similar observations have been made on the *Virgo constellation* and spiral galaxies. Also the Sun presents a red-shift greater than the predicted value by the Einstein effect.

It seems that some of these anomalies can be explained if we consider the Equations (45) in the calculation of the *gravitational mass* of the point of emission.

The expression of the gravitational spectrum shift, supplied by Einstein's theory [37] is given by:

$$\Delta\omega = \omega_1 - \omega_2 = \frac{\phi_2 - \phi_1}{c^2}\omega_1 =$$
$$= \frac{-Gm_{g2}/r_2 + Gm_{g1}/r_1}{c^2}\omega_1 \qquad (111)$$

where ω_1 is the frequency of the light at the point of emission; ω_2 is the frequency at the point of observation; ϕ_1 and ϕ_2 are respectively, the Newtonian gravitational potentials at the point of emission and at the point of observation.

This expression has been deduced from $t = t_0\sqrt{-g_{00}}$ [38] which correlates *own time* (real time), t, with the temporal coordinate x^0 of the space-time ($t_0 = x^0/c$).

When the gravitational field is *weak*, the temporal component g_{00} of the metric tensor is given by $g_{oo} = -1 - 2\phi/c^2$ [39]. Thus we readily obtain:

$$t = t_0\sqrt{1 - 2Gm_g/rc^2} \qquad (112)$$

Curiously, this equation tell us that we can have $t < t_0$ when $m_g > 0$; and $t > t_0$ for $m_g < 0$. In addition, if $m_g = c^2 r/2G$, i.e., if $r = 2Gm_g/c^2$ (*Schwarzschild radius*) we obtain $t = 0$.

Let us now consider the well-known process of stars' *gravitational contraction*. It is known that the destination of the star is directly correlated to its mass. If the star's mass is less than $1.4M_\odot$ (Schemberg -Chandrasekhar's limit), it becomes a *white dwarf*. If its mass exceeds that limit, the pressure produced by the degenerate state of the matter no longer counterbalances the gravitational pressure, and the star's contraction continues. Afterwards occur the reactions between protons and electrons (capture of electrons), where *neutrons* and anti-neutrinos are produced.

The contraction continues until the system regains stability (when the pressure produced by the neutrons is sufficient to stop the gravitational collapse). Such systems are called *neutron stars*.

There is also a critical mass for the stable configuration of neutron stars. This limit has not been fully defined as yet, but it is known that it is located between $1.8M_\odot$ and $2.4M_\odot$. Thus, if the mass of the star exceeds $2.4M_\odot$, the contraction will continue.

According to Hawking [40] collapsed objects cannot have mass less than $\sqrt{\hbar c/4G} = 1.1\times10^{-8}kg$. This means that, with the progressing of the compression, the neutrons cluster must become a cluster of superparticles where the *minimal inertial mass* of the superparticle is:

$$m_{i(sp)} = 1.1\times10^{-8}kg. \qquad (113)$$

Symmetry is a fundamental attribute of the Universe that enables an investigator to study particular aspects of physical systems by themselves. For example, the assumption that space is homogeneous and isotropic is based on *Symmetry Principle*. Also here, by symmetry, we can assume that there are only *superparticles* with mass $m_{i(sp)} = 1.1\times10^{-8}kg$ in the cluster of *superparticles*.

Based on the mass-energy of the superparticles (~10^{18} GeV) we can say that they belong to a putative class of particles with mass-energy beyond the *supermassive* Higgs bosons (the so-called X bosons). It is known that the GUT's theories predict an entirely new force mediated by a new type of boson, called simply X (or X boson). The X bosons carry both electromagnetic and color charge, in order to ensure proper conservation of those charges in any interactions. The X bosons must be extremely massive, with mass-energy in the unification range of about 10^{16} GeV.

If we assume the superparticles *are not hyper*massive Higgs bosons then the possibility of the *neutrons cluster* become a *Higgs bosons cluster* before becoming a *superparticles cluster* must be considered. On the other hand, the fact that superparticles must be so massive also means that it is not possible to create them in any conceivable particle accelerator that could be built. They can exist as free particles only at a very early stage of the Big Bang from which the universe emerged.

Let us now imagine the Universe coming back to the past. There will be an instant in which it will be similar to a *neutrons cluster*, such as the stars at the final state of gravitational contraction. Thus, with the progressing of the compression, the *neutrons* cluster becomes a superparticles cluster. Obviously, this only can occur before 10^{-23}s (after the Big-Bang).

The temperature T of the Universe at the 10^{-43}s< t < 10^{-23}s period can be calculated by means of the well-known expression [41]:

$$T \approx 10^{22}\left(t/10^{-23}\right)^{-1/2} \qquad (114)$$

Thus at $t \cong 10^{-43}\,s$ (at the *first* spontaneous breaking of symmetry) the temperature was $T \approx 10^{32}\,K$ (~10^{19}GeV). Therefore, we can assume that the absorbed electromagnetic energy by each *superparticle*, before $t \cong 10^{-43}\,s$, was $U = \eta kT > 1\times 10^{9}\,J$ (see Equations (71) and (72)). By comparing with $m_{i(sp)}c^2 \cong 9\times 10^{8}\,J$, we conclude that $U > m_{i(sp)}c$. Therefore, *the unification condition* $\left(U\eta_r \cong M_i c^2 > m_i c^2\right)$ is satisfied. This means that, *before* $t \cong 10^{-43}\,s$, *the gravitational and electromagnetic interactions were unified*.

From the *unification condition* $\left(U\eta_r \cong M_i c^2\right)$, we may conclude that the superparticles' *relativistic inertial mass* $M_{i(sp)}$ is:

$$M_{i(sp)} \cong \frac{U\eta_r}{c^2} = \frac{\eta n_r kT}{c^2} \approx 10^{-8}\,kg \qquad (115)$$

Comparing with the superparticles' *inertial mass at rest* (113), we conclude that:

$$M_{i(sp)} \approx m_{i(sp)} = 1.1\times 10^{-8}\,kg \qquad (116)$$

From Equations (83) and (115), we obtain the superparticle's *gravitational mass at rest*:

$$m_{g(sp)} = m_{i(sp)} - 2M_{i(sp)} \cong$$
$$\cong -M_{i(sp)} \cong -\frac{\eta n_r kT}{c^2} \qquad (117)$$

and consequently, the superparticle's *relativistic gravitational mass*, is:

$$M_{g(sp)} = \left|\frac{m_{g(sp)}}{\sqrt{1-V^2/c^2}}\right| = \frac{\eta n_r kT}{c^2\sqrt{1-V^2/c^2}} \qquad (118)$$

Thus, the gravitational forces between two *superparticles*, according to (13), is given by:

$$\vec{F}_{12} = -\vec{F}_{21} = -G\frac{M_{g(sp)}M'_{g(sp)}}{r^2}\hat{\mu}_{21} = \left[\left(\frac{M_{i(sp)}}{m_{i(sp)}}\right)^2\left(\frac{G}{c^5\hbar}\right)(\eta n_r \kappa T)^2\right]\frac{\hbar c}{r^2}\hat{\mu}_{21} \qquad (119)$$

Due to the *unification* of the gravitational and electromagnetic interactions at that period, we have:

$$\vec{F}_{12} = -\vec{F}_{21} = G\frac{M_{g(sp)}M'_{g(sp)}}{r^2}\hat{\mu}_{21} = \left[\left(\frac{M_{i(sp)}}{m_{i(sp)}}\right)^2\left(\frac{G}{c^5\hbar}\right)(\eta\kappa T)^2\right]\frac{\hbar c}{r^2}\hat{\mu}_{21} = \frac{e^2}{4\pi\varepsilon_0 r^2} \qquad (120)$$

From the equation above we can write:

$$\left[\left(\frac{M_{i(sp)}}{m_{i(sp)}}\right)^2\left(\frac{G}{c^5\hbar}\right)(\eta\kappa T)^2\hbar c\right] = \frac{e^2}{4\pi\varepsilon_0} \qquad (121)$$

Now, assuming that:

$$\left(\frac{M_{i(sp)}}{m_{i(sp)}}\right)^2\left(\frac{G}{c^5\hbar}\right)(\eta\kappa T)^2=\psi \qquad (122)$$

Equation (121) can be rewritten in the following form:

$$\psi=\frac{e^2}{4\pi\varepsilon_0\hbar c}=\frac{1}{137} \qquad (123)$$

which is the well-known *reciprocal fine structure constant.*

For $T=10^{32}K$ the Equation (122) gives:

$$\psi=\left(\frac{M_{i(sp)}}{m_{i(sp)}}\right)^2\left(\frac{G}{c^5\hbar}\right)(\eta n_r\kappa T)^2\approx\frac{1}{100} \qquad (124)$$

This value has the same order of magnitude that the exact value (1/137) of the *reciprocal fine structure constant*.

From equation (120) we can write:

$$\left(G\frac{M_{g(sp)}M'_{g(sp)}}{\psi c\vec{r}}\right)\vec{r}=\hbar \qquad (125)$$

The term between parenthesis has the same dimensions that the *linear momentum* $\vec{p}$. Thus (125) tells us that:

$$\vec{p}\cdot\vec{r}=\hbar. \qquad (126)$$

A component of the momentum of a particle cannot be precisely specified without loss of all knowledge of the corresponding component of its position at that time (i.e., a particle cannot be precisely localized in a particular direction without loss of all knowledge of its momentum component in that direction).

This means that in intermediate cases the product of the uncertainties of the simultaneously measurable values of corresponding position and momentum components is *at least of the order of magnitude* of $\hbar$,

$$\Delta p.\Delta r\geq\hbar \qquad (127)$$

This relation, *directly obtained here from the Unified Theory*, is the well-known relation of the *Uncertainty Principle* for position and momentum.

According to Equation (83), the gravitational mass of the superparticles at the *center* of the cluster becomes *negative* when $2\eta n_r kT/c^2>m_{i(sp)}$, i.e., when:

$$T>T_{critical}=\frac{m_{i(sp)}}{2\eta n_r k}\approx 10^{32}K.$$

According to Equation (114) this temperature corresponds to $t_c\approx10^{-43}s$. With the progressing of the compression, more superparticles into the center will have *negative* gravitational mass. Consequently, there will be a critical point in which the *repulsive* gravitational forces between the superparticles with negative gravitational masses and the superparticles with positive gravitational masses will be so strong that an explosion will occur. This is the event that we call the Big Bang.

Now, starting from the Big Bang to the present time. Immediately after the Big Bang, the superparticles' *decompression* begins. The gravitational mass of the most central superparticle will only be positive when the temperature becomes smaller than the critical temperature, $T_{critical}\approx10^{32}K$. At the maximum state of compression (exactly at the Big Bang) the volumes of the superparticles was equal to the elementary volume $\Omega_0=\delta_V d^3_{\boldsymbol{min}}$ and the volume of the Universe was $\Omega=\delta_V(nd_{min})^3=\delta_V d^3_{initial}$ where $d_{initial}$ was the *initial* length scale of the Universe. At this very moment the *average* density of the Universe was equal to the *average* density of the superparticles, thus we can write:

$$\left(\frac{d_{initial}}{d_{min}}\right)^3=\frac{M_{i(U)}}{m_{i(sp)}} \qquad (128)$$

where $M_{i(U)}\approx10^{53}kg$ is the inertial mass of the Universe. It has already been shown that $d_{min}=\tilde{k}l_{planck}\approx10^{-34}m$. Then, from Equation (128), we obtain:

$$d_{initial}\approx10^{-14}m \qquad (129)$$

After the Big Bang the Universe expands itself from $d_{initial}$ up to d_{cr} (when the temperature decrease reaches the critical temperature $T_{critical} \approx 10^{32} K$, and the gravity becomes *attractive*). Thus, it expands by $d_{cr} - d_{initial}$, under effect of the *repulsive* gravity:

$$\bar{g} = \sqrt{g_{max} g_{min}} =$$
$$= \sqrt{\left[G\left(\tfrac{1}{2} M_{g(U)}\right)/\left(\tfrac{1}{2} d_{initial}\right)^2\right] \left[G \tfrac{1}{2} M_{i(U)}/\left(\tfrac{1}{2} d_{cr}\right)^2\right]} =$$
$$= \frac{2G\sqrt{M_{g(U)} M_{i(U)}}}{d_{cr} d_{initial}} = \frac{2G\sqrt{\sum m_{g(sp)} M_{i(U)}}}{d_{cr} d_{initial}} =$$
$$= \frac{2G\sqrt{\chi \sum m_{i(sp)} M_{i(U)}}}{d_{cr} d_{initial}} = \frac{2GM_{i(U)}\sqrt{\chi}}{d_{cr} d_{initial}}$$

during a period of time $t_c \approx 10^{-43} s$ .Thus,

$$d_{cr} - d_{initial} = \tfrac{1}{2}\bar{g}(t_c)^2 = \left(\sqrt{\chi}\right)\frac{GM_{i(U)}}{d_{cr} d_{initial}}(t_c)^2 \qquad (130)$$

Equation (83), gives:

$$\chi = \frac{m_{g(sp)}}{m_{i(sp)}} = 1 - \frac{2Un_r}{m_{i(sp)} c^2} = 1 - \frac{2\eta n_r kT}{m_{i(sp)} c^2} \approx 10^{-32} T$$

The temperature at the beginning of the Big Bang (t=0) should have been much greater than $T_{critical} \approx 10^{32} K$. Thus, χ must be a very big number. Then it is easily seen that during this period, the Universe expanded at an astonishing rate. Thus, there is an evident *inflation* period, which ends at $t_c \approx 10^{-43} s$.

With the progressing of the *decompression* the *superparticles* cluster becomes a neutrons cluster. This means that the neutrons are created *without its antiparticle*, the antineutron. Thus it solves the matter/antimatter dilemma that is unresolved in many cosmologies.

Now a question: How did the *primordial superparticles* appear at the beginning of the Universe?

It is a proven quantum fact that a wave function may collapse, and that, at this moment, all the possibilities that it describes are suddenly expressed in *reality*. This means that, through this process, particles can be suddenly *materialized*.

The materialization of the *primordial superparticles* into a critical volume denotes *knowledge* of what would happen with Universe starting from that *initial condition*, a fact that in the view of the author, points towards the *existence* of a Creator.

It was shown previously the possible existence of *imaginary particles* with imaginary masses in Nature. These particles can be associated with real particles, such as in case of the *photons* and *electrons*, as we have shown, or they can be associated with others imaginary particles by constituting the imaginary bodies. Just as the real particles constitute the real bodies.

The idea that we make about a *consciousness* is basically that of an *imaginary body* containing *psychic energy* and *intrinsic knowledge*. We can relate psychic energy with *psychic mass* (psychic mass= psychic energy/c^2). Thus, by analogy with the real bodies the psychic bodies would be constituted by psychic particles with psychic mass. Consequently, the psychic particles that constitute a consciousness would be equivalent to imaginary particles, and the *psychic mass* , m_Ψ ,of the psychic particles would be equivalent to the *imaginary mass*, i.e.,

$$m_\Psi = m_{g(imaginary)} \qquad (131)$$

Thus, the imaginary masses associated to the *photons* and *electrons* would be *elementary psyche* actually, i.e.,

$$m_{\Psi photon} = m_{g(imaginary)photon} =$$
$$= \frac{4}{\sqrt{3}}\left(\frac{hf}{c^2}\right) i \qquad (132)$$

$$m_{\Psi electron} = m_{g(imaginary)electron} =$$
$$= \tfrac{4}{\sqrt{3}}\left(\frac{hf_{electron}}{c^2}\right) i =$$
$$= \tfrac{4}{\sqrt{3}} m_{i(real)electron}\, i \qquad (133)$$

The idea that electrons have elementary psyche associated to themselves is not new. It comes from the pre-Socratic period. By proposing the existence of psyche associated with matter, we are adopting what is called *panpsychic* posture. Panpsychism dates back to the pre-Socratic period; remnants of organized panpsychism may be found in the Uno of Parmenides or in Heracleitus's Divine Flow. The scholars of Miletus's school were called *hylozoists*, that is, "those who believe that matter is alive". More recently, we will find the panpsychistic thought in Spinoza, Whitehead and Teilhard de Chardin, among others. The latter one admitted the existence of proto-conscious properties at the elementary particles' level.

We can find experimental evidences of the existence of psyche associated to electron in an experiment similar to that commonly used to show the wave duality of light. (Figure 9). One merely substitutes an electron ray (fine electron beam) for the light ray. Just as in the experiment mentioned above, the ray which goes through the holes is detected as a wave if a *wave detector* is used (it is then observed that the interference pattern left on the detector screen is analogous with that produced by the light ray), and as a particle if a *particle detector* is used.

Since the electrons are detected on the other side of the metal sheet, it becomes obvious then that they passed through the holes. On the other hand, it is also evident that when they approached the holes, they had to decide which one of them to go through.

How can an electron "decide" which hole to go through? Where there is "choice", isn't there also *psyche*, by definition? The existence of *psyche mass* associated to material particles can solve the black hole *information paradox* ‡ if we assume that any *information* placed into a black hole is saved into the psyche mass associated to the black hole and after - during the evaporation of the black hole, carried by the psyche masses associated to the evaporated particles (Hawking radiation). Thus, if the material entering the black hole is *a pure quantum state*, the transformation of that state into the mixed state of Hawking radiation does *not* destroy information about the original quantum state because the psyche mass does not lose any structure of the original quantum state in the transformation. In this way, when information goes to into a black hole it is not destroyed and might escape from the black hole during its evaporation.

The idea of psyche mass associated to a black hole leads to conclusion that there has been a psyche mass associated to the Initial Universe.

If the primordial superparticles that have been materialized at the beginning of the Universe came from the collapse of a primordial wave function, then the psychic form described by this wave function must have been generated in a consciousness with a psychic mass much greater than that needed to materialize the Universe (material and psychic).

This giant consciousness, in its turn, would not only be the greatest of all consciences in the Universe but also the *substratum* of everything that exists and, obviously, everything that exists would be entirely contained within it, including *all the spacetime*.

Thus, if the consciousness we refer to contains all the space, its volume is necessarily infinite, consequently having an *infinite psychic mass*.

This means that it contains all the existing psychic mass and, therefore, any other consciousness that exists will be contained in it. Hence, we may conclude that "It" is the *Supreme Consciousness* and that there is no other equal to It: It is *unique*.

Since the Supreme Consciousness also contains *all* time; past, present and future, then, for It the time does not flow as it flows for us.

‡ In 1975, Stephen Hawking showed that black holes eventually evaporate releasing particles containing no information (Hawking radiation). It was soon realized that this prediction created an *information loss*. From the *no hair theorem* one would expect the Hawking radiation to be completely independent of the material entering the black hole. However, if the material entering the black hole were *a pure quantum state*, the transformation of that state into the mixed state of Hawking radiation would *destroy information* about the original quantum state. This violates Liouville's theorem and presents a *physical paradox*. It is a contentious subject for science since it violated a commonly assumed tenet of science—that *information cannot be destroyed*.

Within this framework, when we talk about the Creation of the Universe, the use of the verb "to create" means that "something that was not" came into being, thus presupposing the concept of *time flow*.

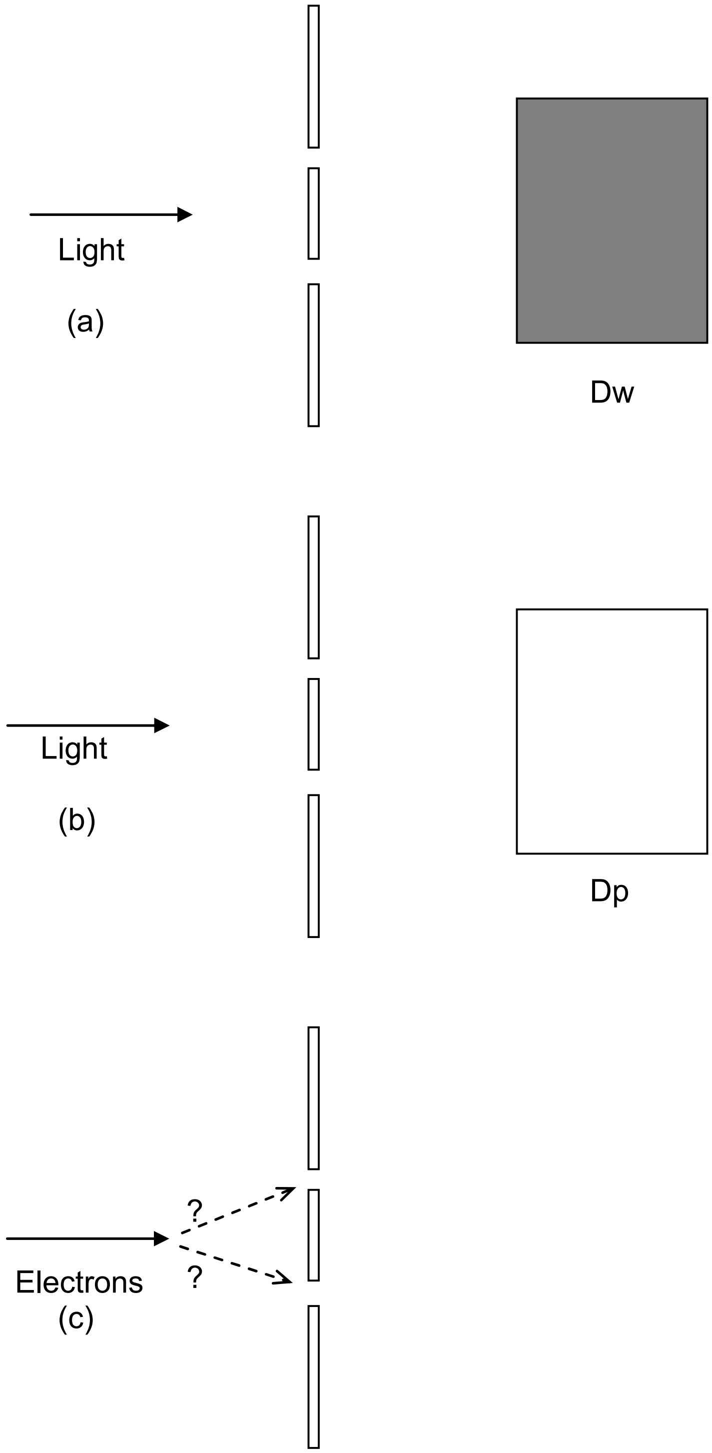


Figure 9: A Light Ray after Going through the holes in a Metal Sheet.

(a) Will be detected as a wave by a wave detector Dw or (b) as a particle if the wave detector is substituted for the wave detector Dp. Electron ray (c) has similar behavior as that of a light ray. However, before going through the holes, the electrons must "decide" which one to go through.

For the Supreme Consciousness, however, the instant of Creation is mixed up with all other times, consequently there being no "before" or "after" the Creation and, thus, the following question is not justifiable: "What did the Supreme Consciousness do before Creation?"

On the other hand, we may also infer, from the above that the existence of the Supreme Consciousness has no defined limit (beginning and end), what confers upon It the unique characteristic of *uncreated* and *eternal*.

If the Supreme Consciousness is eternal, Its wave function Ψ_{SC} shall never collapse (will never be null). Thus, for having an infinite psychic mass, the value of Ψ_{SC}^2 will be always infinite and, hence, we may write that

$$\int_{-\infty}^{+\infty}\Psi_{SC}^2 dV = \infty$$

By comparing this equation with Equation (108) derived from Quantum Mechanics, we conclude that the Supreme Consciousness is simultaneously everywhere (i.e., It is *omnipresent)*.

Since the Supreme Consciousness contains all consciences, it is expected that It also contain *all the knowledge.* Therefore, It is also *omniscient*. Consequently, It *knows* how to formulate well-defined mental images with psychic masses sufficient for its contents to *materialize*. In this way, It can materialize *everything* It wishes (*omnipotence*).

All of these characteristics of the Supreme Consciousness (infinite, unique, uncreated, eternal, omnipresent, omniscient, and omnipotent) coincide with those traditionally ascribed to *God* by most religions.

It was shown in this work that the "virtual" *quanta* of the *gravitational interaction* must have spin 1 and not 2, and that they are "virtual" photons (*graviphotons*) with *zero mass* outside the *coherent* matter. Inside the coherent matter the graviphoton mass is *non-zero*. Therefore, the gravitational forces are also *gauge* forces, because they are yielded by the exchange of "virtual" *quanta* of spin 1, such as the electromagnetic forces and the weak and strong nuclear forces.
Thus, the gravitational forces are produced by the exchange of "virtual" photons. Consequently, this is precisely the *origin of the gravity*.

Newton's theory of gravity does not explain *why* objects attract one another; it simply models this observation. Also Einstein's theory does not explain the origin of gravity. Einstein's theory of gravity only describes gravity with more precision than Newton's theory does.

Besides, there is nothing in both theories explaining the *origin of the energy* that produces the gravitational forces. Earth's gravity attracts all objects on the surface of our planet. This has been going on for well over 4.5 billion years, yet no known energy source is being converted to support this tremendous ongoing energy expenditure. Also is the enormous continuous energy expended by Earth's gravitational field for maintaining the Moon in its orbit - millennium after millennium. In spite of the ongoing energy expended by Earth's gravitational field to hold objects down on surface and the Moon in orbit, why the energy of the field never diminishes in strength or drains its energy source? Is this energy expenditure balanced by a conversion of energy from an unknown energy source?

The energy W necessary to support the effort expended by the gravitational forces F is well-known and given by:

$$W = \int_{\infty}^{r} F dr = -G\frac{M_g m_g}{r}$$

According to the *principle of energy conservation*, this energy expenditure must be balanced by a conversion of energy from another energy type.

The Uncertainty Principle tells us that, due to the occurrence of exchange of *graviphotons* in a time interval $\Delta t < \hbar/\Delta E$ (where ΔE is the energy of the graviphoton), the energy variation ΔE cannot

be detected in the system $M_g - m_g$. Since the total energy W is the sum of the energy of the n graviphotons (i.e., $W = \Delta E_1 + \Delta E_2 + \ldots + \Delta E_n$), then the energy W *cannot be detected as well*. However, as we know it can be converted into another type of energy, for example, in rotational kinetic energy, as in the hydroelectric plants, or in the *Gravitational Motor*, as shown in this work.
It is known that a *quantum* of energy $\Delta E = hf$ which varies during a time interval $\Delta t = 1/f = \lambda/c < \hbar/\Delta E$ (wave period) cannot be experimentally detected. This is an *imaginary* photon or a "*virtual*" photon. Thus, the graviphotons are *imaginary* photons, i.e., the energies ΔE_i of the graviphotons are imaginaries energies and therefore the energy $W = \Delta E_1 + \Delta E_2 + \ldots + \Delta E_n$ is also an *imaginary* energy. Consequently, it belongs to the *imaginary space-time*.

According to Equation (131), imaginary energy is equal to *psychic energy*. Consequently, the *imaginary space-time* is, in fact, the *psychic space-time*, which contains the Supreme Consciousness. Since the Supreme Consciousness has infinite psychic mass, then the *psychic space-time* has *infinite psychic energy*. This is highly relevant, because it confers to the *psychic space-time* the characteristic of *unlimited source of energy*.

This can be easily confirmed by the fact that, in spite of the enormous amount of energy expended by Earth's gravitational field to hold objects down on the surface of the planet and maintain the Moon in its orbit, the energy of Earth's gravitational field never diminishes in strength or drains its energy source.

If an experiment involves a large number of identical particles, all described by the same wave function Ψ, *real* density of mass ρ of these particles in x, y, z, t is proportional to the corresponding value Ψ^2 (Ψ^2 is known as *density of probability*. If Ψ is *complex* then $\Psi^2 = \Psi\Psi^*$. Thus, $\rho \propto \Psi^2 = \Psi.\Psi^*$). Similarly, in the case of psychic particles, the *density of psychic mass*, ρ_Ψ, in x, y, z, will be expressed by $\rho_\Psi \propto \Psi_\Psi^2 = \Psi_\Psi \Psi_\Psi^*$. It is known that Ψ_Ψ^2 is always *real* and *positive* while $\rho_\Psi = m_\Psi/V$ is an *imaginary* quantity. Thus, as the *modulus* of an imaginary number is always real and positive, we can transform the proportion $\rho_\Psi \propto \Psi_\Psi^2$, in equality in the following form:

$$\Psi_\Psi^2 = k|\rho_\Psi| \qquad (134)$$

Where k is a *proportionality constant* (real and positive) to be determined.

In Quantum Mechanics we have studied the *Superpositon Principle*, which affirms that, if a particle (or system of particles) is in a *dynamic state* represented by a wave function Ψ_1 and may also be in another dynamic state described by Ψ_2 then, the general dynamic state of the particle may be described by Ψ, where Ψ is a linear combination (superposition) of Ψ_1 and Ψ_2, i.e.:

$$\Psi = c_1\Psi_1 + c_2\Psi_2 \qquad (135)$$

C*omplex constants* c_1 and c_2, respectively, indicates the percentage of dynamic state, represented by Ψ_1 e Ψ_2 in the formation of the general dynamic state described by Ψ.

In the case of psychic particles (psychic bodies, consciousness, etc.), by analogy, if $\Psi_{\Psi 1}$, $\Psi_{\Psi 2}$,..., $\Psi_{\Psi n}$ refer to the different dynamic states the psychic particle assume, then its general dynamic state may be described by the wave function Ψ_Ψ, given by:

$$\Psi_\Psi = c_1\Psi_{\Psi 1} + c_2\Psi_{\Psi 2} + \ldots + c_n\Psi_{\Psi n} \qquad (136)$$

The state of superposition of wave functions is, therefore, common for both psychic and material particles. In the case of material particles, it can be verified, for instance, when an electron changes from one orbit to another. Before effecting the transition to another energy level, the electron carries out "virtual transitions" [42]; a kind of *relationship* with other electrons before performing the real transition. During this relationship period, its wave function remains "*scattered*" by *a wide region of the space* [43] thus superposing the wave functions of the other electrons. In this relationship the electrons *mutually* influence each other, with the possibility of *intertwining* their wave functions[2]. When this happens, there occurs the so-called *Phase Relationship* according to quantum-mechanics concept.

In the electrons "virtual" transition mentioned before, the "listing" of all the possibilities of the electrons is described, as we know, by *Schrödinger's wave equation.* Otherwise, it is general for material particles. By analogy, in the case of psychic particles, we may say that the "listing" of all the possibilities of the psyches involved in the relationship will be described by *Schrödinger's equation* – for psychic case, i.e.,

$$\nabla^2\Psi_\Psi + \frac{p_\Psi^2}{\hbar^2}\Psi_\Psi = 0 \qquad (137)$$

Because the wave functions are capable of intertwining themselves, the quantum systems may "penetrate" each other, thus establishing an internal relationship where all of them are affected by the relationship, no longer being isolated systems but becoming an integrated part of a larger system. This type of internal relationship, which exists only in quantum systems, was called *Relational Holism* [44].

The equation of *quantization of mass* (33), in the generalized form is expressed by:

$$m_{g(imaginary)} = n^2 m_{g(imagynary)(min)}$$

Thus, we can also conclude that the *psychic mass is also quantized*, due to $m_\Psi = m_{g(imaginary)}$ (Equation 131), i.e.,

$$m_\Psi = n^2 m_{\Psi(min)} \qquad (138)$$

where

$$m_{\Psi(min)} = \tfrac{4}{\sqrt{3}}\left(hf_{min}/c^2\right) i = $$
$$= \tfrac{4}{\sqrt{3}} m_{i(real)min}\ i \qquad (139)$$

[2] Since the electrons are simultaneously waves and particles, their wave aspects will interfere with each other; besides superposition, there is also the possibility of occurrence of *intertwining* of their wave functions.

It was shown that the *minimum quantum* of real inertial mass in the Universe, $m_{i(real)min}$, is given by:

$$m_{i(real)min} = \pm h\sqrt{3/8}\,/cd_{max} =$$
$$= \pm 3.9\times10^{-73}kg \qquad (140)$$

By analogy to Equations (132) and (133), the expressions of the psychic masses associated to the *proton* and the *neutron* are respectively given by:

$$m_{\Psi proton} = m_{g(imaginary)proton} =$$
$$= \tfrac{4}{\sqrt{3}}\left(hf_{proton}/c^2\right) i =$$
$$= \tfrac{4}{\sqrt{3}} m_{i(real)proton}\ i \qquad (141)$$

$$m_{\Psi neutron} = m_{g(imaginary)neutron} =$$
$$= \tfrac{4}{\sqrt{3}}\left(hf_{neutron}/c^2\right) i =$$
$$= \tfrac{4}{\sqrt{3}} m_{i(real)neutron}\ i \qquad (142)$$

The *imaginary* gravitational masses of the atoms must be *very smaller* than their *real* gravitational masses. On contrary, the weight of the bodies would be very different of the observed values.

This fact shows that $m_{g(imaginary)proton}$ and $m_{g(imaginary)neutron}$ must have *contrary* signs. In this way, the *imaginary* gravitational mass of an atom can be expressed by means of the following expression:

$$m_{g(imaginary)atom} = N\left(m_e \pm \left(m_n - m_p\right) + \frac{\Delta E}{c^2}\right)i$$

where, ΔE, is the interaction energy. By comparing this expression with the following expression:

$$m_{g(real)atom} = N\left(m_e + m_p + m_n + \frac{\Delta E}{c^2}\right)$$

Thus,

$$m_{g(imaginary)atom} << m_{g(real)atom}$$

Now consider a monatomic body with *real* mass $M_{g(real)}$ and *imaginary* mass $M_{g(imaginary)}$. Then we have the following:

$$\frac{M_{g(imaginary)}}{M_{g(real)}}=\frac{\Sigma\left(m_{g(imaginary)atom}+\frac{\Delta E_a i}{c^2}\right)}{\Sigma\left(m_{g(real)atom}+\frac{\Delta E_a}{c^2}\right)}=$$

$$=\frac{n\left(m_e\pm(m_n-m_p)+\frac{\Delta E}{c^2}+\frac{\Delta E_a}{c^2}\right)i}{n\left(m_e+m_p+m_n+\frac{\Delta E}{c^2}+\frac{\Delta E_a}{c^2}\right)}\cong$$

$$\cong\left(\frac{m_e\pm(m_n-m_p)+\frac{\Delta E}{c^2}}{m_e+m_p+m_n+\frac{\Delta E}{c^2}}\right)i$$

Since $\Delta E_a << \Delta E$.

The intensity of the gravitational forces between $M_{g(imaginary)}$ and an imaginary particle with mass $m_{g(imaginary)}$ is given by:

$$F=G\,M_{g(imaginary)}m_{g(imaginary)}/r^2=$$

$$\cong\left(\frac{m_e\pm(m_n-m_p)+\frac{\Delta E}{c^2}}{m_e+m_p+m_n+\frac{\Delta E}{c^2}}\right)G\frac{M_{g(real)}i\;m_{g(real)}i}{r^2}$$

Therefore, the *total* gravity is:

$$g_{real}+\Delta g_{(imaginary)}=-G\frac{M_{g(real)}}{r^2}-$$

$$-\left(\frac{m_e\pm(m_n-m_p)+\frac{\Delta E}{c^2}}{m_e+m_p+m_n+\frac{\Delta E}{c^2}}\right)G\frac{M_{g(real)}}{r^2}$$

Thus, the *imaginary* gravitational mass of a body produces an *excess* of gravity acceleration, Δg , given by:

$$\Delta g\cong\left(\frac{m_e\pm(m_n-m_p)+\frac{\Delta E}{c^2}}{m_e+m_p+m_n+\frac{\Delta E}{c^2}}\right)G\frac{M_{g(real)}}{r^2}$$

In the case of *soft* atoms we can consider $\Delta E\cong 2\times10^{-13}\,joules$. Thus, in this case we obtain:

$$\Delta g\cong 6\times10^{-4}G\frac{M_g}{r^2}$$

In the case of the *Sun*, for example, there is an *excess* of gravity acceleration, due to its *imaginary* gravitational mass, given by:

$$\Delta g\cong\left(6\times10^{-4}\right)G\frac{M_{gS}}{r^2}$$

At a distance from the Sun of $r=1.0\times10^{13}m$ the value of Δg is:

$$\Delta g\cong 8\times10^{-10}m.s^{-2}$$

Experiments in the pioneer 10 spacecraft, at a distance from the Sun of about 67 AU or $r=1.0\times10^{13}m$ [45], measured an excess acceleration towards the Sun of:

$$\Delta g=8.74\pm1.33\times10^{-10}m.s^{-2}$$

Note that the general expression for the gravity acceleration of the Sun is:

$$g=\left(1+\approx6\times10^{-4}\right)G\frac{M_{gS}}{r^2}$$

Therefore, in the case of the *gravitational deflection of light about the Sun*, the new expression for the deflection of the light is:

$$\delta=\left(1+\approx6\times10^{-4}\right)\frac{4GM_{gS}}{c^2d}$$

Thus, the increase in δ due to the excess acceleration towards the Sun can be considered *negligible*.

Similarly to the collapse of the real wave function, the collapse of the psychic wave function must suddenly also express in reality all the possibilities described by it. This is, therefore, *a point of decision* in which there occurs the compelling need of realization of the *psychic form*. Thus, this is moment in which the content of the psychic form realizes itself in the space-time. For an observer in space-time, something is *real* when it is under a matter or radiation form. Therefore, the content of the psychic form may realize itself in space-time exclusively under the form of radiation, that is, it does not materialize. This must occur when the *Materialization Condition* is not satisfied, i.e., when the content of the psychic form is undefined (impossible to be defined by its own psychic) or it does not contain enough psychic mass to *materialize*[3] the respective psychic contents.

Nevertheless, in both cases, there must always be a production of "virtual" photons to convey the psychic interaction to the other psychic particles, according to the quantum field theory, only through this type of quanta will interaction be conveyed, since it has an infinite reach and may be either attractive or repulsive, just as electromagnetic interaction which, as we know, is conveyed by the exchange of "virtual" photons.

If electrons, protons and neutrons have psychic mass, then we can infer that the psychic mass of the atoms are *Phase Condensates*[4]*.* In the case of the molecules the situation is similar. More molecular mass means more atoms and consequently, more psychic mass. In this case the phase condensate also becomes more structured because the great amount of elementary psyches inside the condensate requires, by stability reasons, a better distribution of them. Thus, in the case of molecules with very large molecular masses (*macromolecules*) it is possible that their psychic masses already constitute the most organized shape of a Phase Condensate, called Bose-Einstein Condensate[5].

The fundamental characteristic of a Bose-Einstein condensate is, as we know, that the various parts making up the condensed system not only behave as a whole but also *become a whole*, i.e., in the psychic case, the various consciousnesses of the system become a *single consciousness* with psychic mass equal to the sum of the psychic masses of all the consciousness of the condensate. This obviously, increases the available knowledge in the system since it is proportional to the psychic mass of the consciousness. This unity confers an *individual* character to this type of consciousness. For this reason, from now on they will be called *Individual Material Consciousness.*

It derives from the above that most bodies do not possess individual material consciousness. In an iron rod, for instance, the cluster of elementary psyches in the iron molecules does not constitute Bose-Einstein condensate; therefore, the iron rod does not have an individual consciousness. Its consciousness is consequently, much more simple and constitutes just a phase condensate imprecisely structured made by the consciousness of the iron atoms.

The existence of consciousnesses in the atoms is revealed in the molecular formation, where atoms with strong mutual affinity (their consciousnesses) combine to form molecules. It is the case, for instance of the water molecules, in which two Hydrogen atoms join an Oxygen atom. Well, how come the combination between these atoms is always the same: the same grouping and the same invariable proportion? In the case of molecular combinations the phenomenon repeats itself. Thus, the chemical substances either mutually attract or repel themselves, carrying out specific motions for this reason. It is the so-called *Chemical Affinity*. This phenomenon certainly results from a specific interaction between the consciousnesses. From now on, it will be called *Psychic Interaction*.

Mutual Affinity is a dimensionless psychic quantity with which we are familiar and of which we have perfect understanding as to its meaning. The degree of *Mutual Affinity*, A, in the case of two consciousnesses, respectively described by

---

[3] By this we mean not only materialization proper but also the movement of matter to realize its psychic content (including radiation).

[4] Ice and NaCl crystals are common examples of imprecisely-structured *phase condensates*. Lasers, super fluids, superconductors and magnets are examples of phase condensates more structured.

[5] Several authors have suggested the possibility of the Bose-Einstein condensate occurring in the brain, and that it might be the physical base of memory, although they have not been able to find a suitable mechanism to underpin such a hypothesis. Evidences of the existence of Bose-Einstein condensates in living tissues abound (Popp, F.A Experientia, Vol. 44, p.576-585; Inaba, H., New Scientist, May89, p.41; Rattermeyer, M and Popp, F. A. Naturwissenschaften, Vol.68, Nº5, p.577.)

$\Psi_{\Psi 1}$ e $\Psi_{\Psi 2}$, must be correlated to $\Psi_{\Psi 1}^2$ e $\Psi_{\Psi 2}^2$ [6]. Only a simple algebraic form fills the requirements of interchange of the indices, the product,

$$\Psi_{\Psi 1}^2 . \Psi_{\Psi 2}^2 = \Psi_{\Psi 2}^2 . \Psi_{\Psi 1}^2 = = |A_{1,2}| = |A_{2,1}| = |A| \qquad (143)$$

In the above expression, $|A|$ is due to the product $\Psi_{\Psi 1}^2 . \Psi_{\Psi 2}^2$ will be always positive. From Equations (143) and (134) we get:

$$|A| = \Psi_{\Psi 1}^2 . \Psi_{\Psi 2}^2 = k^2 |\rho_{\Psi 1}||\rho_{\Psi 2}| = = k^2 \frac{|m_{\Psi 1}|}{V_1} \frac{|m_{\Psi 2}|}{V_2} \qquad (144)$$

The psychic interaction can be described starting from the psychic mass because the psychic mass is the source of the psychic field. Basically, *the psychic mass is gravitational mass*, $m_\Psi = m_{g(imaginary)}$. In this way, the equations of the gravitational interaction are also applied to the Psychic Interaction. That is, we can use Einstein's General Relativity equations, given by:

$$R_i^k = \frac{8\pi G}{c^4}\left(T_i^k - \tfrac{1}{2}\delta_i^k T\right) \qquad (145)$$

in order to describe the Psychic Interaction. In this case, the expression of the energy-momentum tensor, T_i^k, must have the following form [46]:

$$T_i^k = |\rho_\Psi| c^2 \mu_i \mu^k \qquad (146)$$

[6] Quantum Mechanics tells us that Ψ does not have a physical interpretation nor a simple meaning and also it cannot be experimentally observed. However such restriction does not apply to Ψ^2, which is known as *density of probability* and represents the probability of finding the body, described by the wave function Ψ, in the point x, y, z at the moment t. A large value of Ψ^2 means a strong possibility to find the body, while a small value of Ψ^2 means a weak possibility to find the body.

The psychic mass density, ρ_Ψ, is an imaginary quantity. Thus, in order to homogenize the above equation it is necessary to put $|\rho_\Psi|$ because, as we know, the module of an imaginary number is always real and positive.

Making on the transition to Classical Mechanics [47] one can verify that Equation (19) are reduced to:

$$\Delta\Phi = 4\pi G|\rho_\Psi| \qquad (147)$$

This is, therefore, the equation of the psychic field in *nonrelativistic* Mechanics. With respect to its form, it is similar to the equation of the gravitational field, with the difference that now, instead of the density of gravitational mass we have the density of *psychic mass*. Then, we can write the general solution of Equation (147), in the following form:

$$\Phi = -G\int \frac{|\rho_\Psi| dV}{r^2} \qquad (148)$$

This equation expresses, with nonrelativistic approximation, the potential of the psychic field of any distribution of psychic mass.

Particularly, for the potential of the field of only one particle with psychic mass $m_{\Psi 1}$, we get:

$$\Phi = -\frac{G|m_{\Psi 1}|}{r} \qquad (149)$$

Then the force produced by this field upon another particle with psychic mass $m_{\Psi 2}$ is:

$$|\vec{F}_{\Psi 12}| = |-\vec{F}_{\Psi 21}| = -|m_{\Psi 2}| \frac{\partial \Phi}{\partial r} = = -G\frac{|m_{\Psi 1}||m_{\Psi 2}|}{r^2} \qquad (150)$$

By comparing Equations (150) and (144) we obtain:

$$|\vec{F}_{\Psi 12}| = |-\vec{F}_{\Psi 21}| = -G|A| \frac{V_1 V_2}{k^2 r^2} \qquad (151)$$

In the *vectorial* form the above equation is written as follows

$$\vec{F}_{\Psi 12} = -\vec{F}_{\Psi 21} = -GA \frac{V_1 V_2}{k^2 r^2} \hat{\mu} \qquad (152)$$

V*ersor* $\hat{\mu}$ has the direction of the line connecting the mass centers (psychic mass) of both particles and oriented from $m_{\Psi 1}$ to $m_{\Psi 2}$.

In general, we may distinguish and quantify two types of mutual affinity: *positive* and *negative* (*aversion*). The occurrence of the first type is synonym of psychic *attraction*, (as in the case of the atoms in the water molecule) while the aversion is synonym of *repulsion*. In fact, Equation (152) shows that the forces $\vec{F}_{\Psi 12}$ and $\vec{F}_{\Psi 21}$ are attractive, if A is *positive* (expressing *positive* mutual affinity between the two *psychic bodies*), and repulsive if A is *negative* (expressing *negative* mutual affinity between the two *psychic bodies*). Contrary to the interaction of the matter, where the opposites attract themselves here, the *opposites repel themselves*.

A method and device to obtain images of *psychic bodies* have been previously proposed [48]. By means of this device, whose operation is based on the gravitational interaction and the piezoelectric effect, it will be possible to observe psychic bodies.

Expression (144) can be rewritten in the following form:

$$A = k^2 \frac{m_{\Psi 1}}{V_1} \frac{m_{\Psi 2}}{V_2} \qquad (153)$$

The psychic masses $m_{\Psi 1}$ and $m_{\Psi 2}$ are *imaginary* quantities. However, the product $m_{\Psi 1} . m_{\Psi 2}$ is a *real* quantity. One can then conclude from the previous expression that the degree of mutual affinity between two consciousnesses depends basically on the densities of their psychic masses, and that:

1) If $m_{\Psi 1} > 0$ and $m_{\Psi 2} > 0$ then $A > 0$ (positive mutual affinity between them)

2) If $m_{\Psi 1} < 0$ and $m_{\Psi 2} < 0$ then $A > 0$ (positive mutual affinity between them)

3) If $m_{\Psi 1} > 0$ and $m_{\Psi 2} < 0$ then $A < 0$ (negative mutual affinity between them)

4) If $m_{\Psi 1} < 0$ and $m_{\Psi 2} > 0$ then $A < 0$ (negative mutual affinity between them)

In this relationship, such as occurs in the case of material particles ("virtual" transition of the electrons previously mentioned), the consciousnesses interact mutually, *intertwining* or not their wave functions. When this happens, there occurs the so-called *Phase Relationship* according to quantum-mechanics concept. Otherwise a *Trivial Relationship* takes place*.*

The psychic forces such as the gravitational forces, must be very weak when we consider the interaction between two particles. However, in spite of the subtleties, those forces stimulate the relationship of the consciousnesses with themselves and with the Universe (Equation 152).

From all the preceding, we perceive that Psychic Interaction – unified with matter interactions, constitutes a single *Law* which links things and beings together and, in a network of continuous relations and exchanges, governs the Universe both in its material and psychic aspects. We can also observe that in the interactions the same principle reappears always identical. This *unity of principle* is the most evident expression of *monism* in the Universe.

APPENDIX A: ALLAIS EFFECT EXPLAINED

A Foucault-type pendulum slightly increases its period of oscillation at sites experiencing a *solar eclipse*, as compared with any other time. This effect was first observed by Allais [49] over 40 years ago. Also Saxl and Allen [50], using a torsion pendulum, have observed the phenomenon. Recently, an anomalous eclipse effect on gravimeters has become well-established [51], while some of the pendulum experiments have not. Here, we will show that the Allais gravity and pendulum effects during solar eclipses result from a *shielding effect of the Sun's gravity when the Moon is between the Sun and the Earth*.

The *interplanetary medium* includes interplanetary dust, cosmic rays and hot plasma from the solar wind. Its *density* is inversely proportional to the squared distance from the

Sun, decreasing as this distance increases. Near the Earth-Moon system, this *density* is very low, with values about $5\, protons/cm^3$ $\left(8.3\times10^{-21}\,kg/m^3\right)$. However, this density is *highly variable*. It can be increased up to $\sim 100\, protons/cm^3\ \left(1.7\times10^{-19}\,kg/m^3\right)$ [52].

The *atmosphere of the Moon* is very tenuous and insignificant in comparison with that of the Earth. The *average* daytime abundances of the elements known to be present in the lunar atmosphere, in atoms per cubic centimeter, are as follows: H <17, He 2-40x10^3, Na 70,K 17, Ar 4x10^4, yielding ~8x10^4 total atoms per cubic centimeter $\left(\cong 10^{-16} kg.m^{-3}\right)$ [53]. According to Öpik [54], near the Moon surface, the density of the lunar atmosphere can reach values up to $10^{-12} kg.m^{-3}$.The *minimum* possible density of the lunar atmosphere is in the top of the atmosphere and is essentially very close to the value of the *interplanetary medium*.

Since the density of the interplanetary medium is very small it cannot work as gravitational shielding. However, there is a top layer in the lunar atmosphere with density $\cong 1.3\times10^{-18} kg.m^{-3}$ that can work as a gravitational shielding and explain the Allais and pendulum effects. Below this layer, the density of the lunar atmosphere increases, making the effect of gravitational shielding negligible.

During the solar eclipses, when the Moon is between the Sun and the Earth, two *gravitational shieldings* $Sh1$ and $Sh2$, are established in the top layer of the lunar atmosphere (Figure 1A). In order to understand how these gravitational shieldings work (the gravitational shielding *effect*) see Figure 2 Thus, right after $Sh1$ (inside the system Moon-Lunar atmosphere), the *Sun's gravity acceleration*, $\vec{g}_S$, becomes $\chi\,\vec{g}_S$ where, according to Equation (57) χ is given by:

$$\chi = \left\{1-2\left[\sqrt{1+\left(\frac{n_r^2 D}{\rho c^3}\right)^2}-1\right]\right\} \qquad (1A)$$

The total density of *solar* radiation D arriving at the top layer of the lunar atmosphere is given by:

$$D = \sigma T^4 = 6.32\times10^7 W/m^2$$

Since the temperature of the surface of the Sun is $T = 5.778\times10^3 K$ and $\sigma = 5.67\times10^{-8} W.m^{-2}.K^{-4}$. The density of the top layer is $\rho \cong 1.3\times10^{-18} kg.m^{-3}$ then Equation (1A) gives:

$$\chi = -1.1$$

The *negative* sign of χ shows that $\chi\vec{g}_S$, has *opposite* direction to $\vec{g}_S$. As previously showed (see Figure 2), after the second gravitational shielding $(Sh2)$ the gravity acceleration $\chi\vec{g}_S$ becomes $\chi^2\vec{g}_S$. This means that $\chi^2\vec{g}_S$ has the *same direction* of $\vec{g}_S$. In addition, right after $(Sh2)$ the lunar gravity becomes $\chi\vec{g}_{moon}$. Therefore, the *total gravity acceleration in the Earth* will be given by,

$$\vec{g}' = \vec{g}_\oplus - \chi^2\vec{g}_S - \chi\vec{g}_{moon} \qquad (2A)$$

Since $g_S \cong 5.9\times10^{-3} m/s^2$ and $g_{moon} \cong 3.3\times10^{-5} m/s^2$ Equation (2A), gives

$$\begin{aligned} g' &= g_\oplus - (-1.1)^2 g_S - (-1.1) g_{Moon} = \\ &\cong g_\oplus - 7.1\times10^{-3} m.s^{-2} = \\ &= \left(1-7.3\times10^{-4}\right) g_\oplus \qquad (3A) \end{aligned}$$

This decrease in g increases the period $T = 2\pi\sqrt{l/g}$ of a *paraconical pendulum* (Allais effect) in about,

$$T' = T\sqrt{\frac{g_\oplus}{\left(1-7.3\times10^{-4}\right)g_\oplus}} = 1.00037\ T$$

This corresponds to 0.037% increase in the period, and is roughly the value (0.0372%) obtained by Saxl and Allen during the total solar eclipse in March 1970 [50]. As we have seen, the density of the interplanetary medium near the Moon is *highly variable* and can reach values up to $\sim 100\ protons/cm^3\ \left(1.7\times10^{-19}\,kg/m^3\right)$.

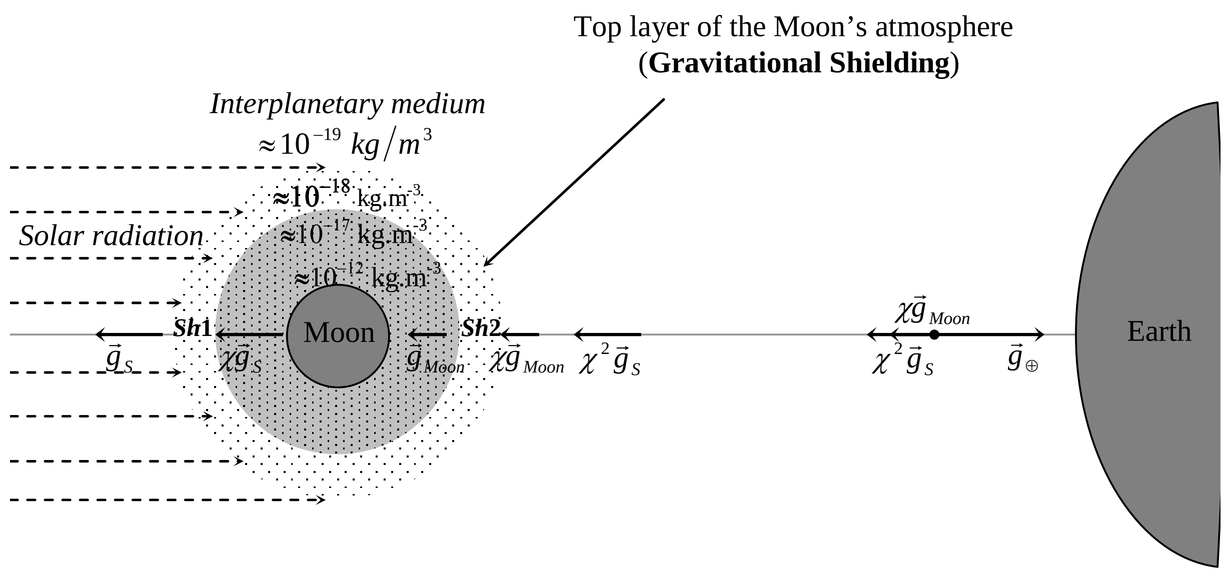


Figure 1A: *Schematic Diagram of the Gravitational Shielding around the Moon.*
(The top layer of the Moon's atmosphere with density of the order of 10^{-18} kg.m^{-3}, produces a gravitational shielding when subjected to the radiation from the Sun. Thus, the solar gravity $\vec{g}_S$ becomes $\chi\ \vec{g}_S$ after the first shielding $Sh1$ and $\chi^2\vec{g}_S$ after the second shielding $Sh2$. The Moon's gravity becomes $\chi\ \vec{g}_{Moon}$ after $Sh2$. Therefore the *total gravity acceleration in the Earth* will be given by $\vec{g}' = \vec{g}_\oplus - \chi^2\vec{g}_S - \chi\vec{g}_{moon}$).

When the density of the interplanetary medium increases, the top layer of the lunar atmosphere can also increase its density, by absorbing particles from the interplanetary medium due to the lunar gravitational attraction. In the case of a density increase of roughly 30% $\left(1.7\times10^{-18}\,kg/m^3\right)$, the value for χ becomes,

$$\chi = -0.4$$

Consequently, we get:

$$\begin{aligned} g' &= g_\oplus - (-0.4)^2 g_S - (-0.4) g_{Moon} = \\ &\cong g_\oplus - 9.6\times10^{-4}\,m.s^{-2} = \\ &= \left(1-9.7\times10^{-5}\right)g_\oplus \qquad (4A) \end{aligned}$$

This decrease in g increases the pendulum's period by about,

$$T' = T\sqrt{\frac{g_\oplus}{\left(1-9.4\times10^{-5}\right)g_\oplus}} = 1.000048\quad T$$

This corresponds to 0.0048% increase in the pendulum's period. Jun's abstract [55] tells us of a relative change less than 0.005% in the pendulum's period associated with the 1990 solar eclipse.

For example, if the density of the top layer of the lunar atmosphere increase up to $2.0917\times10^{-18}\,kg/m^3$, the value for χ becomes:

$$\chi = -1.5\times10^{-3}$$

Thus, we obtain:

$$g' = g_{\oplus} - \left(-1.5\times10^{-3}\right)^2 g_S - \left(-1.5\times10^{-3}\right) g_{Moon} =$$
$$\cong g_{\oplus} - 6.3\times10^{-8} m.s^{-2} =$$
$$= \left(1 - 6.4\times10^{-9}\right) g_{\oplus} \qquad (5A)$$

So, the total gravity acceleration in the Earth will *decrease* during the solar eclipses by about

$$6.4\times10^{-9}\, g_{\oplus}$$

The size of the effect, as measured with a *gravimeter*, during the 1997 eclipse, was roughly $(5-7)\times10^{-9}\, g_{\oplus}$ [56, 57].

The decrease will be even smaller for $\rho \gtrsim 2.0917\times10^{-18}\mathrm{kg.m^{-3}}$. The lower limit now is set by Lageos satellites, which suffer an anomalous acceleration of only about $3\times10^{-13}\, g_{\oplus}$, during "seasons" where the satellite experiences eclipses of the Sun by the Earth [58].

APPENDIX B: FURTHER EXPLAINATION OF QUANTIZED GRAVITY EQUATION

In this appendix we will show why, in the *quantized gravity equation* (Equation 34), $n = 0$ is *excluded* from the sequence of possible values of n. Obviously, the exclusion of $n = 0$, means that the gravity can have only discrete values *different of zero*.

Equation (33) shows that the gravitational mass is *quantized* and given by:

$$M_g = n^2 \left| m_{g(\min)} \right|$$

Since Equation (43) leads to:

$$m_{g(\min)} = m_{i0(\min)}$$

where,

$$m_{i0(\min)} = \pm h\sqrt{3/8}\Big/ c d_{\max} = \pm 3.9\times10^{-73} kg$$

is the *elementary quantum of inertial mass*. Then the equation for M_g becomes:

$$M_g = n^2 \left| m_{g(\min)} \right| = n^2 \left| m_{i0(\min)} \right|$$

On the other hand, Equation (44) shows that:

$$M_i = n_i^2 \left| m_{i0(\min)} \right|$$

Thus, we can write that:

$$\frac{M_g}{M_i} = \left(\frac{n}{n_i}\right)^2 \quad or \quad M_g = \eta^2 M_i \qquad (1B)$$

where $\eta = n/n_i$ is a *quantum number* different of n.

By multiplying both members of Equation (1B) by $\sqrt{1 - V^2/c^2}$ we get:

$$m_g = \eta^2 m_i \qquad (2B)$$

By substituting (2B) into Equation (21) we get:

$$E_n = \frac{n^2 h^2}{8 m_g L^2} = \frac{n^2 h^2}{8 \eta^2 m_i L^2} \qquad (3B)$$

From this equation we can easily conclude that η cannot be *zero* $\left(E_n \to \infty \;\; or \;\; E_n \to \frac{0}{0}\right)$. On the other hand, the Equation (2B) shows that the exclusion of $\eta = 0$ means the exclusion of $m_g = 0$ as a possible value for the gravitational mass. Obviously, this also means the exclusion of $M_g = 0$ (Relativistic mass). Equation (33) tells us that $M_g = n^2 \left| m_{g(\min)} \right|$, thus we can conclude that the exclusion of $M_g = 0$ implies in the exclusion of $n = 0$ since $m_{g(\min)} = m_{i0(\min)} = finite \;\; value$ (*elementary* quantum of mass). Therefore Equation (3B) is only valid for values of n and η different of zero. Finally, from the *quantized gravity equation* (Equation 34),

$$g = -\frac{GM_g}{r^2} = n^2 \left(-\frac{G\left| m_{g(\min)} \right|}{\left(r_{\max}/n\right)^2} \right) =$$
$$= n^4 g_{\min}$$

we conclude that the exclusion of $n = 0$ means that *the gravity* can have only discrete values *different of zero*.

REFERENCES

1. Isham, C.J. 1975. *Quantum Gravity: An Oxford Symposium*. OUP: Oxford, UK.

2. Isham, C.J. 1997. "Structural Problems Facing Quantum Gravity Theory". In: M. Francaviglia, G. Longhi, L. Lusanna, and E. Sorace (eds.) *Proceedings of the 14th International Conference on General Relativity and Gravitation*. 167-209. World Scientific: Singapore.

3. Landau, L. and Lifchitz, E. 1969. *Theorie du Champ*. MIR: Moscow, USSR. Portuguese version (1974) Ed. Hemus: S. Paulo, Brazil. 35.

4. Landau, L. and Lifchitz, E. [3]. 15.

5. Landau, L. and Lifchitz, E. [3]. 35.

6. Landau, L. and Lifchitz, E. [3]. 36.

7. Beiser, A. 1967. *Concepts of Modern Physics*. McGraw-Hill, Portuguese version. 1969. Ed. Poligno: S. Paulo, Brazil. 151.

8. Schiff, L.I. 1981. *Quantum Mechanics*. McGraw-Hill: New York, NY. 54.

9. Braginsky, V.B., C.M. Caves, and K.S. Thorne. 1977. *Phys. Rev.* D15:2047.

10. Landau, L.D. and E.M. Lifshitz. 1951. *The Classical Theory of Fields, 1st Edition.* Addison-Wesley: New York, NY. 328.

11. Forward, R.L. 1961. *Proc. IRE*. 49:892.

12. Tajmar, M. et al. 2006. *Experimental Detection of the Gravitomagnetic London Moment*. gr-qc/0603033.

13. Tate, J., Cabrera, B., Felch, S.B., and Anderson, J.T. 1989. "Precise Determination of the Cooper-Pair Mass". *Phys. Rev. Lett.* 62(8):845-848.

14. Tate, J., Cabrera, B., Felch, S.B., and Anderson, J.T. 1990. "Determination of the Cooper-Pair Mass in Niobium". *Phys. Rev. B*. 42(13):7885-7893.

15. Tajmar, M. and De Matos, C.J. 2006. *Local Photon and Graviton Mass and its Consequences*. gr-qc/0603032.

16. Landau, L. and Lifchitz, E. [3]. 426-427.

17. Hildebrandt, A.F. 1964. "Magnetic Field of a Rotating Superconductor". *Phys. Rev. Lett.* 12(8): 190-191.

18. Verheijen, A.A., van Ruitenbeek, J.M., de Bruyn Ouboter, R., and de Jongh, L.J. 1990. "The London Moment for High Temperature Superconductors". *Physica B*. 165-166, 1181-1182.

19. Verheijen, A.A., van Ruitenbeek, J.M., de Bruyn Ouboter, R., and de Jongh, L.J. 1990. "Measurement of the London Moment for Two High Temperature Superconductors". *Nature*. 345, 418-419.

20. Sanzari, M.A., Cui, H.L., and Karwacki, F. 1996. London Moment for Heavy-Fermion Superconductors". *Appl. Phys. Lett.* 68(26): 3802-3804.

21. Liu, M. 1998. "Rotating Superconductors and the Frame-Independent London Equation". *Phys. Rev. Lett.* 81(15):3223-3226.

22. Jiang, Y. and Liu, M. 2001. "Rotating Superconductors and the London Moment: Thermodynamics Versus Microscopics". *Phys. Rev. B*. 63:184506.

23. Capellmann, H. 2002. "Rotating Superconductors: Ginzburg-Landau Equations". *Europ. Phys. J.* B 25:25-30.

24. Berger, J. 2004. "Nonlinearity of the Field Induced by a Rotating Superconducting Shell". *Phys. Rev.* B 70:212502.

25. Podkletnov, E. and Nieminen, R. 1992. *Physica C*. 203:441.

26. Ference, Jr., M., Lemon, H.B., and Stephenson, R.J. 1956. *Analytical Experimental Physics*. Chicago Press: Chicago, IL. Portuguese version (*Eletromagnetismo*), Ed. Blücher: S. Paulo, Brazil. 73.

27. Modanese, G. 1996. gr-qc/9612022, p.4

28. Quevedo, C.P. 1978. *Eletromagnetismo*. McGraw-Hill: San Paulo, Brazil. 102.

29. Landau, L. and Lifchitz, E. [3]. 324.

30. Landau, L. and Lifchitz, E. [3]. 357.

31. Landau, L. and Lifchitz, E. [3]. 358-359.

32. Landau, L. and Lifchitz, E. [3]. 64.

33. Luminet, J. et al. 2003. *Nature*. 425,593–595.

34. Ellis, G.F.R. 2003. *Nature*. 425, 566–567.

35. Hogan, C., et al. 1999. “Surveying Space-Time with Supernovae”. *Scientific America*. 280. January. 46-51.

36. Landau, L. and Lifchitz, E. [3]. 440-441.

37. Landau, L. and Lifchitz, E. [3]. 333.

38. Landau, L. and Lifchitz, E. [3]. 331.

39. Landau, L. and Lifchitz, E. [3]. 327.

40. Hawking, S.W. 1971. MNRAS. 152:75.

41. Carr, B.J. 1976. *Astrophys. J.* 206:10.

42. Bohm, D. 1951. *Quantum Theory* Prentice-Hall: New York, N.Y. 415.

43. D’Espagnat, B. 1979. “The Question of Quantum Reality”. *Scientific American*. 241:128.

44. Teller, P. 1986. “Relational Holism and Quantum Mechanics”. *British Journal for the Philosophy of Science.* 37:71-81.

45. Anderson, J.D., et al. 2002. “Study of the Anomalous Acceleration of Pioneer 10 and 11”. *Phys. Rev.* D65:082004.

46. Landau, L. and Lifchitz, E. [3]. 116 & 363.

47. Landau, L. and Lifchitz, E. [3]. 363-364.

48. De Aquino, F. 2007. “Gravity Control by means of Electromagnetic Field through Gas at Ultra-Low Pressure”. Physics/0701091.

49. Allais, M. 1959. *Acad. Sci. URSS.* 244:2469; 245:1875; 245:2001; 245:2170; 245:2467.

50. Saxl, E. and Allen, M. 1971. *Phys. Rev*. D3, 823-825.

51. http://science.nasa.gov/newhome/headlines/ast06aug991.htm

52. Martin, M. and Turyshev, S.G. 2004. *Int. J. Mod. Phys*. D13:899-906.

53. Stern, S.A. 1999. *Rev. Geophys*. 37:453.

54. Öpik, E.J. 1957. “The Density of the Lunar Atmosphere”. *Irish Astr. J.* 4(6):186.

55. Jun, L., et al. 1991. *Phys. Rev.* D 44:2611-2613.

56. http://www.spacedaily.com/news/china-01zi.htmll

57. Wang. Q., et al. 2000. *Phys. Rev.* D 62:041101(R).

58. Rubincam, D.P. 1990. *J. Geophys. Res., [Atmos.].* 95:4881.

ABOUT THE AUTHOR

Fran De Aquino formerly worked as Senior-Researcher at the Instituto Nacional de Pesquisas Espaciais - INPE (National Institute for Space Research - INPE , Brazil). He was also Director of the INPE's Space Observatory in São Luís, since the installation of this observatory until 1997.

Currently, he is a Professor at the Universidade Estadual do Maranhão- UEMA (State University of Maranhao, Brazil). He also works as Consultant in science and technology for several international companies. Fran De Aquino's publications list includes several scientific articles and books.

SUGGESTED CITATION

De Aquino, F. 2010. “Mathematical Foundations of the Relativistic Theory of Quantum Gravity”. *Pacific Journal of Science and Technology*. 11(1):173-232.

Pacific Journal of Science and Technology