Statistical Modeling of Mother-to-Child and Heterosexual Modes of Transmission of HIV/AIDS Epidemic.

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ABSTRACT

The spread of the HIV/AIDS epidemic is a major world-wide concern. The epidemic has various mode of transmission. This paper addresses the scope of the epidemic generally, and two of the major modes of transmission (mother–to–child and heterosexual intercourse) were discussed extensively with respect to population distributions (0 – 5) years and 15 years and above. The model was developed by solving the Chapman–Kolmogrov differential equation for Birth–Illness–Death process using open population growth model and a comparative analysis was carried out to investigate the rate of spread of HIV epidemic among the two major modes of transmission discussed with the aid of some mathematical assumptions. The results are presented graphically in this paper.

(Keywords: open population growth mode, Kolmogrov differential equation, mother-to-child transmission, heterosexual transmission, birth-illness-death process)

INTRODUCTION

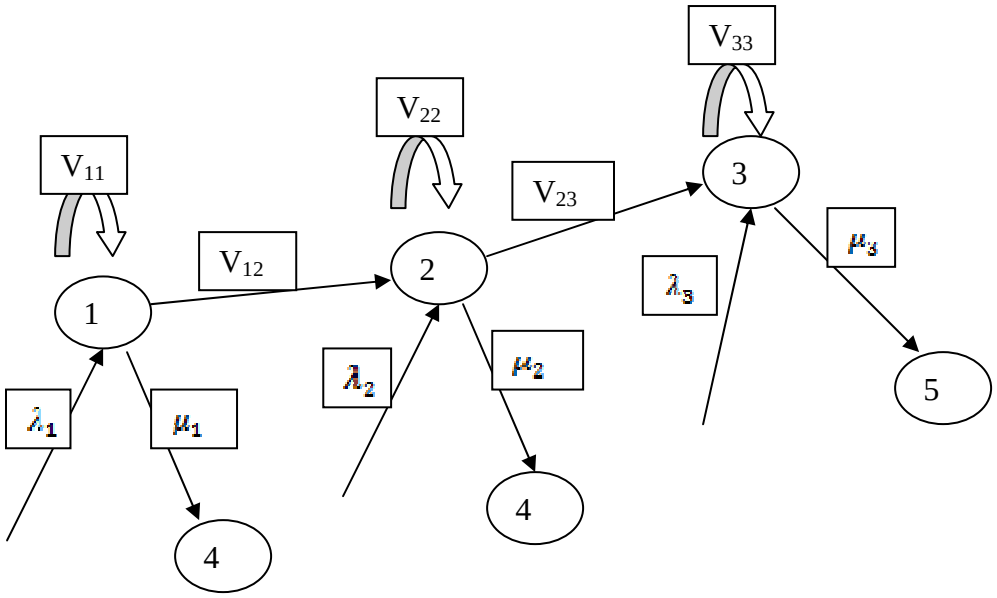
Human Immune–deficiency Virus and Acquired Immune Deficiency Syndrome (HIV/AIDS) which was rarely heard of in the 70s is now a worldwide problem. Efforts to solve this problem has not been made much positive progress. Hence, it becomes a meaningfully examination to find out which among the modes of transmission have the highest rates so that policies will be formulated for the government to control the epidemic. There are various modes of transmission of the epidemic, but two of the major modes of transmission are mother–to–child transmission (MTCT) and heterosexual transmission (HT) (Bashiru, 2005). The term mother–to–child transmission and heterosexual transmission of HIV, is described as Vertical and Horizontal transmission (Michael, 2003, Mugisha, 2003, and Gumel, 2003).

Mugisha (2003), indicates that there is a reduction in the rate of spread of the epidemic but it has not been significant in Africa, Asia, and Latin America. Perhaps this may be due to less access to and systematic distribution of therapeutic drugs.

GOVERNING EQUATION

According to the exposition provided in Ishan V. (1988), the general course of HIV infection and AIDS disease in a susceptible individual is as follows: there is latent period from infection until the individual becomes infectious. This infected can pass on the disease to other susceptible individuals. The period from infection until overt symptoms appear is the incubation period. For many diseases, the individual can be isolated and therefore will be unable to pass on the infection once symptoms appear so that infectivity ceases at this point. For the AIDS diseases, the incubation period after infection with human immunodeficiency virus (HIV) is known to be extremely long. For an individual infected with HIV, antibodies are normally detectable in the blood a few weeks after infection. Such an individual is said to be sero–positive. The individual will eventually die from the HIV infection since no definite cures have so far been discovered for the AIDS disease. This means that we do not need to include a class of “recovered – immune” individuals in the model.

For any population, the pattern of transmission of the HIV infection and AIDS can be described in the transition diagram below.



***Key**: 1: Susceptible 2: Infected 3: AIDS 4: Natural Death 5: AIDS induced Death.*

Fig.1: Open Population Growth Model With Positive Net Migration Of New Susceptible, Infective, and Sero–Positive.

Let S_1 ---S_S denote the health states and let the population sizes of the states be represented at the initial time {0} by a vector:

$$i = \begin{pmatrix} i_1 \\ . \\ . \\ . \\ j_s \end{pmatrix} \qquad \textbf{(1)}$$

and at time (1) by random vector:

$$x_i = \begin{pmatrix} x_{it} \\ . \\ . \\ . \\ x_{st} \end{pmatrix} \qquad \textbf{(2)}$$

So that x_{it} for example, is the population size in state S_i at time t. The value of x_i is represented by a constant vector:

$$j = \begin{pmatrix} j_1 \\ . \\ . \\ . \\ j_s \end{pmatrix} \qquad \textbf{(3)}$$

Where the element j_α are non – negative integers so that we have this conditional probability:

$$p_{ij}(o,t) = p_r\{x_t = j\} \qquad \textbf{(4)}$$

given that $x_0 = i$

for each $\tau\, 0 \le \tau \angle t,$ a change in the population size of each state $S\alpha$ during the time interval $(\tau, \tau + \Delta)$ is assumed to take place according to the following instantaneous probabilities.

$\lambda^*{}_\alpha(\tau)\Delta + 0(\Delta) = p_r$ (the size state S_α will increase by one during the interval $(\tau_1\, \tau + \Delta)$

$\nu^{*}_{\alpha\beta}(\tau)\Delta + 0(\Delta)$ = p_r {one individual will move from state
S_α to S_β during the interval $(\tau_1\ \tau + \Delta)\ \beta \neq \alpha$.

$\mu^{*}_{\alpha}(\tau)\Delta + \mathbf{0}(\Delta) =$ p_r {the size of state S_α will decrease by one during the interval $(\tau_1\ \tau + \Delta)$ through emigration or death.

We let,

$$\nu^{*}_{\alpha\alpha} = -\left[\sum_{\beta \neq \alpha} \nu^{*}_{\alpha\beta}(\tau) + \mu^{*}_{\alpha}(\tau)\right] \qquad \textbf{(5)}$$

and introduce a column vector

$$\delta_\alpha = \begin{pmatrix} \delta_{1\alpha} \\ \vdots \\ \delta_{s\alpha} \end{pmatrix} \qquad \textbf{(6)}$$

The components of δ_α in (1.6) are kronecker deltas, so that $\delta_{\alpha\alpha}$ =1 and $\delta_{\alpha\beta}$ = 0 for $\alpha \neq \beta$ it is easy to see that the probabilities in (1.4) satisfy the following system of Chapman- kolmogrov differential equations such that

$$\frac{d}{dt}\mathbf{P_{ij}}(o,t) = -\mathbf{P_{ij}}(0,t)\sum_{\alpha-1}^{s}(\lambda^{*}_{\alpha}(t) - \mathbf{V}^{*}_{\alpha\alpha}(t)) +$$
$$\sum_{\alpha-1}^{s}\mathbf{P_{ij-\delta\alpha}}(o,t)\lambda^{*}_{\alpha}(t) +$$
$$\sum_{\alpha=1}^{s}\sum_{\beta=1}^{s}\mathbf{P_{ij+\delta\alpha-\delta\beta}}(o,t)\mathbf{V}^{*}_{\alpha\beta}(t) + \sum_{\beta=1}^{s}\mathbf{P_{ij+\delta\alpha}}(o,t)\mu^{*}_{\alpha}(t) \qquad \textbf{(7)}$$

At t=0, the initial conditions are:

$$\mathbf{P_{ii}}(0,0) = 1$$
$$\mathbf{P_{ij}}(0,0) = 0$$
for $\boldsymbol{j \neq i}$

The system of differential Equations in (7) describes the growth of a population in general and various model can be derived from it by making appropriate assumptions regarding:

$\lambda^{*}_{\alpha}(t)$, $\mathbf{V}^{*}_{\alpha\beta}(t)$, and $\mu^{*}_{\alpha}(t)$.

In this model, it is assumed that individuals infected will eventually die from the disease or the disease related cause. We shall assume that the recruitment or immigration intensities, the intensities of transition from the susceptible state to the infective state to sero–positive full blown AIDS state as well as the intensities of death from each state are constant throughout.

An increase in population in each state is due either to immigration or internal migration when $\lambda^{*}_{\alpha}(t)$ is a function of the population size of J_α of state S_α at time t, due to the fact that the model is a Birth –illness – Death process.

Then we assume that:

$$\lambda^{*}_{\alpha}(t) = j_\alpha\lambda_\alpha,\ \ \nu^{*}_{\alpha\beta} = j_\alpha\nu_{\alpha\beta},\ \ \mu^{*}_{\alpha} = j_\alpha\mu_\alpha \qquad \textbf{(8)}$$

where α = 1, 2, ……, 5

where λ_α, $\nu_{\alpha\beta}$ and μ_α are independent of t and the process is time homogeneous. We again define:

$$\nu_{\alpha\alpha} = -\left[\sum_{\substack{\beta=1 \\ \beta\neq\alpha}}^{s}\nu_{\alpha\beta} + \mu_\alpha\right] \qquad \textbf{(9)}$$

$\alpha \neq \beta$, α, β = 1,2, 3, …,5

so that:

1+ $\nu_{\alpha\alpha}\Delta t$ + $O(\Delta t)$ = Pr(the size of state α remains unchanged during the interval (t, t + Δt)

j_α (t)	is the population size of t,
λ_α(t)	is the function of t, it is also known as immigration rate (into stat S_α)
$\nu_{\alpha\beta}$ (t)	is the interval immigration rate (from S_α to S_β)
μ_α (t)	is the emigration rate (from S_α)

The assumptions (8) was used to obtained Equation 10 below:

$$\frac{d}{dt}P_{ij}(o,t) = -P_{ij}(0,t)\sum_{\alpha=1}^{s} j_\alpha(\lambda_\alpha - \nu_{\alpha\alpha}) + \sum_{\alpha=1}^{s} P_{ij-\delta\alpha}(o,t)(j_\alpha - 1)\lambda_\alpha +$$

$$\sum_{\alpha=1}^{s}\sum_{\beta=1}^{s} P_{ij+\delta\alpha-\delta\beta}(o,t)\nu_{\alpha\beta} + \sum_{\alpha=1}^{s} P_{ij+\delta\alpha}(o,t) j_\alpha \mu_\alpha(t) \qquad \textbf{(10)}$$

The above equation can be solve by the introduction of generating function. Multiply equation above by $u_\alpha^{j\alpha}$ and sum over j_α we obtained (11).

$$\sum_{j\alpha}^{s} \frac{d}{dt}P_{ij}(o,t)\, u_\alpha^{j\alpha} = -\sum_{\alpha=1}^{s}(\lambda_\alpha - \nu_{\alpha\alpha})\left[\sum_{j_\alpha=1}^{s} j_\alpha P_{ij}(0,t) u_\alpha^{j\alpha}\right]$$

$$+ \lambda_\alpha \sum_{\alpha=1}^{s}\left[\sum_{j_\alpha=1}^{s}(j_{\alpha-1})P_{ij-\delta\alpha}(0,t) u_\alpha^{j\alpha}\right]$$

$$+ \nu_{\alpha\beta}\sum_{j_\alpha}\left[\sum_{\alpha=1}^{s}\sum_{\beta=1}^{s} j_\alpha P_{ij+\delta\alpha-\delta\beta}(0,t) u_\alpha^{j\alpha}\right]$$

$$+ \mu_\alpha \sum_{J\alpha}\left[\sum_{\alpha=1}^{s} j_\alpha P_{ij+\delta\alpha}(o,t) u_\alpha^{J\alpha}\right] \qquad \textbf{(11)}$$

Setting, $G_{X(t)}(u,t) = \sum_{j1}.....\sum_{js} u_1^{j_1}.....j_\alpha^{j\alpha} P_{ij}(o,t)$ **(12)**

Differentiate (12) with respect to t and u_α respectively and substitute in (11) we have:

$$\frac{\partial Gx_t(u,t)}{\partial t} - P'_{ij}(o,t) = -\sum_{\alpha=1}^{s} u_\alpha \frac{\partial}{\partial u_\alpha} Gx_t(u,t)(\lambda_\alpha - \nu_{\alpha\alpha})$$

$$+ \sum_{\alpha=1}^{s} u_\alpha^2 \frac{\partial}{\partial u_\alpha} Gx_t(u,t)\lambda_\alpha + \nu_{\alpha\beta}\sum_{\alpha=1}^{s}\sum_{\beta=1}^{s} u_\beta \frac{\partial}{\partial u_\alpha} Gx_t(u,t)\lambda_\alpha$$

$$+ \mu_\alpha \sum_{\alpha=1}^{s} \frac{\partial}{\partial \boldsymbol{u}_\alpha} \boldsymbol{Gx_t(u,t)} \qquad \textbf{(13)}$$

Re-arranging we have, $\frac{\partial Gx_t(u,t)}{\partial t} = -\sum_{\alpha=1}^{S} u_\alpha \lambda_\alpha \frac{\partial}{\partial u_\alpha} Gx_t(u,t) + \sum_{\alpha=1}^{S} \nu_{\alpha\alpha} u_\alpha \frac{\partial}{\partial u_\alpha} Gx_t(u,t)$

$$\sum_{\alpha=1}^{S} u_\alpha^2 \frac{\partial}{\partial u_\alpha} Gx_t(u,t)\lambda_\alpha + \sum_{\alpha=1}^{S}\sum_{\beta=1}^{S} \nu_{\alpha\beta} u_\beta \frac{\partial}{\partial u_\alpha} Gx_t(u,t)$$

$$+ \mu_\alpha\left[\sum_{\alpha=1}^{S} \frac{\partial}{\partial u_\alpha} G\chi_t(u,t)\right] + \boldsymbol{P'_{ij}(o,t)} \qquad \textbf{(14)}$$

Considering the initial conditions in (8):

$\boldsymbol{P_{ij}(0,0) = 0}$ and $\boldsymbol{P_{ii}(0,0) = 1}$ at t = 0. **(15)**

Then we have, $$\frac{\partial Gx_t(u,t)}{\partial t} = \sum_{\alpha=1}^{S} u_{\alpha}^2 \lambda_\alpha \frac{\partial}{\partial u_\alpha} Gx_t(u,t) - \sum_{\alpha=1}^{S} u_\alpha \lambda_\alpha \frac{\partial}{\partial u_\alpha} Gx_t(u,t)$$
$$+ \sum_{\alpha=1}^{S} \nu_{\alpha\alpha} u_\alpha \frac{\partial}{\partial u_\alpha} Gx_t(u,t) + \sum_{\alpha=1}^{S} \sum_{\beta=1}^{S} \nu_{\alpha\beta} u_\beta \frac{\partial}{\partial u_\alpha} Gx_t(u,t)$$
$$+ \sum_{\beta=1}^{S} \mu_\alpha \frac{\partial}{\partial \boldsymbol{u}_\alpha} \boldsymbol{Gx_t(u,t)} \quad \textbf{(16)}$$

Collecting the like terms and rearranging algebraically, we have:

$$\frac{\partial Gx_t(u,t)}{\partial t} = -\sum_{\alpha=1}^{S} u_\alpha \lambda_\alpha (1-u_\alpha) \frac{\partial}{\partial u_\alpha} Gx_t(u,t) + \sum_{\alpha=1}^{S} u_\alpha \nu_{\alpha\alpha} \frac{\partial}{\partial u_\alpha} Gx_t(u,t)$$
$$+ \sum_{\alpha=1}^{S} \sum_{\beta=1}^{S} \nu_{\alpha\beta} u_\beta \frac{\partial}{\partial u_\alpha} Gx_t(u,t) + \sum_{\alpha=1}^{S} \mu_\alpha \frac{\partial}{\partial \boldsymbol{u}_\alpha} \boldsymbol{Gx_t(u,t)} \quad \textbf{(17)}$$

Let, $Z_\alpha = 1 - u_\alpha$ **(18)**

Recall from Equation (9) that $$\nu_{\alpha\alpha} = -\left[\sum_{\substack{\beta=1 \\ \alpha=1}} \nu_{\alpha\beta} + \mu_\alpha\right] \quad \textbf{(19)}$$

Differentiating Equation (18) with respect to Z_α and u_α, then equate them we have $\partial Z_\alpha = \partial u_\alpha$ **(20)**

and let:

$$\mu_\alpha = -\sum_{\alpha=1}^{S} \nu_{\alpha\beta} \quad \textbf{(21)}$$

Substitute (18), (19), (20), and (21) into (16) we obtained:

$$\frac{\partial Gx_t(Z,t)}{\partial t} = -\sum_{\alpha=1}^{S} \lambda_\alpha (1-Z_\alpha) Z_\alpha \frac{\partial}{\partial z_\alpha} Gx_t(Z,t)$$
$$- \sum_{\alpha=1}^{S} \sum_{\beta=1}^{S} Z_\beta \nu_{\alpha\beta} \frac{\partial}{\partial z_\alpha} Gx_t(Z,t) \quad \textbf{(22)}$$

Re-arranging we have:

$$\frac{\partial Gx_t(Z,t)}{\partial t} = -\sum_{\alpha=1}^{S} \left[\lambda_\alpha (1-Z_\alpha) Z_\alpha + \sum_{\beta=1}^{S} Z_\beta \nu_{\alpha\beta}\right] \frac{\partial}{\partial z_\alpha} Gx_t(z,t) \quad \textbf{(23)}$$

Finding the general solution of the homogeneous partial differential equation above, Flower (2005),

"every function $u(x, y, z) = 0$ which satisfies $\frac{dx}{P}, \frac{dy}{Q}, \frac{dz}{R}$ is a solution of the partial differential equation $P\frac{\partial z}{\partial x} + Q\frac{\partial z}{\partial y} = R$ ".

The auxiliary equation of (23) is,

$$\frac{dt}{1} = -\frac{dz_\alpha}{\left[\lambda_\alpha(1-Z_\alpha)Z_\alpha + \sum_{\beta=1}^{S} Z_\beta \nu_{\alpha\beta}\right]} = \frac{\partial}{0} Gx_t(z,t) \quad \textbf{(24)}$$

Considering the equation below,

$$\frac{dt}{1} = -\frac{dz_\alpha}{\left[\lambda_\alpha(1-Z_\alpha)Z_\alpha + \sum_{\beta=1}^{S} Z_\beta \nu_{\alpha\beta}\right]} \quad \textbf{(25)}$$

Simplifying, we obtained:

$$\frac{dZ_\alpha}{\partial t} = -\lambda_\alpha(1-Z_\alpha)Z_\alpha - \sum_{\beta=1}^{S} Z_\beta \nu_{\alpha\beta} \quad \textbf{(26)}$$

Where $\alpha = 1, 2, 3\ldots$ and $\beta = 1, 2, 3, \ldots$ Also for $\frac{dt}{1} = \frac{\partial}{0} Gx_t(z,t)$ **(27)**

In this research work the differential equation arrived at equation (26) will be applied in solving the following special HIV/AIDS mode of transmission.

(1) Mother-to-child transmission
(2) Heterosexual transmission

ASSUMPTIONS AND NOTATIONS FOR THE MODELS

Let:

S(t): denote the number of persons in group S at time (t)

I(t): denote the number of persons in group I at time (t).

A(t): denote the number of persons in group AIDS at time (t)

It is reasonable to assume that the beginning of the epidemic at t = 0, that S(0) is large, that I(0) is fairly small, and that A(0) = 0.

At time t, let N(t) represent the size of the population therefore the total population consist of N(t) = S(t)t + I(t) + A(t).

(A) If the population size is n (n > 0) at time t, during the small interval of time (t, t + Δt). The probability that "Birth" [an increase to the population] will occur is $\lambda_n(t)\Delta t + 0(\Delta t)$. the probability of "no Birth" occurring in that small interval is $1 - \lambda_n(t)\Delta t + 0(\Delta t)$ and the probability of more than "One Birth" occurring is $0(\Delta t)$. "Birth" occurring in (t, t + Δt) are independent of time since in the last occurrence.

(B) The probability that "death will occur in a small interval of time (t, t + Δt) is $\mu_n(t)\ \Delta t + 0(\Delta t)$. the probability of "no death" occurring is $1 - \mu_n(t)\ \Delta t + 0(\Delta t)$. and the

probability that more than one "one death" occurs is $0(\Delta t)$. "Death" occurring in $(t, t + \Delta t)$ are independent of time since the last occurrence.

(C) n = 0 is an absorbing state of the process.

(D) For the same population size, the "birth" and "death" occur independently of each other.

(i) Let the birth rate for sexually mature persons be λ per person per time. Thus the probability that a birth will occur in the heterosexual population during the time interval $(t, t + \Delta t)$ is $\lambda\Delta t + 0(\Delta t)$.

(ii) Let the death (death unrelated to HIV/AIDS or emigration rate (migrate out of the population because of fear of HIV/AIDS be μ_k per person per time, where k = 1, 2, 3 (the different age groups have different per capita mortality rates).

(iii) Let the immigration rate for the sexually mature person be m per time, this is independent of the population.

(iv) Let the sexual contact rate between a mutually sexual S person and an I person be W where $W \geq 0$.
A given a sexual contact between an S person and and I person during $(t, t + \Delta t)$, we let δ be the probability that this I person will transmit the AIDS virus to the S person.

This event converts the S person to an I person $w\delta = \sqrt{w_m \delta_m w_f \delta_f}$ where $w_m\delta_m$ is the probability that an I mate transmit the AIDS virus to an S female and $w_f\delta_f$ is the probability that an I female transmit the AIDS virus to an S male.

- Let the rate of which an infected mother does not transmitting the HIV virus to the newborn be β.
- Let P_1 (Survival rate) be the proportion of children, born free of the HIV, that survive the age group 0 – 5 years.
- P2 be the proportion of children attain age 5 years, survive through the age group (5 – 15)years.

(v) Let the transition rate from infection to AIDS case be Y.

Mother-To-Child Transmission

The study population consist of the pre – school age group (0 – 5) years, these are the children born of infected Mothers in group (15 and above) years .

The population is divide into those children born free of HIV virus but can contact the virus from their mothers through breast milk (susceptibles), those who contact the virus from their infected mother (infectives) and the former infective who develop full blown symptoms AIDS. Bashiru (2005). The mode of transmission in this group is mother–to–child transmission (MTCT). The virus may be transmitted to the new born babies during the pregnancy (*in utero*), labor delivery (through contamination by blood or other fluids during birth) or after the child's birth during breast feeding (Abdulkarim and Ndakwo, 2007; Mugisha, 2003; and Waema and Olowofeso, 2005).

Mugisha (2003), found that among infected infants who are not breastfed, about two – thirds of the case of MTCT occur around the time of delivery and the rest during the pregnancy (mostly during the last two months). In populations where breast feeding is the norm, it account for more than one third of all transmission. Thus the rate of transmission from uninfected to infected Mother will transmit the virus to either the fetus or newborn *in utero* during or shortly after delivery which is 21 – 43%

$S_1(t)$ SUSCEPTIBLE

The probability that there are n individuals in the infective population during the time interval $\{t, t + \Delta t\}$ is equal to probability

1 That there are " n" individuals by time t and nothing happen during the time interval $\{t, t + \Delta t\}$.

2 That there are "n – 1" individuals by time t and 1 is added by HIV

transmission, immigration or mother-to-child transmission during the time interval $\{t, t+\Delta t\}$.

3 That there "n + 1" individuals by time t and 1 dies or coverts to AIDS during the time interval $\{t, t+\Delta t\}$.

The change in population size during the time interval (t , t + Δt) is governed by the following conditional probabilities.

$Pr(s_1 (t + t\Delta) = n+1/s_1(t) = ny = ns_3\lambda\Delta t + nI_3\beta\lambda\Delta t + 0(\Delta t)$

$Pr(s_1 (t + t\Delta) \geq n+2/s_1(t) = ny = 0(\Delta t)$

$Pr\{s_1 (t + t\Delta) = n-1/s_1(t) = n\}= nP_1s_1 + ns_1\mu_1\Delta t + 0(\Delta t)$

$Pr(s_1 (t + t\Delta) \leq n-2/s_1(t) = n\ y = 0(\Delta t)$

$Pr\{s_1 (t + t\Delta) = n/s_1(t) = n\}= 1 - ns_3\lambda\Delta t - nI_3\beta\lambda\Delta t - nP_1s_1 - ns_1\mu_1\Delta t - 0(\Delta t)$

Now , $\lambda_n(t) = nS_3\lambda + nI_3\beta\lambda$ **(28)**

$I_1(t)$ INFECTION

The " birth" are the new infections from infected mother-to-child and "death" are the children who develop AIDS symptoms or die.

The change in population size during the time interval (t, t + Δt) is governed by the following conditional probabilities.

$Pr\{X(t + \Delta t) = n + 1/X(t) = n\} = nI_3(1 - \beta)\ m\lambda\Delta t + 0(\Delta t)$

$Pr\{X(t + \Delta t) \geq n + 2/X(t) = n\} = 0(\Delta t)$

$Pr\{X(t + \Delta t) = n - 1/X(t) = n\} = nI_1\ \mu_1\Delta t + nI_1\ \gamma\Delta t + 0(\Delta t)$

$Pr\{X(t + \Delta t) \leq n - 2/X(t) = n\} = 0(\Delta t)$

$Pr\{X(t + \Delta t) = n/X(t) = n\} = 1- nI_3(1 - \beta)\ m\lambda\Delta t - nI_1\gamma\Delta t - nI_1\mu_1\Delta t - 0(\Delta t)$

Now, $\lambda_n(t) = nI_3(1-\beta)m\lambda$ **(29)**

$A_1(t)$ (SYMPTOMATIC) AIDS CASE

The change in population size during the time interval (t, t+Δt) is governed by the following conditional probabilities.

$Pr\{X(t + \Delta t) = n + 1/X(t) = n\} = nI_1\gamma\Delta t + 0(\Delta t)$

$Pr\{X(t + \Delta t) \geq n + 2/X(t) = n\} = 0(\Delta t)$

$Pr\{X(t + \Delta t) = n - 1/X(t) = n\} = nA_1\ \mu_1\Delta t + 0(\Delta t)$

$Pr\{X(t + \Delta t) \leq n - 2/X(t) = n\} = 0(\Delta t)$

$Pr\{X(t + \Delta t) = n/X(t) = n\} = 1- nI_3\gamma\Delta t - nA_1\mu_1\Delta t - 0(\Delta t)$

Now, $\lambda_n(t) = nI_1\gamma$ **(30)**

HETEROSEXUAL TRANSMISSION

Here we consider a population consisting of adults (15 and more years). Bashiru (2005) stated that it is this group that is sexually mature and active, and therefore, capable of reproduction. It is this group that is responsible for the horizontal transmission of the HIV virus through heterosexual activities and for vertical transmission by infected mothers to their children.

Since age group 2 consists of HIV free population and it is the survivors of this subgroup over the development period (5,15) that generate age group 3, here we include the survivors in the susceptible model.

$S_3(t)$ SUSCEPTIBLE

The probability that there are n individuals in the infective population during the time interval $\{t, t+\Delta t\}$ is equal to probability:

1 That there are "n" individuals by time t and nothing happen during the time interval $\{t, t+\Delta t\}$.

2 That there are "n – 1" individuals by time t and 1 is added by HIV transmission, immigration or mother-to-child transmission during the time interval $\{t, t+\Delta t\}$.

3. Those there "n + 1" individuals by time t and 1 dies or coverts to AIDS during the time interval $\{t, t + \Delta t\}$.

The change in population size during the time interval (t, t + Δt) is governed by the following condition probabilities.

Pr{S_3(t + Δt) = n + 1/ S_3(t) = n} = $P_2S_2^*\Delta t$+ 0(Δt)

Pr{S_3 (t + Δt) $\geq$ n + 2/ S_3 (t) = n} = 0(Δt)

Pr{ S_3 (t + Δt) = n - 1/S_3(t) = n} = nS_3(wδ + μ_3)Δt + 0(Δt)

Pr{S_3 (t + Δt) $\leq$ n - 2/S_3(t) = n} = 0(Δt)

Pr{ S_3(t + Δt) = n/ S_3(t) = n} = 1- n$P_2S_2^*\Delta t$ - nS_3(wδ + μ_3)Δt - 0(Δt)

Now, $\lambda_n(t) = P_2S_2^*\Delta t$ **(31)**

I_3(t) (ASYMPTOMATIC)INFECTED

The change in population size during the time interval (t, t + Δt) is governed by the following conditional probabilities:

Pr{I_3(t + Δt) = n + 1/ I_3(t) = n} = nI_3w$\delta\Delta$t + 0(Δt)

Pr{I_3 (t + Δt) $\geq$ n + 2/ I_3 (t) = n} = 0(Δt)

Pr{I_3 (t + Δt) = n - 1/I_3(t) = n} = n$I_3\mu_3\Delta$t + n$I_3\gamma\Delta$t + 0(Δt)

Pr{I_3 (t + Δt) $\leq$ n - 2/I_3(t) = n} = 0(Δt)

Pr{ I_3(t + Δt) = n/ I_3(t) = n} = 1- nI_3w$\delta\Delta$t - n$I_3\gamma\Delta$t – n$I_3\mu_3\Delta$t - 0(Δt)

Now, $\lambda_n(t) = nI_3w\delta$ **(32)**

A_3(t) (SYMPTOMATIC) AIDS CASE

The change in population size during the time interval (t, t + Δt) is governed by the following conditional probabilities:

Pr {A_3(t + Δt) = n + 1/ A_3(t) = n} = n$I_3\gamma$ + 0(Δt)

Pr {A_3 (t + Δt) $\geq$ n + 2/ A_3 (t) = n} = 0(Δt)

Pr {A_3 (t + Δt) = n - 1/A_3(t) = n} = n$A_3\mu_3\Delta$t + 0(Δt)

Pr {A_3 (t + Δt) $\leq$ n - 2/A_3(t) = n} = 0(Δt)

Pr {A_3(t + Δt) = n/ A_3(t) = n} = 1- n$I_3\gamma\Delta$t - n$A_3\mu_3\Delta$t - 0(Δt)

Now, $\lambda_n(t) = nI_3\gamma$ **(33)**

Recall Equation (26) for the application.

For mother–to–child, substitute Equation (28), (29), and (30) into (26) and also (31), (32), and (34) into (26) for heterosexual transmission. Thus, we arrived at 3 systems differential equations below for mother-to-child transmission.

$$\frac{dZ_1}{dt} = (nS_3\lambda + nI_3\beta\lambda)Z_1^2 - \left[(nS_3\lambda + nI_3\beta\lambda) + \nu_{11}\right]Z_1 - \nu_{12}Z_2$$

$$\frac{dZ_2}{dt} = nI_3(1-\beta)m\lambda Z_2^2 - \left[nI_3(1-\beta)m\lambda - \nu_{22}\right]Z_2 - \nu_{23}Z_3$$

$$\frac{dZ_3}{dt} = nI_1\gamma z_3^2 - \left[nI_1\gamma - \nu_{33}\right]Z_3$$

also for heterosexual transmission:

$$\frac{dZ_1}{dt} = P_2S_2^*z_1^2 - \left[P_2S_2^* - \nu_{11}\right]Z_1 - \nu_{12}Z_2$$

$$\frac{dZ_2}{dt} = nI_3 w\delta Z_2^2 - \left[nI_3 w\delta - \nu_{22}\right]Z_2 - \nu_{23}Z_3$$

$$\frac{dZ_3}{dt} = nI_3 \gamma Z_3^2 - \left[nI_3 \gamma - \nu_{33}\right]Z_3$$

Each of the 3 systems differential equations above was solved by Iterative Numerical approach. Using classical Runge Kutta method with aid of Visual Basic programming software develops. Boundary conditions and initial values for the models are assumed. In this research work the following assumptions are made:

$\lambda = 0.05$, $\boldsymbol{N(0) = 120{,}000{,}000}$, $\boldsymbol{S_\alpha(t) = \frac{S_\alpha(t)}{N(0)}}$, $\boldsymbol{I_\alpha(t) = \frac{I_\alpha(t)}{N(0)}}$ and $\boldsymbol{A_\alpha(t) = \frac{A_\alpha(t)}{N(0)}}$

where $\alpha = 1,2,3,- - - - - - -$

$\nu_{11} = \nu_{12} = \nu_{13} = \nu_{22} = \nu_{23} = \nu_{33} = 0.4$

RESULTS AND DISCUSSION

The result are presented graphically below,

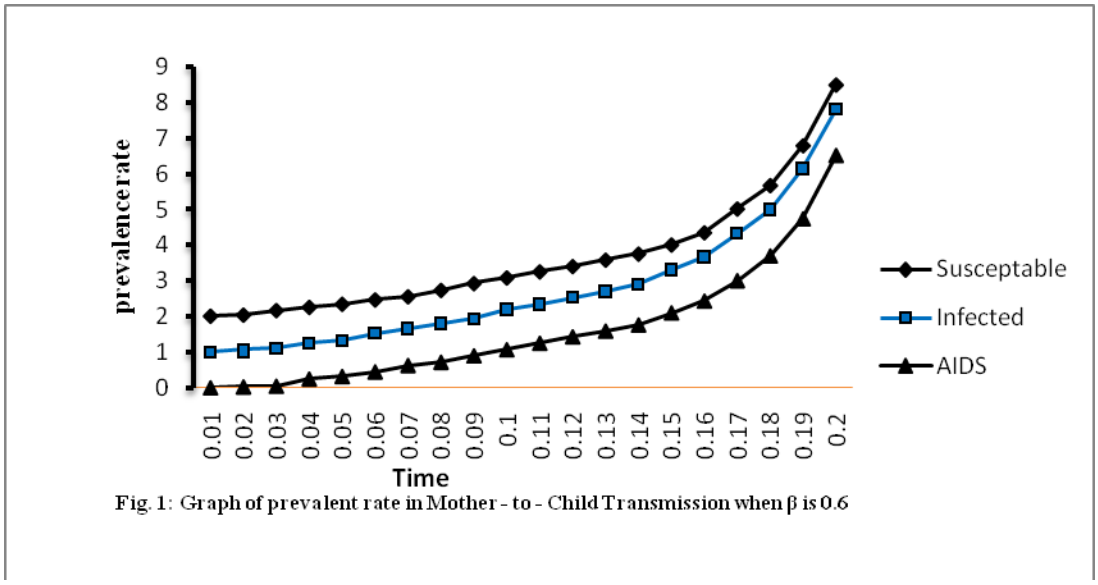


Fig. 1: Graph of prevalent rate in Mother - to - Child Transmission when β is 0.6

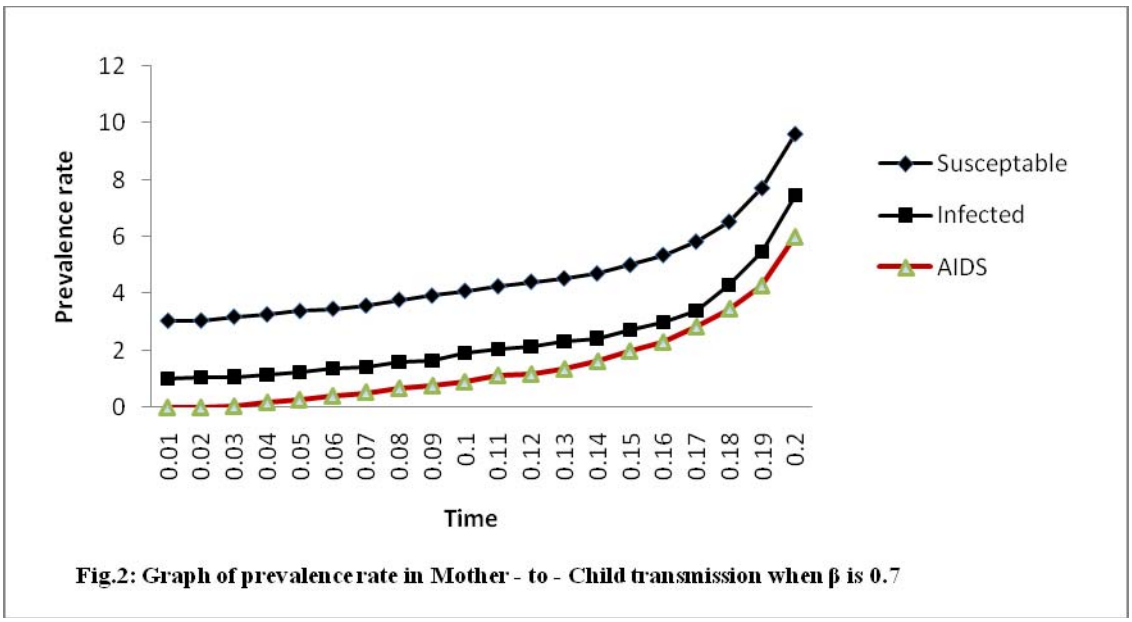


Fig.2: Graph of prevalence rate in Mother - to - Child transmission when β is 0.7

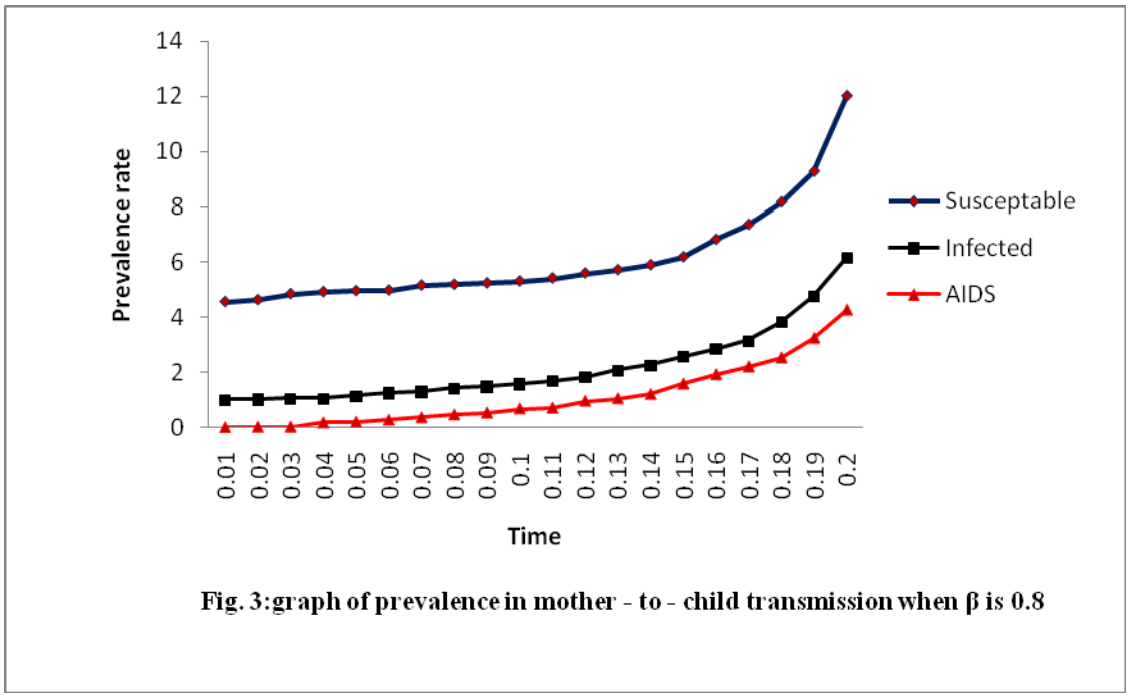


Fig. 3:graph of prevalence in mother - to - child transmission when β is 0.8

From Figure 1 and Figure 2 above, it was observed that when β was increase to 0.7 in Figure 2, there is little changes in the infected and AIDS prevalence rate but we recorded a positive progress when compared with that of Figure 1 because at Time (t) is 0.2 Susceptible is 8.51, infected is 7.81 and AIDS is 6.50 while in Figure 2, Susceptible is 9.60, infected is 7.46 and AIDS is 6.00. Its also observed that as β increase to 0.8 in Figure 3. The Susceptible rate move up drastically while in the infected and AIDS cases they are moving down at Time (t) is 0.01. Susceptible rate move up 4.56 while infected and AIDS cases are 1.0006 and 0.0025, respectively. Mugisha (2003) states that vertical transmission of HIV/AIDS has been the principal cause of 80 –

90 % of HIV- infected children, however there is no doubt that the treatment of pregnant women with antiretroviral recombinants has reduced the transmission to some low level as it indicate in the Figure 3 this is likely occurs in the developed countries.

In Figures 4 and 5 there are no traces of any changes, the susceptible, infected and AIDS prevalence in both figures are very close with constant δ (probability that infected person will transmit the AIDS virus to the Susceptible person) of 0.1. From the result of the analysis, one interesting observation is that as the β increases, it reduces (1 – β).

Since β is the rate at which the baby born will not be infected by the infected mother, the susceptible rate is increasing while the infected rate and AIDS cases are decreasing. The decrease in MTCT was as a result of an appropriate opportunity for educating women about HIV/AIDS when they go for an antenatal visit during pregnancy.

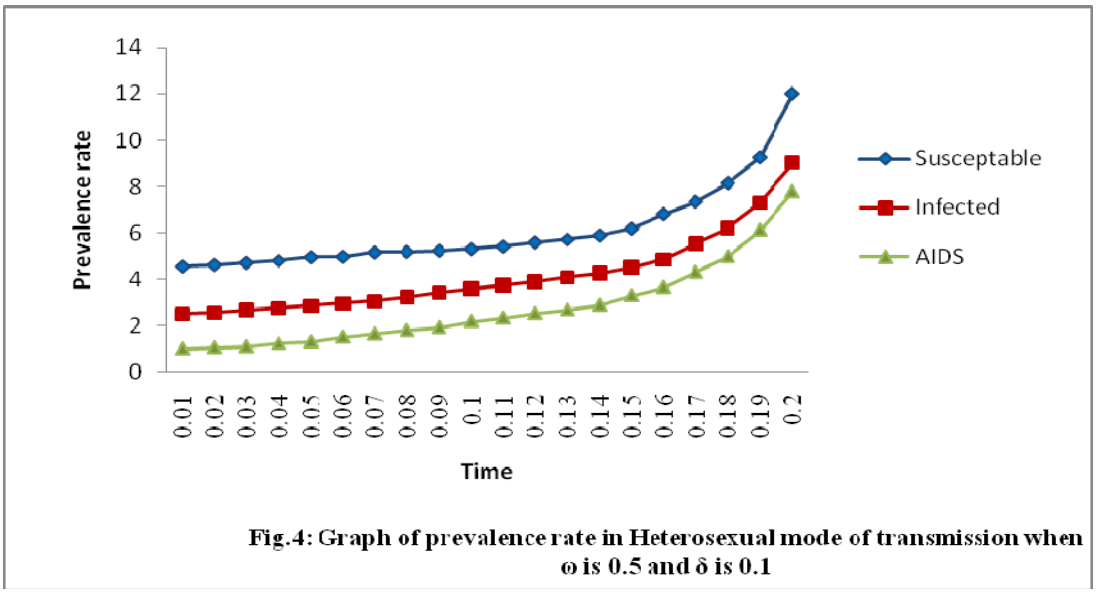


Fig.4: Graph of prevalence rate in Heterosexual mode of transmission when ω is 0.5 and δ is 0.1

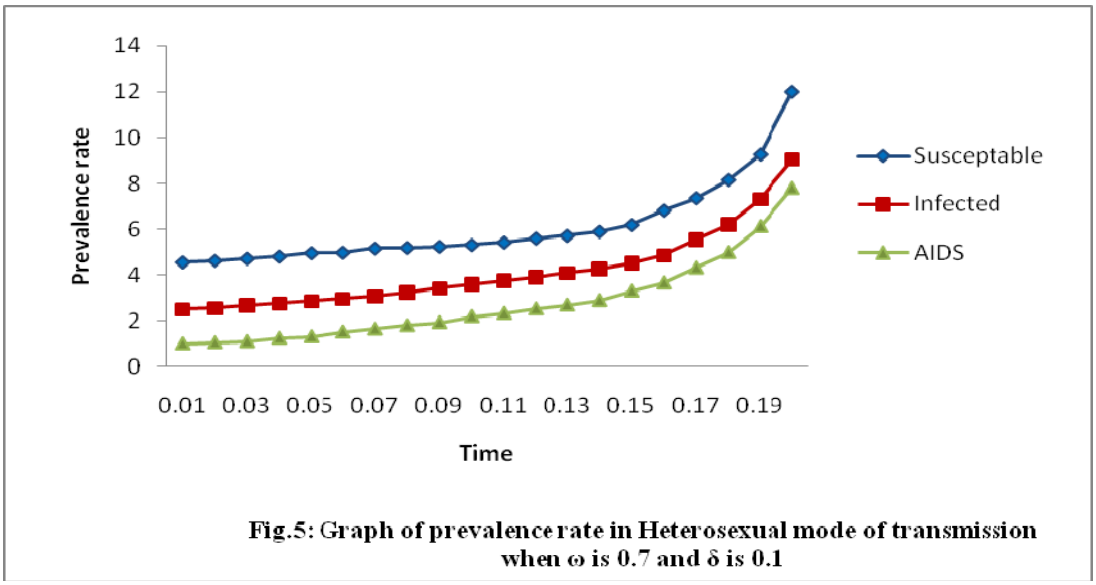


Fig.5: Graph of prevalence rate in Heterosexual mode of transmission when ω is 0.7 and δ is 0.1

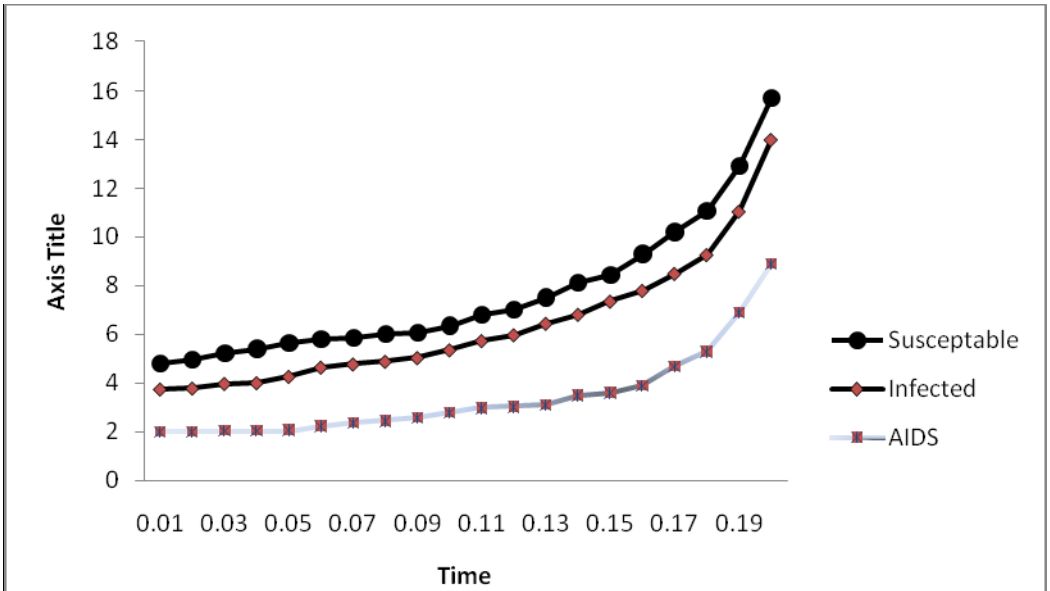


Fig. 6: Graph of prevalence rate in Heterosexual mode of transmission when ω is 0.7 and δ is 0.4

Furthermore, some religions organization have instituted a practice of mandatory blood testing among potential marital partners, although not in conformity with the national policy on HIV/AIDS. Where either partner is found positive, the couple is usually counseled on the implications for such marriage. HIV positive women and those who have developed AIDS are less likely to become pregnant.

However in heterosexual transmission, there are no significant observable changes in Figure 4 and Figure 5, when the rate of sexual contact is varied from 0.5 to 0.7 with constant infected rate of 0.1. the possible reason for this observation is probably due to use of condom, the spread of the virus depend more on the number of sexual contacts with different sexual partners per units time. The use of condoms reduces δ by a factor of 0.90, if condoms are used properly and subsequently increases in the sexual rate (ω) because individual would think they are protected through the use of condoms. In fig 6, when the infected rate (δ) was increased from 0.1 to 0.4 the prevalent rate for Susceptible is moving up slowly while the prevalence rate of infected and AIDS are more pronounced. One possible reason for this trend is the uncared attitude and the possibility of failure in the use of condoms.

CONCLUSION

Mode of transmission of the HIV/AIDS are commonly seen in the mother–to–child transmission and heterosexual transmission, we have discussed the two modes of transmission with respect to number of people in the various stages of HIV/AIDS disease progression.

The result of the prevalence rate in mother–to–child mode of transmission and the heterosexual mode of transmission for the various values of β (the rate at which an infected mother does not transmitting HIV Virus to the new born baby), ω (sexual contact rate between a mutually sexual susceptible person and infected person) and δ (probability that infected person will transmit the AIDS virus to the susceptible person) are presented.

From the result of the analysis, it was discovered that as time increases for (MTCT) the susceptible rate increases while the infected rate and AIDS cases decreases. However, in the heterosexual transmission, there are no significant observable changes in the rate of spread of the epidemic when the rate of sexual contact varied from 0.5 to 0.1 with constant infected rate of 0.1. this shows that heterosexual mode of transmission gives higher rate of the epidemic.

RECOMMENDATION

The results from the model confirm that the epidemic is spreading mostly by heterosexual and interventions should be extended to other mode of transmission.

Over the years, various statistical models have been used to make estimates of the burden of HIV/AIDS. Understanding the magnitude and future trends of the HIV/AIDS epidemics is a necessary pre – requisite for proper planning and mobilization of resources for its prevention and control. Since 1998, United Nation Program on HIV/AIDS (UNAIDS) has been conducting global, regional and national HIV/AIDS estimates. This exercise allows national AIDS program to estimate the burden of disease in the country, evaluate the HIV/AIDS situation and plan better the need for services to mitigate the impact of the epidemic.

- Since the explosive epidemic is from heterosexual and young people (age 15 – 24 years) constitute a large proportion of those infected and the high HIV/AIDS prevalence amongst women 15 – 24 years suggest a high incidence of infection still occurring. There is need to focus intervention programs towards this sub – population.

- There is need to focus activities on girls within schools to empower them to protect themselves from HIV infection. HIV/AIDS education in schools should be adopted as a strategy to address this issue.

- The current care and support activities especially the anti – retroviral program need to be scale–up to meet the increasing demand of large number of AIDS cases in the country.

- Sentinel sero–surveys should be done at every two years interval to enable the country asses the direction of the epidemic.

- Behavioral survey should be continued so as to generate data that can explain the various factors driving the epidemic in the various health zones in the country and inform intervention nation-wide.

- Voluntary counseling and testing services need to be intensified in antenatal clinics, this will act as an entry into the preventive mother–to-child transmission (PMTCT) program.

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