Empirical Formulations for Inter-Layer Precipitable Water Vapor in Nigeria.

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ABSTRACT

Analysis of Variance (ANOVA) technique used on daily upper air climatological data, spanning over a decade, obtained over three meteorological stations in Nigeria, suggests that regression relations of the form W = α(LPW) + β (α, β constants) exists between low-level, mid-level, and total column tropospheric precipitable water vapor. Tests carried out on the models using daily soundings made over the stations in 1990 for Lagos, 1983 for Minna and 1991 for Kano gave encouraging results as established by the use of Kolmogorov-Smirnov tests. Comparing the performances of the models in this present study with the earlier proposed models, which is generally of the form W = $\alpha(\text{LPW})^{\beta}$ (α, β constants), improved performances were noted at almost all the stations.

(Keywords: ANOVA, regression, climatological, tropospheric, precipitable)

INTRODUCTION

The hydrological cycle has an important role in global climate change (e.g. Allen and Ingram 2002; Kaufman *et al.* 2002). At current concentrations, water vapor is the most important green house gas in the atmosphere being the gas that absorbs most solar radiation (Kiehl and Trenberth, 1997). Water vapor can lead to global warming as it is both a symptom and a cause of the green house effect. It continually cycles through evaporation and condensation, transporting heat energy around the earth and between the surface and the atmosphere.

Water vapor in the atmosphere allows the short wavelength radiation of the sun to pass through the atmosphere, but traps the long wavelength radiation emitted by the Earth's surface. This trapped radiation causes air temperature to increase so that air can then sustain large amount of water vapor, thereby increasing the green house effect (IPCC 2001; Soden *et al.* 2002).

The dependence of microwave attenuation on water vapor in the atmosphere, which is significant to satellite communication, remote sensing and radio astronomy (Hogg and Guirand, 1979; Reber and Swope, 1972) calls for a good assessment of it in the path of signal propagation. Also, considering its importance to meteorological and hydrological studies, agronomy and ecology, detailed information about its distribution at the different atmospheric layers is very crucial. Various studies over the years have focused attention on the subject of precipitable water evaluation (e.g. Adedokun, 1986, 1983; Balogun and Adedokun, 1985; Adeyemi and Adeyemi, 2005).

This paper describes the analysis of data on column precipitable water with a view to examine the relationship between the low level, middle level upper level and the total column precipitable water; as well as to observe the relevance, or otherwise of the earlier models over the three radiosonde stations in tropical Nigeria. The stations are Lagos, a coastal station, Minna, an inland station and Kano, a sub-sahelian station.

DATA AND DATA ANALYSIS TECHNIQUES.

The daily upper air data obtained for the period 1977-1991 were used in the computations reported in this paper. These data were obtained from the archives of the Department of Meteorological Services, Federal Ministry of Aviation, Oshodi, Lagos, Nigeria. Since it has been the characteristics of data obtained in West Africa where many large gaps are observed due to dearth of data, we have therefore combined the data obtained to a 365-day period for the purpose of this study. This was done by finding the daily average of all the available data for each of the stations. This combination (i.e. averaging) is also of necessity in view of the stringency of the KS test being applied to the actual and model generated series where any attempt to simulate artificially a missing datum would lead to further departure from reality.

The data, as available and obtained for a fourteen year period, were those used in making the computations for the stations shown in Table 1. It should be noted that the selected stations are representative of the regions they belong in Nigeria. The radiosonde observations for 1200GMT were used for precipitable water vapor content (Wg/cm^2) computations for the three stations. These have been computed for surface-850mb, 850-700mb, 700-500mb, 500-400mb and 400-300mb, at each of the stations. Mean values of these quantities at the different levels were then calculated as shown below:

Precipitable Water Vapor Calculations: The precipitable water is the vertically integrated total mass of water vapor per unit area for a column of atmosphere. Therefore

$$W = \int_{z=0}^{z_{top}} \rho_v dz = \int_{z=0}^{z_{top}} q_v \rho dz = -\frac{1}{g}\int_{z=0}^{z_{top}} \bar{q}_v dp = -\frac{1}{g}\sum_{i=1}^{N} \bar{q}_v \Delta p_i \quad \textbf{(1)}$$

where q_v = specific humidity for the layer considered, W is in g/cm^2, ρ_v = water vapor density, and Δp is pressure gradient.

The specific humidity q in Equation 1 above is calculated from (Balogun and Adedokun, 1985)

$$q = 0.622\ (e-p)/p \quad \textbf{(2)}$$

where e(mb) = vapor pressure and p(mb) is total atmospheric pressure.

Equation (1) was integrated over the five intervals listed below to obtain W_a, W_b, W_c, W_d ,W_e. W_a, (i.e., precipitable water vapor content between surface and 850mb); W_b, (i.e., precipitable water vapor content between 850 and 700mb pressure level); W_c, (i.e., precipitable water vapor content between 700 and 500mb); W_d, (i.e., precipitable water vapor content between 500 and 400mb), and W_e, (implying precipitable water vapor content between 400 and 300mb pressure level).

The low-level tropospheric water vapor content is LPW = W_a.

The mid-level tropospheric water vapor content, MPW, was calculated from:

$$MPW = W_b + W_c \quad \textbf{(3)}$$

The upper-level tropospheric water vapor content, W_u, was estimated from:

$$UPW = W_d + W_e \quad \textbf{(4)}$$

The total column precipitable water vapor content W_T was calculated from:

$$TPW = -\frac{1}{g}\int_{1000}^{300} \bar{q} dp = LPW + MPW + UPW \quad \textbf{(5)}$$

TPW is the total moisture content in an atmospheric column of unit cross-sectional area which extends from the earth's surface to the top of the atmosphere (Balogun and Adedokun, 1985). The top of the atmosphere considered here, is the 300mb pressure level because it is assumed that atmospheric moisture content decreases rapidly to zero above this level.

RESULT AND DISCUSSION

Vertical Distribution of Precipitable Water: Figure 1 (a, b, and c) shows the distribution of monthly mean values of low-level precipitable water (LPW), middle level precipitable water (MPW), upper – level precipitable water (UPW) and total column precipitable water (TPW) for the period under investigation at the different stations.

The coastal station of Lagos produced a double peak structure for all precipitable water vapor parameters with the exception of the upper level precipitable water (UPW) that are uniformly low across seasons (see Figure 1a). This distribution gave rise to maxima in May and September with a dip in August, for LPW, MPW and TPW. The midland station of Minna and the sub-sahelian station of Kano gave rise to a single peak structure for all parameters except UPW, attaining maximum in August/September. The dip, observed in August and at Lagos, which is as a result of low amount of precipitable water vapor in the atmosphere can be associated with the well known 'little dry season' (LDS) period-a common occurrence in the coastal part of West Africa (Adeyemi, 2005, Balogun 1981). This occurs when the Intertropical discontinuity (ITD) line is at its most northerly position.

Table 1: Geographic and Climatic Characteristics of the Stations.

Station	Location/Vegetation	Latitude	Longitude	Elevation (m)	Climate
Lagos	Equatorial/coastal	$6^0 28^1$N	$3^0\ 28^1$ E	19	humid
Minna	Middle belt/savannah	$9^0\ 37^1$ N	$6^0 30^1$E	249	Partially humid
Kano	North/sub-sahelian	12^0 2N	$8^0 30^1$E	480	Semi-arid

The occurrence of the LDS has been likened to several factors such as coastal upwelling and the northern advance of the subtropical high pressure system of the southern Atlantic Ocean (Balogun and Adedokun, 1985). At the two northern stations of Minna and Kano, the low-level precipitable water (LPW) is seen to be lower than the middle level precipitable water (MPW). This is in contrast to what obtains in the coastal station of Lagos where LPW is higher than MPW.

The observation at Lagos where LPW is higher than MPW may be explained simply by the influx of water vapor into the lower level of the atmosphere through continuous evaporation from the sea surface. The observation at the two northern stations of Minna and Kano, where LPW is lower than MPW may be due to incessant surface heating experienced at the two stations causing water vapor to be transported to the higher layers of the atmosphere through buoyancy (Adeyemi 2004, Aro, 1975).

Model Development: In order to justify the development of linear regression model between the low-level precipitable water (LPW) and other column precipitable water described above, we have obtained for each station, correlations of (LPW, MPW) (LPW, UPW) and LPW, TPW), and have found that significant positive correlations exist for the linear relationship at all the stations most especially between LPW, MPW and TPW. Therefore, the linear regression model was then applied to investigate the fit of the relationships.

We, hence, obtained an analysis of variance (ANOVA) table (not shown) for each station. Based on the resulting F-ratios, the null hypothesis test was carried out. Table II shows the best fit parameters α and β, obtained for each station as well as the coefficient of determination (CV%) and the probability 'p' at which the null hypothesis was rejected for the regression relationship between LPW and MPW, LPW and UPW, LPW and TPW.

From Table 2, it can be observed that the slope of the regression relations (MPW, LPW) and (TPW, LPW) increases northward. The coefficient of determination is higher in the midland station of Minna than in the Coastal and subsahelian stations. Minna therefore, seems to enjoy higher inter-layer relationship than other stations. The null hypothesis is rejected at $p > 0.05$ for all cases. This then implies that for the relations (UPW, LPW) at Lagos and Kano stations, where $p > 0.05$, the relations are insignificant.

The value of the coefficient of determination (CV) for the linear relations in these stations also corroborates this fact. The value of CV for these stations implies that LPW can only account for 5%, and 2.5% of the variation of UPW at Lagos, and Kano respectively. This is normally to be expected, since water vapor present in that level of atmosphere is small compared to the one at the lower level.

The Reliability of the Model: To verify the reliability of the models obtained in Table 2 above, the developed models were used to evaluate the tropospheric precipitable water from soundings obtained in 1990 for Lagos, 1983 for Minna and 1991 for Kano. The results of these as compared with the actual values obtained from the surface – 300 mb integration, which are then averaged, are as shown in Table III. Fairly good agreements are observed between the actual and model calculated values of the water vapor parameters in Lagos, Minna and Kano. Plots of the actual versus calculated precipitable water for all the stations, obtained by using the various versions of the proposed models are presented in Figures 2(a, b and c) – 4(a, b. and c).

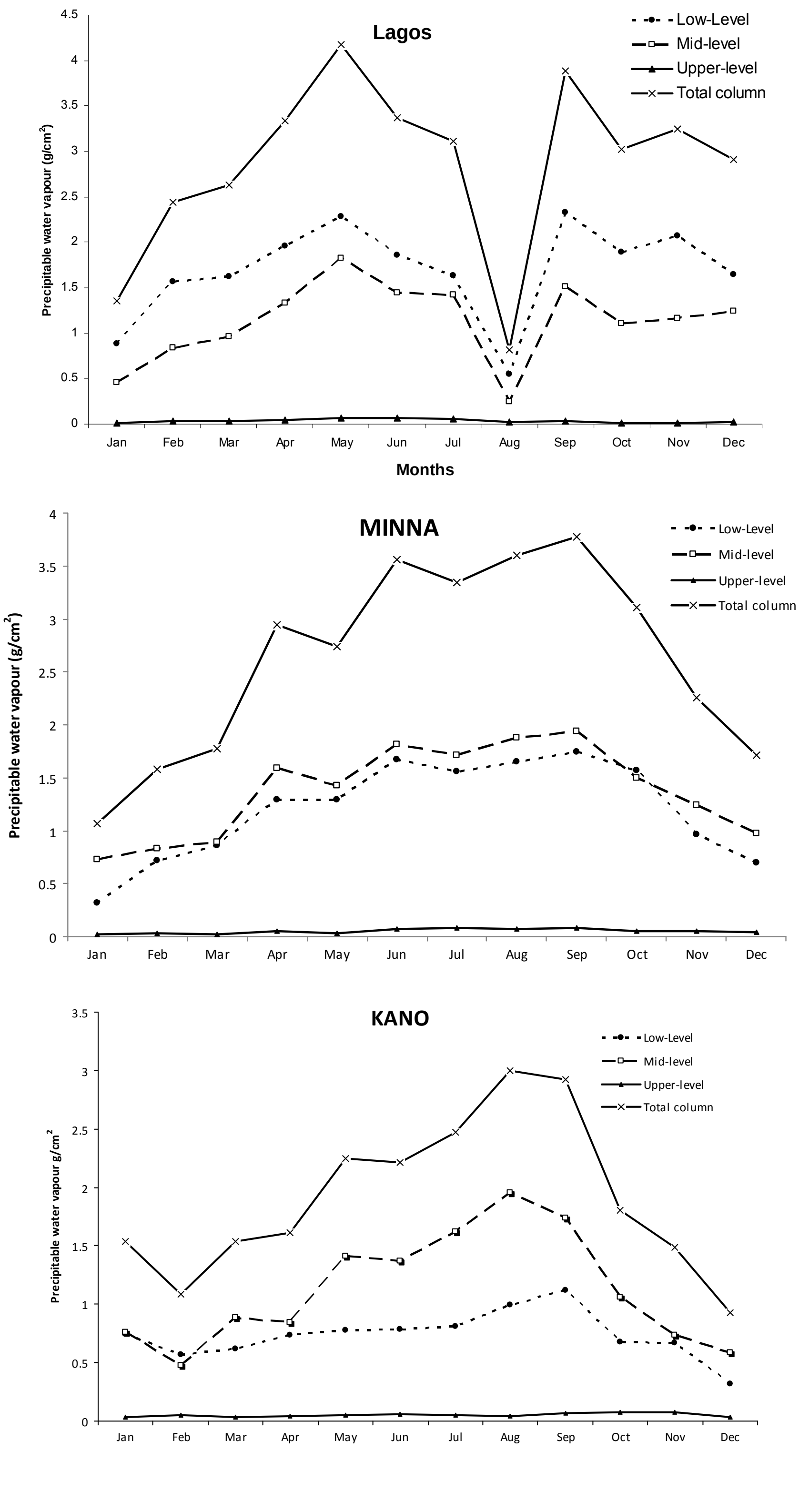

Figure 1: Precipitable Water Vapor Distribution at (a) Lagos, (b) Minna, and (c) Kano.

Table 2: Values of Best Fit Parameters α and β in the Regression Equation of Precipitable Water at Different Levels on Low Level Precipitable Water.

Station	N	MPW=α(LPW) + β				UPW=α(LPW) + β				TPW=α(LPW) + β			
		α	β	CV %	p-value	α	β	CV %	p-value	α	β	CV %	p-value
Lagos	365	0.701	-0.053	65.0	0.0000	0.010	0.018	5.00	0.3000	1.710	-0.035	91.0	0.0000
Minna	365	0.860	0.355	84.0	0.0000	0.025	0.021	27.0	0.0300	1.880	0.376	96.0	0.0000
Kano	365	1.880	-0.253	65.0	0.0000	0.020	0.040	2.50	0.3000	2.890	-0.220	81.0	0.0070

+CV stands for coefficient of variation (R^2)

As can be seen in the Figures, the linear regressions showing the relationship between the actual and observed values for all the stations have shown that UPW was poorly monitored at Lagos and Kano with coefficients of determination (CV) being 2% and 6%, respectively, whereas at Minna it was fairly well monitored with CV = 43%. MPW and TPW on the other hand, were well monitored at all the stations (with CV ranging between ~60% and ~90%). This CV value is seen to be higher at Minna and Kano than at Lagos, a coastal station. This may be attributed to the fact that coastal stations experience high local variability in their weather conditions.

Comparing Figures 2 (a and c) – 4 (a, and c) with each other, the linear regressions showing the relationship between the actual and observed values for (LPW, MPW) and (LPW, TPW) in Lagos gave negative slopes whereas at other stations, positive slopes are observed. The results obtained at all the stations regarding the performances of the linear regressions involving (LPW, MPW) and (LPW, TPW), produced some note worthy deviations that featured occasionally. Hence, we have therefore, subjected the actual and model results to the Kolmogorov – Smirnov test.

The Kolmogorov Smirnov (KS) test is used to determine whether or not any agreement exists between the actual distribution function F_N (x) and each model-generated series F_C(x) (Kendall and Stuart, 1979, and Adedokun, 1989). The maximum deviation between F_N (x) and F_C(x) is defined by:

$$D_N = \max\left|F_N(x) - F_c(x)\right| \quad (6)$$

where x is ordered such that:

$0 \le x_1 \le x_2 \le x_3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots \le x_N$ for N observations

If Ω is number of values ≤ x, then

$$F_N(x) = \frac{\Omega}{N} \quad (7)$$

According to Kendall and Stuart (1979), critical values of D_N for 5 percent significant level is:

$$D_{N(\alpha^1=0.05)} = \frac{1.3581}{\sqrt{N}} \quad (8)$$

Agreement exists between F_N(x) and F_C(x) if $D_N \le D_{N\ (\alpha\ =0.05)}$ for the significant level under consideration. On applying this test to the evaluated MPW and TPW values at the three stations, we obtained the results in Table 4. Going by the results shown in Table 4, the actual and model functions for MPW and TPW are related at all the stations.

Comparison with the Results of an Earlier Model: The results of research work on intra/ inter layer relationship of precipitable water vapor carried out about two decades ago over West Africa by Adedokun established that a good relationship exists between low-level precipitable water (LPW) mid level precipitable water (MPW) and total column precipitable water (TPW). The log-log relationship was used resulting in the power model of the form W = α $(LPW)^{\beta}$. These models developed by Adedokun for the northern and southern zones of West Africa and for mid and total column precipitable water have been applied to evaluate daily mean precipitable water for the period given above. Their results were then compared with those obtained from the present models. The results are as presented in Table 3.

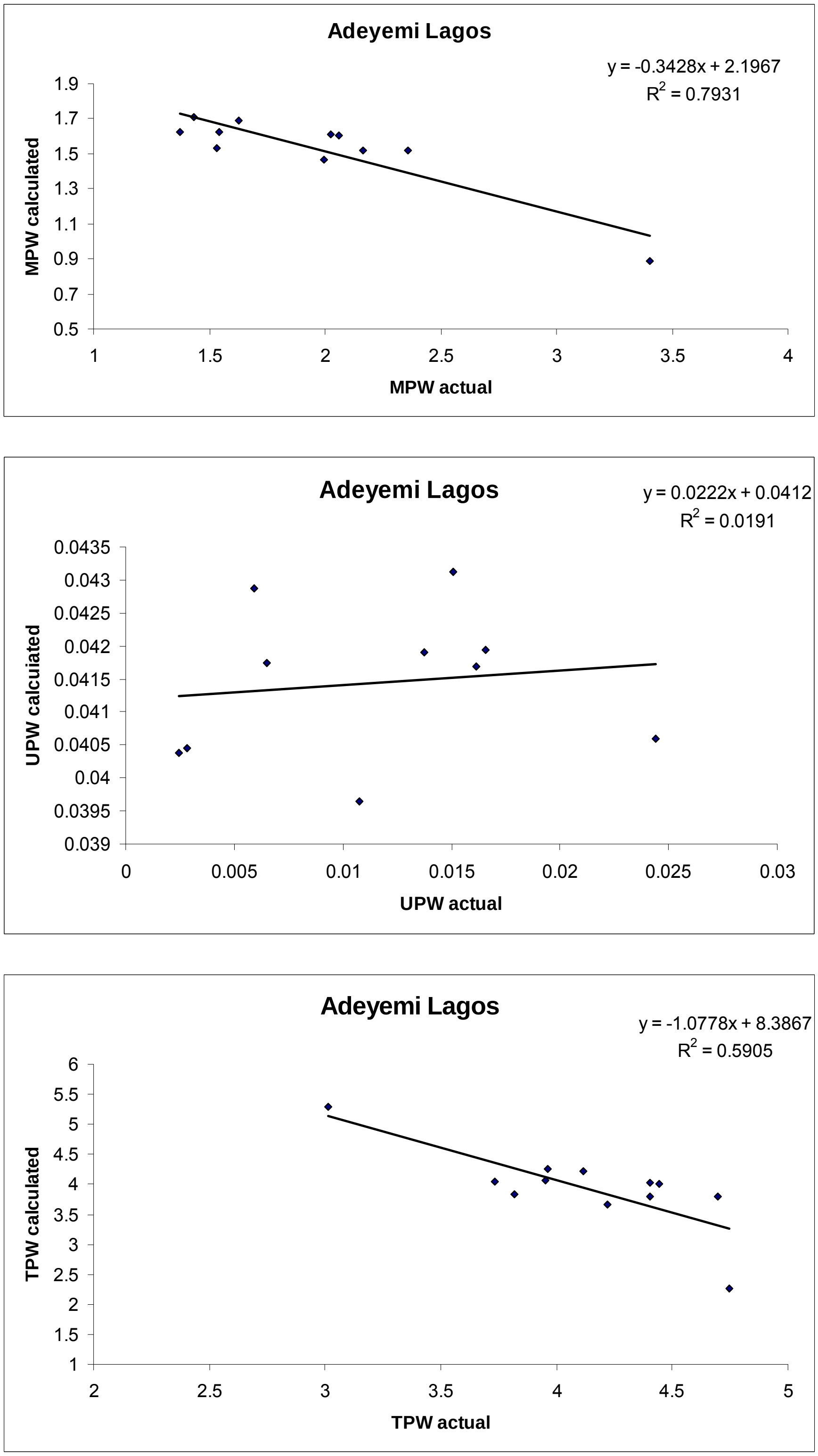

Figure 2: Plots of Calculated versus Actual Precipitable Water for a) Mid-level b) Upper-level and c) Total Column at Lagos.

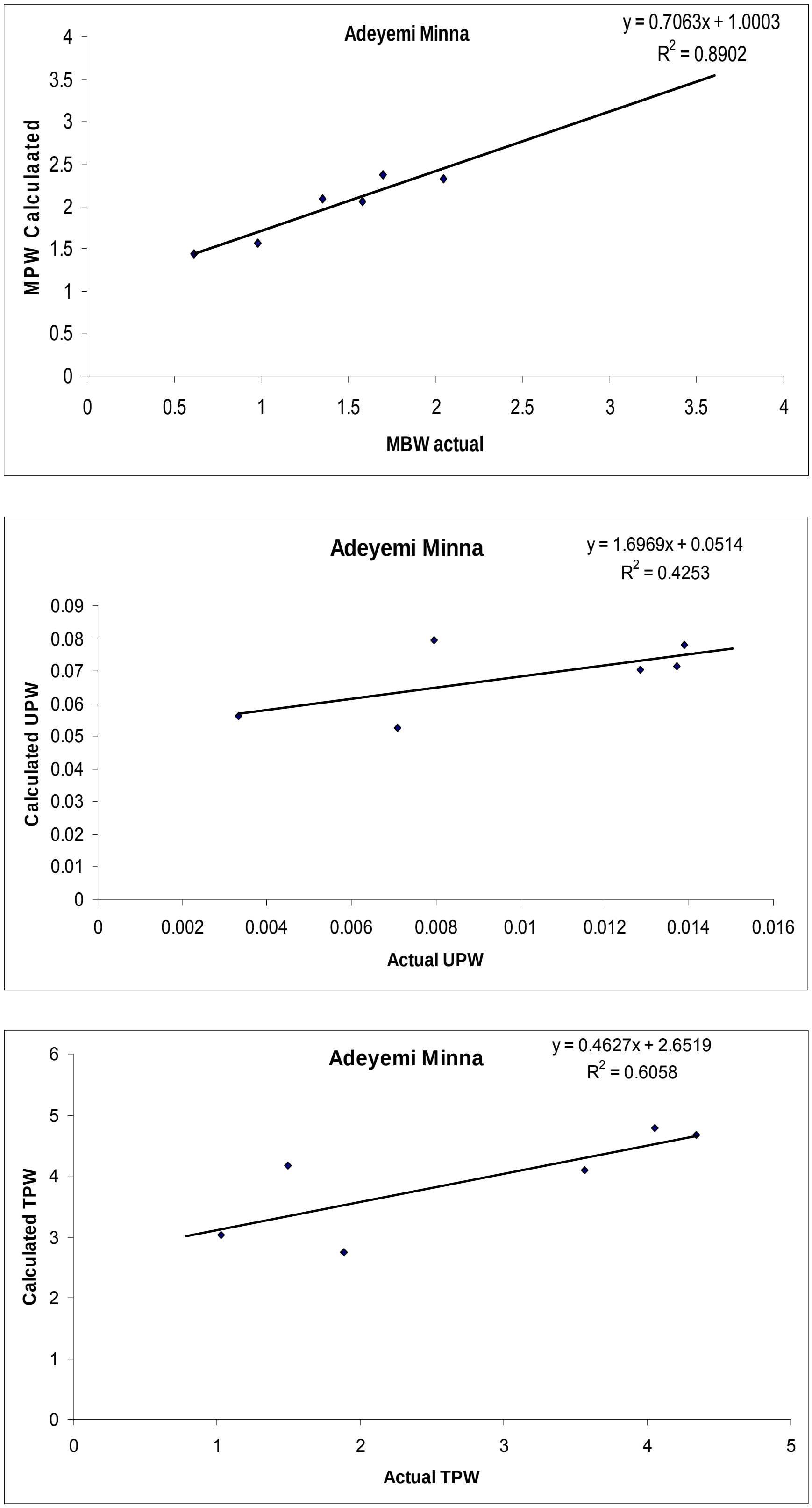

Figure 3: Plots of Calculated versus Actual Precipitable Water for a) Mid-level b) Upper-level and c) Total Column at Minna.

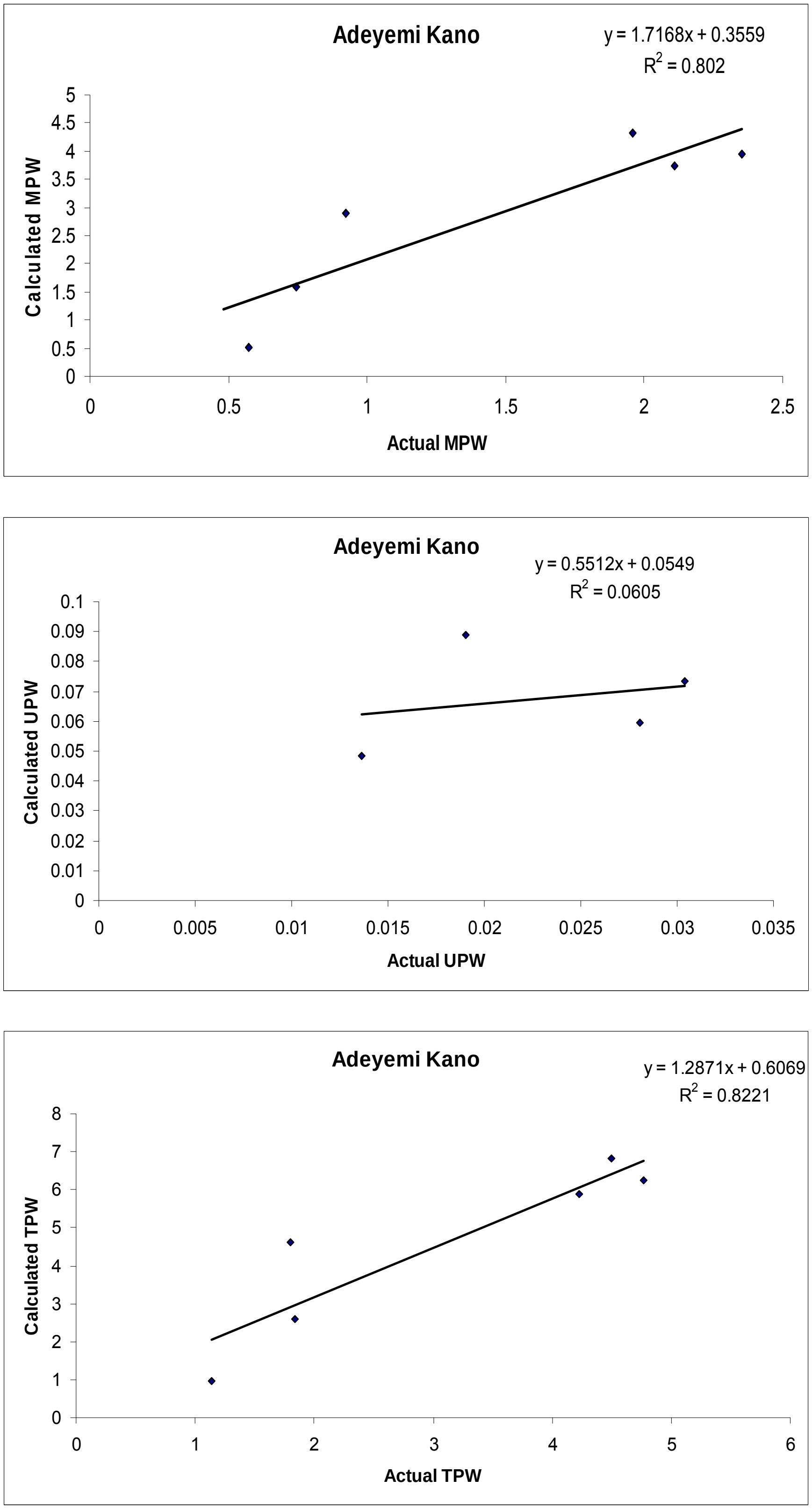

Figure 4: Plots of Calculated versus Actual Precipitable Water for a) Mid-level b) Upper-level and c) Total Column at Kano.

Table 3: Comparison Between the Models of Adedokun and the Present Study in the Evaluation of Mean MPW and TPW.

Stations	Actual MPW	Present Study	Err%	Adedokun 1983	Err%	Actual TPW	Present Study	Err%	Adedokun total 2	Err%
Lagos	1.956	1.576	19.0	2.746	29.0	4.126	3.940*	5.00	3.135	24.0
Minna	1.481	1.974	24.0	2.155	31.0	2.400	3.915	18.0	2.449*	2.00
Kano	2.350	2.834	17.0	1.827	22.5	2.986	4.526	15.0	2.076	30.0

** The underlined cases are the results which are closest to the actual values.*

Table 4: Result of Kolmogorov-Smirnov Test on Actual and the Various Models' Evaluation of MPW and TPW.

Stations	Values of $D_{N(0.05)}$	D_N values for the MPW models		Values of $D_{N(0.05)}$	D_N values for the TPW models	
		Adedokun	Present Study		Adedokun	Present Study
Lagos	0.410	0.254	0.229	0.392	0.272	0.207
Minna	0.410	0.236	0.328	0.392	0.300	0.300
Kano	0.410	0.192	0.215	0.392	0.372	0.372

As shown in Table 3, Adedokun models performed fairly well over all the stations with high error of estimates on most occasions. Of them all, only his TPW model for Minna displayed lower percentage error of estimate (~2%) than those of the present study.

It can also be gathered from Table 3 that Adedokun models overestimated MPW at Lagos and Minna with error of estimates 29% and 31% respectively. At Kano the model underestimated MPW with error 22.5%. TPW, on the other hand, was underestimated by Adedokun model at Lagos and Kano with error of 24 and 30 percent respectively; while at Minna, overestimation was observed with 2% error of estimate.

The models developed in the present study, when applied to MPW for the three stations also overestimated the values on most occasions with percentage error of estimates 19, 24 and 17 for Lagos, Minna, and Kano respectively. For TPW, underestimation is prevalent with percentage error 5, 18, and 15 in the same order. It should be noted that in a relationship of the type depicted here, lower variability often results when the analysis are based on monthly means. When a shorter time period is used (e.g. daily or hourly period), a high variability exists (Bolsenga, 1965; Adedokun 1983; Reitan, 1963). This may be responsible for the high percentage error observed in the results discussed here at almost all the stations, since daily sounding data were used for the computation.

From the foregoing, the models developed in this study performed comparably well with those of Adedokun (1983). On most occasions, they yielded results with lower percentage error of estimate than those of Adedokun (1983).

We have therefore noted with satisfaction that the various versions of the models here proposed did well, each in its own station, except for Minna which is better fitted with Adedokun version of the TPW model. Improved performances over the earlier models proposed by Adedokun have therefore been noted at almost all the stations.

CONCLUSION

Using an ANOVA technique, a linear relationship of the form $W = \alpha(LPW) + \beta$, where W can be replaced by MPW (the mid-level precipitable water) or TPW (the total column precipitable water), has been established from upper air climatological data observed over three stations in Nigeria. This is consequent upon the good correlation existing between low and mid-tropospheric levels of precipitable water on the one hand and low level and total column precipitable water vapor on the other hand.

Owing to the difference in precipitation climatologies of the stations as has been established by similar studies involving precipitation and water vapor (Obasi, 1965; Ilesanmi 1971; Adedokun, 1986; Adeyemi, 2005), no single model could be developed for the three stations. We have therefore, separately found for each station, suitable versions of this proposed model.

These relations, when used to evaluate, on a daily basis, precipitable water obtained over the stations in 1990 for Lagos; 1983 for Minna and 1991 for Kano yielded encouraging results. Comparing these results with the results obtained by the earlier models developed along this line based on a log-log relation of the form $W = \alpha(LPW)^{\beta}$ where W could either be MPW or TPW with α and β constants, was applied to the same period as stated above, the result from the present study gave improved performances over the earlier proposed models at almost all the stations.

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