# Maintainability of Departmentalized Manpower Structures in Markov Chain Model.

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## ABSTRACT

We consider a manpower structure made up of departments and grades. Movement from one department to another is referred to as transfer. A net effect of transfer is defined and conditions for maintainability of structures in Markov framework established based on this.

(Keywords: manpower structure, transfer, department, maintainability, wastage)

## INTRODUCTION

Numerous researchers, like Bartholomew (1973), Bartholomew *et al.* (1991), Davies (1975,1981,and 1982) have made appreciable pioneering contributions in the area of statistical manpower planning and control, especially in homogeneous Markov models. Also Vassiliou (1998) and a host of other referenced authors in the same paper have extensively studied control in other contexts like in non-homogeneous and semi-Markov manpower models. In all, different aspects of maintainability, (that is the maintenance of a given manpower structure through recruitment or promotion, so that it remains the same at subsequent accounting periods), have been widely covered.

Of the three factors of manpower flow – recruitment, promotion, and wastage, only recruitment and promotion have been accepted as good control factors on the ground that exercising control through wastage (for instance, dismissal/retrenchment and inducement to leave) is undesirable and unacceptable (Price *et al.,* 1980 and Bartholomew, 1982).

In this work, we consider the condition for maintainability with an aspect of manpower planning and control that gives the management a desirable and acceptable free hand to what looks like wastage control (quasi-wastage control). This is concerned with the maintainability of a graded manpower system made up of subunits or departments. These departments are assumed to be alike in terms of grade structure, and the grades are the actual promotion carders of units of the system. In this system, the factor of interest on which condition for maintainability is placed then becomes transfer of units across the departments, Ossai (2008). This exercise is desirable and acceptable in any such system, and can bring about balance or control in the system.

We first present the seemingly complex semi-Markov system in a simple and tractable Markov framework, by defining the state of the Markov chain to be the departments as well as the length of stay or grade within the departments. This kind of argument for simple and practically usable, yet representative, manpower models can be seen in works by McClean (1991), Barnet and Ellison (1998), Davies (1982), and others.

## THE MODEL

We consider a system made up of k departments, $d_1$, $d_2$, …$d_k$ and assume that transition or transfer is possible between any two departments, and that these departments have the same configuration with respect to grade structure. For any department $d_i$, i = 1,2,…, k, let there be u grades with the states of these grades denoted by g such that g =1,2,…, u.

We make the following definitions: $n_{i(g)}(t)$ is the number of units in grade g in department i at time t; $n_i(t) = [n_{i(1)}(t), n_{i(2)}(t), \ldots, n_{i(u)}(t)]$; i.e., a row vector of $n_{i(g)}(t)$'s for department i; $n_e(t) = [n_1(t), n_2(t), \ldots, n_k(t)]$, a 1 x uk row vector of $n_i(t)$'s; $P_{i\,j(g,h)}$ is the probability of transferring from $d_{i(g)}$, (grade g in department i,) to $d_{j(h)}$ (grade h in department $_j$ ), j = 1,2, …, k+1, where k+1 is the state of having left, h = 1,2,…, u; $w_{i(g)} = p_{i,k+1(g)}$ is the wastage probability from $d_{i(g)}$;

$w_i = \{w_{i(1)}, w_{i(2)}, ..., w_{i(u)}\}$, i.e. a 1 x u row vector of $w_{i(g)}$'s for department i; $w_e = \{w_1, w_2, ..., w_k\}$, a 1 x uk vector of $w_i$'s; $r_{i(g)}$ is the probability of allocation of a recruit into $d_{i(g)}$; $r_i = \{r_{i(1)}, r_{i(2)}, ..., r_{i(u)}\}$, i.e. a 1 x u row vector of $r_{i(g)}$'s; $r_e = \{r_1, r_2, ..., r_k\}$, a 1 x uk row vector of $r_i$'s.

## DEDUCTIONS FROM THE DEFINITIONS

The total number of workers in the system at time t, denoted by $N_e(t)$, is given by:

$$N_e(t) = \sum_{i=1}^{k} \sum_{g=0}^{u} n_{i(g)}(t)$$

Since for a transition over a period t, for a particular grade g, a worker in $d_i$ must either stay in the same grade g in $d_i$, move to another grade h in $d_i$, move to another grade h in another department $d_j$, or leave the system entirely, we have that for all $_i$ and g:

$$\sum_{h=1}^{u} p_{ii(g,h)} + \sum_{\substack{h=1 \\ \text{for } j=1}}^{u} p_{ij(g,h)} + \sum_{\substack{h=1 \\ \text{for } j=2}}^{u} p_{ij(g,h)} + \ldots + \sum_{\substack{h=1 \\ \text{for } j=i-1}}^{u} p_{ij(g,h)}$$

$$+ \sum_{\substack{h=1 \\ \text{for } j=i+1}}^{u} p_{ij(g,h)} + \ldots + \sum_{\substack{h=1 \\ \text{for } j=k}}^{u} p_{ij(g,h)} + p_{i,k+1(g)} = 1$$

Or more compactly,

$$\sum_{h=1}^{u} p_{ii(g,h)} + \sum_{\substack{j=1,h=1 \\ j \neq i}}^{k+1,u} p_{ij(g,h)} = 1$$

Hence, for any i, $\sum_{h=1}^{u} p_{ii(g,h)} = 1 - \sum_{\substack{j=1,h=1 \\ j \neq i}}^{k+1,u} p_{ij(g,h)}$ is the probability of the event of not leaving department $d_i$ from grade g.

Let $D_{ii}$ represents the intra-departmental transition probabilities, from one grade to another in $d_i$, and $D_{ij}$ represents the inter-departmental transfer probabilities from di to another department $d_j$, then, from the foregoing we can write that:

$$D_{ii} = \begin{bmatrix} p_{ii(1,1)} & p_{ii(1,2)} & \cdot & \cdot & \cdot & p_{ii(1,u)} \\ p_{ii(2,1)} & p_{ii(2,2)} & \cdot & \cdot & \cdot & p_{ii(2,u)} \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ p_{ii(u,1)} & p_{ii(u,2)} & \cdot & \cdot & \cdot & p_{ii(u,u)} \end{bmatrix}$$

and

$$D_{ij} = \begin{bmatrix} p_{ij(1,1)} & p_{ij(1,2)} & \cdot & \cdot & \cdot & p_{ij(1,u)} \\ p_{ij(2,1)} & p_{ij(2,2)} & \cdot & \cdot & \cdot & p_{ij(2,u)} \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ p_{ij(u,1)} & p_{ij(u,2)} & \cdot & \cdot & \cdot & p_{ij(u,u)} \end{bmatrix}$$

Therefore, the transition probability matrix (tpm) of the entire manpower system can be written as

$$P_e = \begin{bmatrix} D_{11} & D_{12} & \cdot & \cdot & \cdot & D_{1k} \\ D_{21} & D_{22} & \cdot & \cdot & \cdot & D_{2k} \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ D_{k1} & D_{k2} & \cdot & \cdot & \cdot & D_{kk} \end{bmatrix}$$

It is now possible, by the foregoing definitions and deductions, to express the basic fixed-size Markov model (Bartholomew *et al.* (1991)) for the manpower system as:

$$n_e(t) = n_e(t-1)P_e + n_e(t-1)w_e'r_e \qquad \textbf{(1)}$$

We note that $P_e$, which is a uk x uk sub-stochastic matrix, with $D_{ii}$ as the diagonal entries and $D_{ij}$ as the off-diagonal entries, can take different forms depending on the assumption about the mode of transition within, and transfer across, the departments.

To investigate the effect of transferring of workers (transfer) on the maintainability of the manpower structure, we define what can be called the net effect of transfer at time t by the vector a(t), and incorporate this in (1). Actually a(t) expresses the expected quantitative net difference due to transfer in each department at time t. We proceed by breaking the overall tpm, $P_e$, into two parts as: $P_e = T + P$, where:

$$T = \begin{bmatrix} 0 & D_{12} & . & . & . & D_{1k} \\ D_{21} & 0 & . & . & . & D_{2k} \\ . & & & & & . \\ . & & & & & . \\ . & & & & & . \\ D_{k1} & D_{k2} & . & . & . & 0 \end{bmatrix},$$

and

$$P = \begin{bmatrix} D_{11} & 0 & . & . & . & 0 \\ 0 & D_{22} & . & . & . & 0 \\ . & & & & & . \\ . & & & & & . \\ . & & & & & . \\ 0 & 0 & . & . & . & D_{kk} \end{bmatrix}$$

Note that P as above is defined as $P = \text{diag}(D_{ii})$, i = 1,2, …,k; that is a form of diagonal matrix with diagonal entries $D_{ii}$'s. As can be observed, T contains all the information about transfer (inter-departmental transitions) while P contains all the information about intra-departmental transitions. Because of the special interest on the component of transfer, and to satisfy the definitions in our formulations we shall observe the condition that the sub matrices $D_{ii}$'s in $P_e$ remain the same under matrix transposition. Also the stochastic condition is that $(T + P)\mathbf{1}' + w_e' = \mathbf{1}'$, where $\mathbf{1}' = (1,1,\ldots,1)'$, (i.e., a row vector of one's). These conditions shall be maintained throughout this work.

Now, provided $P_e$, D's, T and P are all square matrices, $a_1(t)$ which gives at time t the expected difference between the number of units which come into each department through transfer and the number which go out through transfer can be defined as:

$$a_1(t) = n_e(t)T - n_e(t)\{\text{diag}(\mathbf{1}T')_l\},$$
$$(l = 1,2,\ldots,v.\ (v = uk))$$
$$= b_1(t) - C_1(t)$$

where, $b_1(t) = n_e(t)\left(T^*\right)$,

$$C_1(t) = n_e(t)\{\text{diag}(\mathbf{1}T')_l - T'\}$$

and $T^* = T - T'$

We note that though $a_1(t)$ has been expressed as $b_1(t) - C_1(t)$, both $b_1(t)$ and $C_1(t)$ are mere algebraic expressions used to facilitate the mathematics involved in this work. Where necessary, the results involving $a_1(t)$ may be expressed in terms of these terms.

As a further insight into the property of $a_1(t)$, we note that other departments through transfer gain units lost by any department through transfer. In effect the total net effect of transfer is expected to be zero. This is reflected in the following proposition.

**Proposition 1**

For any 1 x v structure $n_e(t)$ and v x v inter-departmental transition matrix T, the sum of the entries in $a_1(t)$ is zero independent of $n_e(t)$, $n_e(t)$ not necessarily zero.

Proof: Let the matrix T be represented generally by $T = \{t_{ij}\}, i, j = 1,2,\ldots,v$. Then $T\mathbf{1}'$ is the column vector of $\sum_{j=1}^{v} t_{ij},\ i = 1,2,\ldots,v$; while $\mathbf{1}T'$ is the row vector of $\sum_{j=1}^{v} t_{ij},\ i = 1,2,\ldots,v$ so that $\text{diag}(\mathbf{1}T')_l\mathbf{1}',\ l = 1,2,\ldots,v$ is also the column vector of $\sum_{j=1}^{v} t_{ij},\ i = 1,2,\ldots,v$.

Hence, $T\mathbf{1}' = \text{diag}(\mathbf{1}T')_l \mathbf{1}'$. But $a_1(t)\mathbf{1}' = n_e(t)\{T\mathbf{1}' - \text{diag}(\mathbf{1}T')_l \mathbf{1}'\} = n_e(t).\{0\} = 0$ irrespective of $n_e(t)$.

Apart from giving credence to the definition of $a_1(t)$, Proposition1 also gives an easy check for the correctness of the values of any computed $a_1(t)$.

To incorporate $a_1(t)$ in the model, (1) can be resolved as:

$$n_e(t) = n_e(t-1)(T-T') + n_e(t-1)(P+T') + n_e(t-1)w_e'r_e$$

or,

$$n_e(t) = n_e(t-1)T^* + n_e(t-1)P_e' + n_e(t-1)w_e'r_e \quad \textbf{(2)}$$

From the forgoing definition, (2) can now be written as:

$$n_e(t) = b_l(t-1) + n_e(t-1)P_e' + n_e(t-1)w_e'r_e \quad \textbf{(3)}$$

## MAINTAINABILITY OF THE MANPOWER STRUCTURE

In manpower planning, a structure is simply maintainable if it is possible to return to the same structure at the next step. Usually, this is achieved by exercising control on the tpm, $P_e$, or the recruitment vector $r_e$. In this way we shall establish conditions, in terms of $a_1(t)$, for the structure to be maintainable.

To talk about a maintainable manpower structure what readily comes to mind is a fixed-size system. Sometimes, however, the system is either expanding or contracting so that the total size is no more fixed. In this case, instead of fixed-total size, we talk of a fixed relative size Bartholomew *et al.* (1991), and then a quasi- maintainable structure. Also, since a fixed-size system is a special case of growth system, with growth rate zero, we present the analysis in this latter case.

Let $\beta$ denote the rate of change in the system. If $N_e(t)$ is the total size of the system, then change in the system can be written as:
$\Delta N_e(t) = \beta N_e(t) = n_e(t)\mathbf{1}'\beta$

Since $n_e(t)$ is changing, $N_e(t)$ is also changing. Hence, to talk of a stable structure we define a new structure of the system based on $n_e(t)$ and $N_e(t)$ as:

$m_e(t) = n_e(t)\{n_e(t)\mathbf{1}'\}^{-1}$

The new structure defined by $m_e(t)$ (the stock proportion) can attain a stable or steady state, even though $n_e(t)$ and $N_e(t)$ are changing in size. This will happen when the relative changes of $n_e(t)$ with $N_e(t)$ are such that the values of $m_e(t)$ remain the same.

By the above definitions the basic equation for the manpower system is given by:

$$n_e(t) = n_e(t-1)P_e + n_e(t-1)w_e'r_e + \Delta N_e(t-1)r_e$$

or

$$n_e(t) = n_e(t-1)P_e + n_e(t-1)w_e'r_e + n_e(t-1)\mathbf{1}'\beta r_e \quad \textbf{(4)}$$

If we combine (2) and (4) then:

$$n_e(t) = n_e(t-1)T^* + n_e(t-1)P_e' + n_e(t-1)w_e'r_e + n_e(t-1)\mathbf{1}'\beta r_e$$

Therefore,

$$n_e(t)\mathbf{1}'m_e(t) = n_e(t-1)\mathbf{1}'m_e(t-1)T^* + n_e(t-1)\mathbf{1}'m_e(t-1)P_e' + n_e(t-1)\mathbf{1}'m_e(t-1)w_e'r_e + n_e(t-1)\mathbf{1}'\beta r_e$$

Since $n_e(t)\mathbf{1}' = (1+\beta)n_e(t-1)\mathbf{1}'$, then:

$$(1+\beta)n_e(t-1)\mathbf{1}'m_e(t) = n_e(t-1)\mathbf{1}'m_e(t-1)T^* + n_e(t-1)\mathbf{1}'m_e(t-1)P_e' + n_e(t-1)\mathbf{1}'m_e(t-1)w_e'r_e + n_e(t-1)\mathbf{1}'\beta r_e$$

which gives:

$$(1+\beta)m_e(t) = m_e(t-1)T^* + m_e(t-1)P_e' + m_e(t-1)w_e'r_e + \beta r_e$$

or,

$$(1+\beta)\mathrm{m_e}(t) = \mathrm{b_2}(t-1) + \mathrm{m_e}(t-1)\left\{\mathrm{P}'_e + (w'_e + \mathbf{1}'\beta)\mathrm{r_e}\right\} \quad \textbf{(5)}$$

where, $\mathrm{b_2}(t-1) = \mathrm{m_e}(t-1)\mathrm{T}^*$.

We observe that $\mathrm{b_2}(t-1)$ in (5) has a relationship with $\mathrm{a_1}(t-1)$ (from $b_1(t)$ in (3)), of the form:

$$\begin{aligned}\mathrm{b_2}(t-1) &= \mathrm{a_1}(t-1)\left\{\mathrm{n_e}(t-1)\mathbf{1}'\right\}^{-1} \\ &\quad + \mathrm{C_1}(t-1)\left\{\mathrm{n_e}(t-1)\mathbf{1}'\right\}^{-1} \\ &= \mathrm{a_2}(t-1) + \mathrm{C_2}(t-1)\end{aligned}$$

where, $\mathrm{a_2}(t-1) = \mathrm{a_1}(t-1)\left\{\mathrm{n_e}(t-1)\mathbf{1}'\right\}^{-1}$ and $\mathrm{C_2}(t-1) = \mathrm{C_1}(t-1)\left\{\mathrm{n_e}(t-1)\mathbf{1}'\right\}^{-1}$

Hence $\mathrm{a_2}(t)$ can be referred to as the proportional net effect of transfer at time, t.

**Proposition 2**

For any 1 x v structure $\mathrm{n_e}(t)$ and v x v inter-departmental transition matrix T, the sum of the entries in $\mathrm{a_2}(t)$ is zero, independent of $\mathrm{n_e}(t)$, $\mathrm{n_e}(t)$ not necessarily zero.

**Proof:**

This is obvious from Proposition 1, because if $\mathrm{a_1}(t)\mathbf{1}' = 0$ independent of $\mathrm{n_e}(t)$ then $\mathrm{a_2}(t)\mathbf{1}' = \mathrm{a_1}(t)\left\{\mathrm{n_e}(t)\mathbf{1}'\right\}^{-1}\mathbf{1}' = 0$, independent of $\mathrm{n_e}(t)$.

Since $m_e(t) = n_e(t)\{ n_e(t)\ \mathbf{1}'\}^{-1}$, Proposition 2 also implies that the sum of entries in $\mathrm{a_2}(t)$ is zero independent of $m_e(t)$.

**CONDITIONS OF MAINTAINABILITY BY $\mathbf{a_2(t)}$ IN RECRUITMENT CONTROL**

From (5), the structure $m_e$ is maintainable if it satisfies the steady-state condition given by setting $m_e(t) = m_e(t-1) = m_e$ (see, for instance, Georgiou and Tsantas (2002), Uche and Ossai(2008)). That is if,

$$(1+\beta)\mathrm{m_e} = \mathrm{b}_2^* + \mathrm{m_e}\left\{\mathrm{P}'_e + (w'_e + \mathbf{1}'\beta)\mathrm{r_e}\right\} \quad \textbf{(6)}$$

where $\mathrm{b}_2^* = \mathrm{m_e}\mathrm{T}^*$

Hence, under recruitment control we look for an $r_e$ such that, from (6):

$$\mathrm{r_e} = \left\{\mathrm{m_e}(\mathrm{I}-\mathrm{P}'_e) + \beta\mathrm{m_e} - \mathrm{b}_2^*\right\}\left\{\mathrm{m_e}(w'_e + \mathbf{1}'\beta)\right\}^{-1} \quad \textbf{(7)}$$

The condition required for any such $r_e$ to satisfy as a well defined manpower recruitment vector is that $r_e$ must be a probability vector. By the above development, the two conditions that achieve this are that:

(i) $\left\{\mathrm{m_e}(\mathrm{I}-\mathrm{P}'_e) + \beta\mathrm{m_e} - \mathrm{b}_2^*\right\}\mathbf{1}' = \mathrm{m_e}(w'_e + \mathbf{1}'\beta)$

and (ii) $\mathrm{m_e}(\mathrm{I}-\mathrm{P}'_e) + \beta\mathrm{m_e} - \mathrm{b}_2^* \geq 0$.

It can easily be shown that the condition in (i) is always satisfied so that, from (7), a sufficient condition for the structure $m_e$ to be maintainable in the present context is that:

$\mathrm{b}_2^* \leq \mathrm{m_e}(\mathrm{I}-\mathrm{P}'_e) + \beta\mathrm{m_e}$. Or, in terms of $\mathrm{a}_2^*$, $\mathrm{a}_2^* \leq \mathrm{m_e}(\mathrm{I}-\mathrm{P}'_e) + \beta\mathrm{m_e} - \mathrm{C}_2^*$, that is

$$\mathrm{a}_2^* \leq \mathrm{m_e}\left\{\mathrm{diag}\left(\mathbf{1}(\mathrm{I}-\mathrm{T}')\right)_l - \mathrm{P}\right\} + \beta\mathrm{m_e} \quad \textbf{(8)}$$

Equation (8) means that a sufficient condition for any structure $m_e$ to be maintainable is that the proportional net effects of transfer in each department, quantified by $\mathrm{a}_2^*$, is equal or less than some value which is a simple function of the given structure.

It is easy to show that in systems with growth rate β = 0, where $n_e$ replaces $m_e$, the condition in (8) changes by only replacing $a_2^*$ with $a_1^*$ and $m_e$ with $n_e$ .

## CONDITIONS OF MAINTAINABILITY BY $a_2(t)$ IN PROMOTION CONTROL

In this case, we look for a $P_e$ such that, from (6),

$$m_e P_e' = m_e(I - w_e' r_e) + \beta(m_e - r_e) - b_2^* \quad \textbf{(9)}$$

Since the elements of $m_e$ and $P_e$ are all nonnegative, so are the elements of $m_eP_e$. Hence, for any $P_e$ that satisfies the stochastic condition earlier defined the existing condition for $m_e$ to be maintainable through promotion is that $m_e(I - w_e' r_e) + \beta(m_e - r_e) \geq 0$,

or $\quad m_e \geq \{(m_e w_e' + \beta) r_e\}\{1+\beta\}^{-1} \qquad$ **(10)**

The result in (10) is due to the fact that:

$$m_e P_e = m_e P_e' + b_2^* = m_e(I - w_e' r_e) + \beta(m_e - r_e) \geq 0$$

since $m_e P_e \geq 0$.

The maintainability condition in (10) simply means that if this condition is satisfied, a $P_e$ exists that maintains the structure $m_e$. But there can be many such $P_e$'s. Hence, there is the need for some characterization of such matrices. One of such characterization can be given, in terms of the proportional net effect of transfer, $b_2^*$, as $b_2^* \leq m_e(I - w_e' r_e) + \beta(m_e - r_e)$. This implies that:

$$a_2^* \leq m_e(I - w_e' r_e) + \beta(m_e - r_e) - C_2^* \quad \textbf{(11)}$$

Equation (11) results from the fact that $m_e P_e - b_2^* = m_e P_e'$, and since $m_e P_e' \geq 0$, $m_e P_e - b_2^* \geq 0$, which implies that $m_e(I - w_e' r_e) + \beta(m_e - r_e) - b_2^* \geq 0$.

Any $P_e$ that will maintain a maintainable structure $m_e$ must therefore satisfy the condition in (11).

Hence, $a_2^*$ or $b_2^*$ can be utilized as a planning tool for realizing a $P_e$ that will maintain a structure through promotion in departmentalized manpower systems.

## AN ILLUSTRATIVE EXAMPLE

We illustrate the above with a university manpower system having two departments – academic (AD) and non academic (NAD) –, with two grades per department – junior staff (JS) and senior staff (SS); that is, i,j,g and h go from 1 to 2. The current structure of the system, to be maintained, with a contraction rate β = -0.03, is:

| | AD | | NAD | |
|---|---|---|---|---|
| | JS | SS | JS | SS |
| $n_e =$ | 656 | 654 | 1720 | 1027 |

$$n_e = \begin{bmatrix} 656 & 654 & 1720 & 1027 \end{bmatrix}$$

with

| | AD | | NAD | |
|---|---|---|---|---|
| | JS | SS | JS | SS |
| $w_e =$ | .0116 | .0447 | .0275 | .1199 |

$$D_{11} = \begin{bmatrix} .9490 & .0106 \\ 0 & .9219 \end{bmatrix}, \quad D_{22} = \begin{bmatrix} .9569 & .0062 \\ 0 & .8636 \end{bmatrix}$$

$$D_{12} = \begin{bmatrix} .0232 & .0056 \\ 0 & .0334 \end{bmatrix}, \quad D_{21} = \begin{bmatrix} .0065 & .0029 \\ 0 & .0165 \end{bmatrix}$$

Hence,

$$P_e = \begin{bmatrix} .9490 & .0106 & .0232 & .0056 \\ 0 & .9219 & 0 & .0334 \\ .0065 & .0029 & .9569 & .0062 \\ 0 & .0165 & 0 & .8636 \end{bmatrix} \text{ and}$$

$m_e$ = [0.1621,0.1593,0.4249,0.2537] .

The result from the foregoing discussion can be used to show that the current structure of the above system is maintainable. In recruitment control, this requires checking that (8) is satisfied for $m_e$; that is checking if $a_2^* \leq m_e\left\{\operatorname{diag}\left(\mathbf{1}(I - T')\right)_l - P\right\} + \beta m_e$.

Now,

$$T = \begin{bmatrix} 0 & 0 & 0.0232 & 0.0056 \\ 0 & 0 & 0 & 0.0334 \\ 0.0065 & 0.0029 & 0 & 0 \\ 0 & 0.0165 & 0 & 0 \end{bmatrix},$$

$$P = \begin{bmatrix} 0.9490 & 0.0106 & 0 & 0 \\ 0 & 0.9219 & 0 & 0 \\ 0 & 0 & 0.9569 & 0.0062 \\ 0 & 0 & 0 & 0.8636 \end{bmatrix},$$

so that

$a_2^* = [-0.0019, 0.0001, -0.0002, 0.0020]$. Also, $m_e\left\{\text{diag}\left(\mathbf{1}(I - T')\right)_l - P\right\} + \beta m_e$ has the value given by [0.0013,0.0006,0.0016,0.0202].

Comparing, we conclude that the given manpower structure is maintainable. It can be seen also that the computed value of $a_2^*$ satisfies Proposition 2, which, in one sense of its importance, ensures that the computations are correct.

## CONCLUSION

We have presented a model of departmentalized and graded manpower system in a simple and tractable Markov framework. The probability entries in the sub-matrices $D_{ii}$ and $D_{ij}$ can, in practice, be estimated using any simple method, such as the maximum likelihood method Bhat (1971).

The introduced concept of net effects of transfer has been shown to establish the maintainability condition in recruitment control. It also gives a characterization of $P_e$'s that will maintain a given structure in promotion control. However, this characterization is not sufficient for realizing any such $P_e$. This is because, even though every $P_e$ that will maintain a given structure must satisfy this condition, not every $P_e$ that satisfies this condition will maintain the structure. This is an issue for further investigation.

The importance of this development is that judgment on the possibility of maintaining a given departmentalized manpower structure can be made using the effect of transfer on the stock numbers of the various grades.

The elements of the vector $a_2(t)$ (or $a_1(t)$), the component of the effect of transfer, can take any real value. Judgment is based on all the elements being considered together and no one element is solely sufficient for this consideration. However, the extent to which the value of each element constrains the maintainability of the manpower structure, seen by comparing the left hand side and the right hand side of Equation (8), can give further insight into the freedom the management has to control the structure. This defines the importance of each department (or grade in each department) in the maintenance of the entire system.

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