Parametric Approach to Detergent Production Using a Developed Algorithm.

Adamu Wakili, M.Sc.*, Prof. M.S. Sesay, and M.S. Atureta, Ph.D.

Mathematical Sciences Program, Abubakar Tafawa Balewa University,
Bauchi, Nigeria.

*E-mail: adamou_wakili@yahoo.com

ABSTRACT

This paper examines parametric linear programming problem (PLPP) with interval in the coefficients of the objective function and the compositions of Slash, Bry- tex, White plus, Action and Bonus and production of detergent in NASCO Company in Jos, Nigeria. The formulated problems from NASCO are tested using the computer program by varying the parameter at regular interval and obtaining the corresponding values of the objective function.

(Keywords: parametric analysis, discretilization, optimization, interval, algorithm)

INTRODUCTION

Mathematical programming often involves unknown parameters and the task of parametric optimization is, in principle, to solve the mathematical program for each possible value of these unknown parameters. Discretization of the parameter, range is not rigorous in general, since there is no guarantee of optimality between mesh points. Moreover, discretization on a fine mesh is a very expensive procedure, especially for high dimension parameter spaces. Algorithms for parametric optimization typically divide the parameter range into regions of optimality [2]. For each critical region either the problem is infeasible or a quantitatively invariant solution, typically a smooth function of the parameters, is optimal [6].

Parametric optimization has several applications including waste management [3], fleet planning [4], model-predictive control [5], and process synthesis under uncertainty [8]. Recently works have been done on this area resulting in good feasible solutions of parametric mixed-integer linear programming (PMILP), interval objective linear programming (IOLP) and parametric linear programming (PLP) to generate cuts of the programming problems.

In operations research, researchers have addressed parameter variations in mathematical programs at two levels; sensitivity analysis, which characterizes the change of the solution with respect to small perturbations of the parameters; and parametric programming, where characterization of the solution for a full range of parameter values is sought. More precisely, programs which depend on only one scalar parameter are referred to as parametric program, while problems depending on a vector of parameters are referred to as multi-parametric programs [13].

The first method for solving parametric linear programs was proposed by Gass and Saaty, since then, extensive research has been devoted to sensitivity and parametric analyses in various field of applications [2, 3].

In this article, one focus on parametric linear programming where the tool for decision making is useful when the value of the parameter is known .The parametric linear programming will be tested on the production of detergent in the production company (NASCO Ltd.) later in the article. It will be noted that positive increase in the values of the parameter in the objective function will result in the corresponding increased in the values the optimization problem.

MODEL FORMULATION

Parametric linear programming is applied to a problem after an initial solution has been found. The revised simplex method algorithm used for parameterization is often applied to the primal problem. Consider the typical linear programming problem:

$$\begin{aligned} &MaxZ = CX \\ &subject\ \ to\ \ the\ \ restrictions \\ &Ax = b \\ &x \geq 0 \\ &where\ C = [C_i, C_{i+1}],\ i = 1,...,4. \end{aligned} \quad \textbf{(1)}$$

C is an row vector, A is an mxn matrix, x and b are nx1 column vector and 0 is an nx1 vector of zeroes.

If the matrix B is composed of m linearly independent columns of A, then any column a_i of A can be written as a linear combination of the columns of B,

$$a_i = By_i \quad \textbf{(2)}$$

This matrix B also provides a basis solution to the simultaneous linear equation represented as follows:

$$x_B = B^{-1}b \quad \textbf{(3)}$$

Corresponding to a basis solution x_B, there is an association vector; C_B which is composed of m elements from vector C.

INDUSTRIAL APPLICATION

NASCO Household is another branch of NASCO group of Companies, it is located at No. 4k Yakubu Gawon Way, Jos, Plateau State of Nigeria along Bukuru – expressway, Jos South Local Government.

NASCO Group of Companies with its headquarters in Jos, Plateau State, started around early 1960. Being a private company, it was owned by Ahmed Nusurdeen Group of Companies. Then it was referred to as the Northern Nigeria Fiber Product Ltd in 1963, it was named NASCO Group of companies.

The Northern Nigerian Fibre Product Ltd has grown into the following: NASCO Foods, Ltd., NASCO Park, Ltd., NASCO Management and Services, NASCO Fibre, Ltd., and NASCO Household Product, Ltd.

NASCO group of companies has now acquired more technologies in terms of producing soaps, detergent, sulphuric acid, biscuit, carpets, cornflakes and so on; it also produces many packing materials, which are widely used in our homes and industries.

NASCO group of companies has the following divisions in its operations:

a) Manufacturing,
b) Marketing,
c) Services,
d) Trading.

NASCO Household produces different types of detergents such as; slash, Bry-tex, whits plus, action, and Bonus of various weights and densities. All depend on their weight in gram (g).

Quality Control: The quality control department is the main heart of NASCO household. It is aimed at controlling every activity carried out in NASCO household product. The department is further divided into three (3) major parts (units).

a) Detergent and suphonic analysis section
b) Soap analysis section
c) Raw material analysis section.

The company relies on information from the quality control department and these activities carried out by this department are basically chemical analysis.

Composition of Detergent: The process is that of sulphuric acid (ARC) is neutralized by a base usually NaOH (Sodium hydroxide) to give paste. This reaction is occurring in the neutralization chamber. The paste is now transferred into the crutcher where additives are added such as Kaolin soda ash (Na_2CO_3), sodium sulphate (Na_2SO_4), carboxyl methyl cellulose (CMC), sodium silicate (Na_2SiO_2), sodium tripoly phosphate (STPP), perfume, and color.

The substances are thoroughly mixed by the crutcher to produce slurry. Slurry is detergent in liquid form; the slurry is then pumped into the dryer for drying and then it is conveyed to the storage tanks ready to be packaged. The various brands of detergent are differentiated by their dye (color), perfume and Alkyl Benzene Sulphonate (ABS) content. The presence of benzene and the branch alkyl group determines the bio-degradation of detergent.

Detergent Composition and Functions:

a) ABS – Active Ingredient (Mathor)
b) Dye – colour agent
c) Kaolin – filter
d) Na_2SO_4 – Makes power bulky (builder)
e) Na_2SO_3 – Anti corrosion to detergent
f) Na_2CO_3 – Softening agent for water
g) Optical brightener (OB) – Brightening agent carboxyl methyl cellulose (CMC) – dirt suspending agent anting redepository agent
h) Sodium to luence suphates (STS) – Anti caking agent
i) Sodium tripoly phosphate (STPP) – builder
j) Form quest – forming agent
k) Perfume – scent agent

Objectives of NASCO Household Product Limited: The major aims and objectives of the NASCO household product limited are:

- To meet the needs of Nigerians and Africans at large in terms of soap and detergent.
- To reduce the unemployment problem facing our society
- To produce quality products that will meet international standard
- To increase the rate of the state labour market
- To encourage the private sector and small scale industries set a good example [10].

Standard Form of an LP Problem: Consider the linear programming with interval coefficients in the objective function:

$$Max\{\sum_{i=1}^{m} Q_i X_i = P(x)\} \qquad \textbf{(4)}$$
$$s.t\ \ x \in X = \{x \in R^m : Ax = b, x \geq 0\}$$

where the coefficients of Q_i are closed intervals, that is $[\underline{Q_i}, \overline{Q_i}]$, $\underline{Q_i}, \overline{Q_i} \in R^m$, $i = 1,2,...m$. The interval objective linear programming problem (IOLP) above has the constraints as in the conventional linear programming problems.

The value of the P(x) can be determined since:

$$x \geq 0, and\ \underline{P_i}(x) = \sum_{i=1}^{m} \underline{Q_i} x_i \ \ , \overline{P_i} = \sum_{i=1}^{m} \overline{Q_i} x_i\ .$$

Now introduce generalized preference relations for which the following notations are needed d_0, d_1 as cuts of the interval.

Let $D = [\underline{n}, \overline{n}]$ be an interval and let d_0, d_1 be fixed numbers with $d_0, d_1 \in [0,1]$ such that $d_0 < d_1$. Then d_0, d_1-cut of the interval D is defined as follows:

$$D/[d_0, d_1] = [\underline{n} + d_0(\overline{n} - \underline{n}),\ \underline{n} + d_1(\overline{n} - \underline{n})]$$

The parametric programming problem becomes:

$$Max\{\sum_{i=1}^{m} [\underline{Q_i} + t(\overline{Q_i} - \underline{Q_i})] x_i \qquad \textbf{(5)}$$
$$s.t\ \ x \in X,\ t \in [d_0, d_1]$$

PROBLEM

The NASCO company ltd produces five brands of detergent as in Table I. The production capacity in kilogram (Kg) of each of the five products (slash, Bry-tex, white plus, action and bonus) is 1.5 hours daily.

Table 1: Showing Differences and Various Brands of Detergent.

Brand	Colour	% ABS	Density	Package
Slash	Blue	16±5%	160 – 200	10kg & 5kg
Bry-tex	White	20±5%	170 – 220	200g & 5kg
White plus	White	22±5%	170 – 250	40g
Action	Blue	18±5%	150 – 200	200g
Bonus	White	20±5%	180 – 250	200g

The production policy of the machine (subsequent production) is that 90% of the production of slash is at most 10% of the remaining products, the production of slash and Bry-tex is 80% as much as the production of the remaining products which is 20% , 60% of the production of bonus is at least 40% the production of action and finally 80% of the production of bonus is at least 20% of the production of the remaining products (slash, Bry-tex, white plus and action).

The profit of each of the products; Slash, Bry-tex, White plus, Action and Bonus are estimated to be [145,150], [100,120], [76, 90], [80,100], and [50, 70], respectively, in Naira.

Determine the daily production schedules that will maximize the estimated profit of each of the products.

Determine the profit that maximizes the production if each package contains 100 units of the five products.

FORMULATION

Let the units (kg) of $x_{1=}$ slash,$X_{2=}$Bry-tex ,$X_{3=}$White Plus , $X_{4=}$ Action and $X_{5=}$ Bonus detergent produced
MaxZ= [145, 150] x_1 + [100, 120] x_2 + [76, 90] x_3 + [80, 100] x_4 + [50, 70]x_5

Subjected to the constraints,

$x_1 + x_2 + x_3 + x_4 + x_5 \leq 1.5$

$x_1 \leq 10\% (x_1 + x_2 + x_3 + x_4 + x_5)$

$x_1 + x_2 \leq 20\% (x_1 + x_2 + x_3 + x_4 + x_5)$

$x_5 \geq 40\% (x_4 + x_5)$

$x_5 \geq 20\% (x_1 + x_2 + x_3 + x_4 + x_5)$

$x_1, x_2, x_3, x_4, x_5 \geq 0$.

Using parametric transformation $t \in [d_0, d_1]$

$$\Rightarrow \; q = d_0 + t(d_1 - d_0)$$

From Equation 5 above:

$$MaxZ = [145 + 5t]x_1 + [100 + 20t]x_2 + [76 + 14t]x_3 + [80 + 20t]x_4 + [50 + 20t]x_5$$

$$\begin{aligned} \textit{subjected to the constraints} \quad & x_1 + x_2 + x_3 + x_4 + x_5 \leq 1.5 \\ & 0.9x_1 - 0.1x_2 - 0.1x_3 - 0.1x_4 - 0.1x_5 \leq 0 \\ & 0.8x_1 + 0.8x_2 - 0.2x_3 - 0.2x_4 - 0.2x_5 \leq 0 \\ & 0.6x_5 - 0.4x_4 \geq 0 \\ & -0.2x_1 - 0.2x_2 - 0.2x_3 - 0.2x_4 + 0.8x_5 \geq 0 \\ & x_1, x_2, x_3, x_4, x_5 \geq 0 \end{aligned}$$

Here $t \in [0,1]$

RESULTS

Output of the problem (a)

Value of Parameter(t)	*x_1*	*X_2*	*x_3*	*X_4*	*x_5*	*Value of Objective Function Z(₦)*
t=0	*0.15*	*0.15*	*0.45*	*0.45*	*0.30*	*121.050*
t=0.1	*0.15*	*0.15*	*0.45*	*0.45*	*0.30*	*124.455*
t=0.2	*0.15*	*0.15*	*0.45*	*0.45*	*0.30*	*126.861*
t=0.3	*0.15*	*0.15*	*0.45*	*0.45*	*0.30*	*128.465*
t=0.4	*0.15*	*0.15*	*0.45*	*0.45*	*0.30*	*131.870*
t=0.5	*0.15*	*0.15*	*0.45*	*0.45*	*0.30*	*134.475*
t=0.6	*0.15*	*0.15*	*0.45*	*0.45*	*0.30*	*136.880*
t=0.7	*0.15*	*0.15*	*0.45*	*0.45*	*0.30*	*138.485*
t=0.8	*0.15*	*0.15*	*0.45*	*0.45*	*0.30*	*141.000*
t=0.9	*0.15*	*0.15*	*0.45*	*0.45*	*0.30*	*144.405*
t=1	*0.15*	*0.15*	*0.45*	*0.45*	*0.30*	*147.000*

Output of the problem (b)

Value of Parameter(t)	*x_1 (shash)*	*x_2 (bry-tex)*	*x_3 (plus)*	*x_4 (action)*	*x_5 (bonus)*	*Value of Objective Function Z(₦)*
T=0	*15*	*15*	*45*	*45*	*30*	*12105.00*
T=0.1	*15*	*15*	*45*	*45*	*30*	*12445.50*
T=0.2	*15*	*15*	*45*	*45*	*30*	*12686.10*
T=0.3	*15*	*15*	*45*	*45*	*30*	*12846.50*
T=0.4	*15*	*15*	*45*	*45*	*30*	*13187.0*
T=0.5	*15*	*15*	*45*	*45*	*30*	*13447.50*
T=0.6	*15*	*15*	*45*	*45*	*30*	*136.880*
T=0.7	*15*	*15*	*45*	*45*	*30*	*13848.50*
T=0.8	*15*	*15*	*45*	*45*	*30*	*14100.00*
T=0.9	*15*	*15*	*45*	*45*	*30*	*14440.50*
T=1	*15*	*15*	*45*	*45*	*30*	*14700.00*

ANALYSIS OF RESULTS

From the Table 1 (b) above for output of problem I it indicates that the production of x_1, x_2 ,x_3 ,x_4 and x_5 will be multiplied by 100 in order to give the exact value:

In 1(a) this means that 15 units of shash , 15 units of bry-tex, 45 units of white plus, 45 units of action and 30 units of bonus detergent should be produced to give a profit of 12105.00 and so on from the table above with varying parameter, t. As t varies also the values of the objective function vary proportionately.

Also from output in problem II table summaries all the units produced of soap to maximize the profit of the company.

CONCLUSION

Any company that want efficient and organized production schedules should employed the use of this method of linear programming. It is accurate and fast in identifying all the production schedules of a company.

REFERENCES

1. Balagurusamy. 2006. *Programming with JAVA (A Primer) , 2nd Edition.*

2. Banerjee, I and M.G. Ierapetriton. 2003. “Parametric Process Synthesis for General Nonlinear Models. *Computers and Chemical Engineering*. 27(10):1499-1512.

3. Borrelli, F., Bemporad, A., and M. Morari. 2003. “Geometric Algorithm for Multiparametric Linear

Programming". *Journal of Optimization Theory and Applications*. 118(3):515-540.

4. Choo, E.U. and Atkins, D.R. 1982. "Bicriteria Linear Fractional Programming". *Journal of Optimization Theory and Applications*. 36(2):203-220.

5. Galperin, E. and Jimenez Guerra, P. 2001. "Duality of Nonscalarized Multiobjective Linear programs (Dual Balance, Level Sets, and Dual Clusters of Optimal Vectors)". *Journal of Optimization Theory and Applications*. 108(1):109-137.

6. Jenkins, L. 1982. "Parametric Mixed Integer Programming: Application to Solid Waste Management. *Management Science.* 28(11):1270-1284.

7. Kanti Swarup, Gupta, P.K., and Man Mohan. 2007. *Tracts in Operations Research, 13th Edition*.

8. Magnanti, T.L. and Orlin, J.B. 1988. "Parametric Linear Programming and Anti-Cyclying Pivoting Rules". *Mathematical Programming*. 41:317-325, North-Holland.

9. Martin, D.H. 1975. "On the Continuity of the Maximum in Parametric Linear Programming". *Journal of Optimization Theory and Applications*. 17(3/4): 205-210.

10. NASCO. 2008. *NASCO Household Manual on Soap, Detergent and Raw Material*. NASCO Ltd. Jos, Nigeria.

11. Sharma, J.K. 2007. *Operations Research Theory and Applications ; 3rd Edition*. New Delhi, India.

12. Simon, R., Philip, H., and Micheal, E. 1999. *The Complete JAVA 2 Certificate Study Guide*. http://www.sybex.com.

13. Thuan, L.V. and Luc, D.T. 2000. "On Sensitivity in Linear Multiobjective Programming". *Journal of Optimization Theory and Applications*.107(3):615-626.

ABOUT THE AUTHORS

Adamu Wakili is Lecturer II in Department of Mathematical Sciences, Gombe State University, Gombe and is currently running a Ph.D. program in Abubakar Tafawa Balewa University Bauchi at the Department of Mathematics program. The Ph.D. work is on parametric linear programming and its applications.

Professor M.S. Sesay is a professor of Mathematics in the Department of Mathematics Programme in Abubakar Tafawa Baleaw University, Bauchi and his area of specialization is Operations Research.

Dr. M.S. Atureta is a senior lecturer in the Department of Mathematics Program in Abubakar Tafawa Baleaw University, Bauchi and his area of specialization is Operations Research.

SUGGESTED CITATION

Wakili, A., M.S. Sesay, and M.S. Atureta. 2009. "Parametric Approach to Detergent Production Using a Developed Algorithm". *Pacific Journal of Science and Technology.* 10(2):289-294.

Pacific Journal of Science and Technology