Application of Queuing Theory to Waiting Time of Out-Patients in Hospitals.

R.A. Adeleke*, O.D. Ogunwale, and O.Y. Halid.

Department of Mathematical Sciences, University of Ado-Ekiti, Ado-Ekiti, Ekiti State, Nigeria.

E-mail: waleod@yahoo.com

ABSTRACT

This paper considers the waiting of patients in university health centers as a single-channel queuing system with Poisson arrivals and exponential service rate where arrivals are handled on a first come first serve basis. Hence, the m/m/1 queuing system is however proposed.

The average number of patients, the average time spent by each patient as well as the probability of arrival of patients into the queuing system will be obtained.

(Keywords: single channel queuing system, Poisson arrivals, exponential service rates)

INTRODUCTION

The existence of any nation is a function of the survival of its citizens and in turn a function of adequate health care programs of its citizenry. Health, no doubt has a great influence on the economy. As a result, the government established hospitals, primary health care centers, federal medical centers, university teaching hospitals, and so on, to improve the health care of Nigerians at affordable cost.

Also, several health policies and laws were formulated and adopted to make these aforementioned centers functional. These health policies also brought about the establishment of global and local health organization such as World Health Organization (WHO). United Nations International Children Emergency Fund (UNICEF) and a host of others.

Despite all these efforts, it should be noted that, there are still some avoidable problems which undermine their success of this sector. One of the most frequent of them is the problem of waiting lines (queues) found in hospitals.

Queuing is a very volatile situation which cause unnecessary delay and reduce the service effectiveness of establishments. Apart from the time wasted, it also leads to breakdown of law and order. Many lives and property had been lost in queues at filling stations in the past.

Queues in hospitals often have severe consequences. For instance, delay in treatment of asthma, diabetes, and cardiac disease patients often lead to complications and eventual death. (*The World Bank Illustrated Home Medical Encyclopedia*, 1998). In light of this, there is need for a critical evaluation of patient waiting time as well as reducing or eliminating it.

METHODOLOGY

Data on arrival times, time service begins, time service ends, and departure time of 100 patients was collected over 14 days. This data will enable us to obtain the arrival rate, the service rate, and the traffic intensity of the patients using results from the birth and death model (which is synonymous to arrival and departure).

MODEL SPECIFICATION

The m/m/1 Queue (Single-Channel Queuing System). In this queuing system, the customers arrive according to a Poisson process with rate λ.

The time it takes to serve every customer is an exponential random variable with parameter μ.

The service times are mutually independent and further independent of the inter arrival times. When a customer enters an empty system, his service starts at once and if the system is non-empty, the incoming customer joins the queue.

When a service completion occurs, a customer from the queue if any, enters the service facility at once to get served.

THEOREM

The process $(X(t),\ t \geq 0)$ is a birth and death process with birth rate $\lambda_i = \lambda\ \forall\ \ i \geq 0$ and death rate $\mu_i = \mu\ \ \forall\ \ i \geq 1$.

Proof

Because of the exponential distribution of the inter arrival times and of the service times, it is obvious that $(X(t),\ t \geq 0)$ is a Markov process. On the other hand, since the probability of having two events (departure and arrival) in the interval if time $(t, t+h)$ is $0(h)$. We therefore have:

$$\left.\begin{array}{l} P[X(t+h) = i+1 \mid X(t) = i] = \lambda h + 0(h)\ \forall\ \ i \geq 0 \\ P[X(t+h) = i-1 \mid X(t) = i] = \mu h + 0(h)\ \forall\ \ i \geq 1 \\ P[X(t+h) = i \mid X(t) = i] = 1 - (\lambda + \mu)h + 0(h)\ \forall\ \ i \geq 1 \\ P[X(t+h) = i \mid X(t) = i] = 1 - \lambda h + 0(h)\ ,\ \ i = 0 \\ P[X(t+h) = i \mid X(t) = i] = 0(h)\ ,\ \ |\, j - i \,| \geq 2 \end{array}\right\} \qquad \textbf{(1)}$$

This shows that $(X(t),\ t \geq 0)$ is a birth and death process.

Suppose $\pi(i),\ i \geq 0$ is the density function of the number of customers in the system at a steady-state, the balanced equation for this birth and death process is:

$$\lambda\ \pi(0) = \mu\ \pi(1) \qquad \textbf{(2)}$$

Generally,

$$(\lambda + \mu)\pi(i) = \lambda\pi(i-1) + \mu\pi(i+1)\ \forall\ \ i \geq 1 \qquad \textbf{(3)}$$

We define $\rho = \dfrac{\lambda}{\mu}$ **(4)**

referred to as the traffic intensity called the mean quantity of work brought to the system per unit time.

A direct application of (4) yields a stationary queue-length density function of an m/m/1 queue given by:

$$\pi(i) = (1-\rho)\rho^i,\ i \geq 0,\ \rho < 1 \qquad \textbf{(5)}$$

Therefore, the stability condition $\rho < 1$ simply says that the system is stable if the work that is brought to the system per unit time is strictly smaller than the processing rate.

It is noteworthy that the queue will empty infinitely many times when the system is stable. That is, $i = 0$ from (5)

$$\pi(0) = 1 - \rho\ > 0 \qquad \textbf{(6)}$$

We observe that (5) which is the density function of the queue length in a steady-state is a geometric distribution. We then find,

$$E(x) = \frac{\rho}{1-\rho} \qquad \textbf{(7)}$$

called the mean number of customers.

We also observe that $E(x) \to \infty \ as \ \rho \to 1$, so that in practice if the system is not stable, then the queue will explode. We also find from (5) that:

$$V(x) = \frac{\rho}{(1-\rho)^2} \quad \textbf{(8)}$$

Again, we find from (5) that the probability that the queue exceed say i customers in steady-state is

$$P(x \geq i) = \rho^i \quad \textbf{(9)}$$

We adopt a single-channel queuing system (m/m/1) with Poisson arrivals and exponential service rate and arrivals are handled on a first come first serve basis. In this queuing system, the average arrival rate is less than the average service rate (i.e., $\lambda < \mu$).

If $\lambda < \mu$, there would be an unending queue. The following formulas are developed for this system. The average number of customers on the queue at any given time t is:

$$\frac{\rho^2}{1-\rho} \quad \textbf{(10)}$$

The average number of customers waiting to be served at any time t is:

$$\frac{1}{1-\rho} \quad \textbf{(11)}$$

The average number of customers in the system is:

$$\frac{\rho}{1-\rho} \quad \textbf{(12)}$$

The average time in queue (before service is rendered) is:

$$\frac{\rho}{\mu(1-\rho)} \quad \textbf{(13)}$$

The average time in the system (on queue and receiving service) is:

$$\frac{1}{\mu(1-\rho)} \quad \textbf{(14)}$$

RESULTS

The arrival times as well as the time service began and ended for 100 patients in the University of Ado-Ekiti, Health Center were observed and recorded between 8.00am and 5.00pm.

A total of 14days were used for the data collection. The waiting times and service times were obtained by subtracting arrival time from the time service began for each day. Similarly, service time was found by subtracting time service began from when it ended. Therefore:

- Total waiting times = 945 minutes
- Total service times = 798 minutes taken place in 945 mins and service was rendered in 798 mins.
- This follows the arrival of 100 patients

We arrive at the following;

1. The arrival rate $\lambda = \dfrac{\text{total of patients}}{\text{total waiting times}} \ \dfrac{100}{945} = 0.1058$

2. The service rate $\mu = \frac{\text{total no of patients}}{\text{total service times}} = \frac{100}{798} = 0.1253$

3. Thus, traffic intensity $\rho = \frac{\lambda}{\mu} = 0.8444$

4. The average number of patients queue = $\frac{\rho^2}{1-\rho} = 4.5823 = 5$

5. The average number of patients in queue when queue exist = $\frac{1}{1-\rho} = 6.4267 \approx 6$

6. The average number of patients in the systems = $\frac{\rho}{1-\rho} = 5.4267 \approx 5$

7. The average time in queue [before service is rendered $\frac{\rho}{\mu(1-\rho)} = 43.3099 \approx 43 \min utes$

Alternatively, we calculate it as $\frac{\text{Average no of patients in the system}}{\text{the service rate}} = \frac{5.4267}{0.1253} \approx 43 \min utes$

8. The average time of the system (on queue and receiving treatment) $\frac{1}{\mu(1-\rho)} = 51.2908 \approx 51 \min utes$

9. Average time of service = Average time in the system –average time in queue

= 8 minutes

10. The probability of queueing on arrival =Traffic intensity = $\frac{\lambda}{\mu} = 0.8444$

11. The probability of not queuing on arrival =$1-\rho$ =0.1556

12. The probability that there are n patient in the system $(1-\rho)\rho^n$. Hence, the probability that there are 2 patients in the systems =0.1109

13. The probability that there are more than n patients in the system is ρ^n. Hence, the probability that there are more than 2 patients in the system is 0.7130

DISCUSSION OF RESULTS

The traffic intensity ρ =0.8444 is the probability of patients queuing on arrival which clearly indicates a higher possibility of patients waiting for treatment since the doctor is busy rendering service to a patient that has earlier arrived. This also shows that the service in the health centre is not 100% efficient.

It then follows that there will always be queue since result (8) is greater than (7). That is the average time in the queue system (both on queue receiving service) is greater than the average time in queue before service is rendered.

CONCLUSION AND RECOMMENDATION

The queue theory is a useful statistical technique for solving peculiar problems. Its applications in the organization are indispensable. The queuing problems encountered at the University of Ado-Ekiti Health Centre is similar to what is encountered in older centers as well as other government hospitals across the country. Excessive waste of time in the hospitals or health centers may lead to patients' health complications and in some cases eventual death which may be avoided.

As a result, it is recommended that more doctors should be deployed to these centers so as to convert the single-channel queuing units to multi-channel queuing units. This will offer service on arrival. It is also recommended that more health care centers should be created to take care of all categories of patients (students or members of staff) in the university community. Again, more paramedical officers should be deployed to these centers. This will take care of patients' preliminary tests or service before they see the doctors. This will reduce the service time spent by the doctors in attending to patients and hence the service efficiency.

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