Nonlinear Trigonometric Transformation Time Series Modeling.

K.A. Bashiru[1*], O.E. Olowofeso[2], and S.A. Owabumoye[2]

[1]Mathematical and Physical Sciences Department, Osun State University, PMB. 4494 Osogbo, Nigeria.
[2]Mathematical Sciences Department, Federal University of Technology, PMB 704 Akure, Nigeria.

E-mail: kehindeadekunle2@yahoo.com
Telephone: 23408034997776

ABSTRACT

The study examined Nonlinear Trigonometric Transformation as well as Augmented Nonlinear Trigonometric Transformation with polynomial of order two. The two models were practically tested and compared using daily mean temperature for 6 major towns in Nigeria with different rate of missing values. The results were used to determine the consistency and efficiency of the models formulated.

(Keywords: nonlinear time series, polynomial, consistency, efficiency, missing value, model and forecasting)

INTRODUCTION

Time Series Analysis is an important technique used in many disciplines, such as physics, engineering, finance, economics, meteorology, biology, medicine, hydrology, oceanography, and geomorphology (Terasvirta and Anderson, 1992). This technique is mainly used to infer properties of a system by the analysis of a measured time record (data) Priestley (1988). This is done by fitting a representative model to the data with an aim of discovering the underlying structure as closely as possible. Traditional time series analysis is based on assumptions of linearity and stationarity. However, there has been a growing interest in studying nonlinear and non-stationary time series models in many practical problems because the nature of many phenomena in physics, economics, and finance is inherently non-linear.

The first, and the simplest reason for this, is that many real world problems do not satisfy the assumptions of linearity and/or stationarity. For example, the financial markets and trends that are influenced by the climatic factor like daily temperature, amount of rainfall and intensity of sun are the areas where there is a greater need to explain behaviors that are far from being even approximately linear. Therefore, the need for the further development of the theory and applications for nonlinear models is essential.

In general time series analysis, Box and Jenkins (1970) and Brock and Potter (1993) are known for numbers of nonlinear features such as cycles, asymmetries, bursts, jumps, chaos, thresholds, hetero-scedasticity and mixtures of these have to be taken into account. A problem arises directly from a suitable definition of the nonlinear model because not every model is linear. This class clearly encompasses a large number of possible choices.

For many real time series data, nonlinear models are more appropriate than linear models for accurately describing the dynamic of the series and making multi-step-ahead forecast (see for example, Tsay, 1986; Barnett, Powell, and Tauchen, 1991; and Olowofeso, 2006).

As many applications in financial, physics, engineering, economics, meteorology, biology, medicine, hydrology, oceanography, and geomorphology, etc., data are nonlinear (Bates and Watts, 1988; De Gooijer and Kumar, 1992: and Sugihara and May,1990). Nonlinear models are appropriate for forecasting and accurately describing returns and volatility. Since there are an enormous number of nonlinear models available for modeling and forecasting economic time series, choosing the best model for a particular application is daunting (Robinson, 1983).

Non-linear time series analysis is a rapidly developing area and there have been major developments in model building and forecasting (De Gooijer and Kumar, 1992).

RESEARCH METHODOLOGY

The model of Gallant (1981) was adopted. "Augmented Nonlinear Parametric Time Series Model (ANPTSM)" while the second model was formulated model based on the Least Square Method Modified Nonlinear Trigonometric Transformation Time Series Model (MNTTTSM).

DATA COLLECTION

Data used in this work were daily mean of temperature from 1987 to 1996 for Ikeja, Ibadan, Ilorin, Minna, and Zaria. The data were collected from Meteorological Centre- Oshodi Lagos.

MODEL FORMULATION

Considering the format below:

t/i	1	2	3	4	5	6	7	8	9	10	11	12	**$\Sigma x_i/12$**
1	$x_{1,1,k}$	$x_{1,2,k}$	$x_{1,3,k}$	$x_{1,4,k}$	$x_{1,5,k}$	$x_{1,6,k}$	$x_{1,7,k}$	$x_{1,8,k}$	$x_{1,9,k}$	$x_{1,10,k}$	$x_{1,11,k}$	$x_{1,12,k}$	$\overline{X}_1$
2	$x_{2,1,k}$	$x_{2,2,k}$	$x_{2,3,k}$	$x_{2,4,k}$	$x_{2,5,k}$	$x_{2,6,k}$	$x_{2,7,k}$	$x_{2,8,k}$	$x_{2,9,k}$	$x_{2,10,k}$	$x_{2,11,k}$	$x_{2,12,k}$	$\overline{X}_2$
.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.
t	$x_{t,1,k}$	$x_{t,2,k}$	$x_{t,3,k}$	$x_{t,4,k}$	$x_{t,5,k}$	$x_{t,6,k}$	$x_{t,7,k}$	$x_{t,8,k}$	$x_{t,9,k}$	$x_{t,10,k}$	$x_{t,11,k}$	$x_{t,12,k}$	$\overline{X}_t$
$^*_{1,k}$	$x^*_{2,k}$	$x^*_{3,k}$	$x^*_{4,k}$	$x^*_{5,k}$	$x^*_{6,k}$	$x^*_{7,k}$	$x^*_{8,k}$	$x^*_{9,k}$	$x^*_{10,k}$	$x^*_{11,k}$	$x^*_{12,k}$	$\Sigma x^*_{i,k}/12$	

for a particular year. In this Model, up to 9 years is considered and the model is formulated base on the data as shown below:

t/i_k	1	2	3	.	.	.	i_k	**$\Sigma x_i/ik$**
1	$x_{1,1,1}$	$x_{1,2,1}$	$x_{1,3,1}$	.	.	.	$x_{1,i,k}$	$\overline{X}_{1,k}$
2	$x_{2,1,1}$	$x_{2,2,1}$	$x_{2,3,1}$	.	.	.	$x_{2,i,k}$	$\overline{X}_{2,k}$
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
t	$x_{t,1,1}$	$x_{t,2,1}$	$x_{t,3,1}$	.	.	.	$x_{t,i,k}$	$\overline{X}_{t,k}$
	$x^*_{1,k}$	$x^*_{2,k}$ $x^*_{3,k}$	.	.	.	$x^*_{i,k}$	$\Sigma x^*_{i,k}/ik = X_*$	

ASSUMPTION AND NOTATION FOR THE MODELS:

Let:

$X_{t,i,k}$: denotes the value of occurrence for day t of Month i in the year k.
$\overline{X}_{t,k}$: denote the mean occurrence for day t of year k.
$X^*_{i,k}$: denotes mean occurrence for month i of year k.
X^{*y}_K : denotes overall yearly mean for the sampled Month.
X^{*m}_i : denotes overall monthly mean for the sampled year.
t: denotes the position of the day from the first day of the Month. $1 \leq t \leq 31$
t_i: denotes the sum of days in month i for $1 \leq i \leq 12$
t_{ik}: denotes the sum of days from the initial sampled Month of initial sampled year to month i of year k.
t_i^*: denotes the sum of days from the initial sampled Month to Month i
k: denotes the position of a particular year from an initial sample year for $-\infty \leq k \leq \infty$
n: denotes the number of sampled years.
m: denotes the number of sampled months.
X_*= Grand Mean occurrence for k year(s) examined.

The first model was reviewed based on the assumption that the sum of the occurrences are to be presented monthly, where i^{th} Month represents the Month i for $1 \le i \le 12$ which is to be modeled using the number of days in each month i.e

	Jan	Feb	Mar	April	May	Jun	Jul	Aug	Sept	Oct	Nov	Dec	Jan	…
i	1	2	3	4	5	6	7	8	9	10	11	12	13	… i_k
t	31	$28^1/_4$	31	30	31	30	31	31	30	31	30	31	31	… t_i
t_i	31	$59^1/_4$	$90^1/_4$	$120^1/_4$	$151^1/_4$	$181^1/_4$	$212^1/_4$	$243^1/_4$	$273^1/_4$	$304^1/_4$	$334^1/_4$	$365^1/_4$	$396^1/_4$	…t

AUGMENTED NONLINEAR PARAMETRIC TIME SERIES MODEL (ANPTSM)

Basically, Trigonometric (Sine & Cosine) transformation augmented with Polynomial of order two was applied to formulate the model across the year i.e Monthly mean sample and the Least Square Method are used for estimating its Parameters as described below:

Let the equation be of the form:

$$X_{t,i,k} = a_1 + a_2\, t\mathrm{Sin}(t_{ik}) + a_3\, t^2 \mathrm{Cos}(t_{ik}) + \varepsilon_i \quad 1 \le i \le 12 \tag{1}$$

The expected value of $X_{t,i,k}$ is $X^*_{i,k}$ then the equation can be reformed as below to estimate the parameters; a_1, a_2 and a_3 using Least Square Method.

$$X^*_{i,k} = a_1 + a_2\, t_i\mathrm{Sin}(t_{ik}) + a_3\, t_i^2 \mathrm{Cos}(t_{ik}) + \varepsilon_i \quad 1 \le i \le 12 \tag{2}$$

$$\varepsilon_i = X^*_{i,k} - (a_1 + a_2\, t_i\, \mathrm{Sin}(t_{ik}) + a_3\, t_i^2 \mathrm{Cos}(t_{ik}))) \tag{3}$$

Let $\boldsymbol{\Sigma}\varepsilon_i^2 = S$

$$S = \boldsymbol{\Sigma}(X^*_{i,k} - (a_1 + a_2 t_i\mathrm{Sin}(t_{ik}) + a_3 t_i^2 \mathrm{Cos}(t_{ik})))^2 \tag{4}$$

Differentiate (4) with respect to a_1, a_2, a_3, a_3,

as $\frac{\partial S}{\partial a_1} \to 0$ we obtained

$$\boldsymbol{\Sigma} X^*_{i,k} = ma_1 + a_2\boldsymbol{\Sigma}t_i\mathrm{Sin}(t_{ik}) + a_3\boldsymbol{\Sigma}t_i^2 \mathrm{Cos}(t_{ik}) \tag{5}$$

where m is the number of the monthly sample mean examined. Similarly:

as $\frac{\partial S}{\partial a_2} \to 0$ we obtained
$$\boldsymbol{\Sigma}t_i\mathrm{Sin}(t_{ik})\, X^*_{i,k} = a_1\boldsymbol{\Sigma}t_i\mathrm{Sin}(t_{ik}) + a_2\boldsymbol{\Sigma}t_i^2\mathrm{Sin}^2(t_{ik}) + a_3\boldsymbol{\Sigma}t_i^3 \mathrm{Sin}(t_{ik})\mathrm{Cos}(t_{ik}) \tag{6}$$

as $\frac{\partial S}{\partial a_3} \to 0$ we obtained
$$\boldsymbol{\Sigma}t_i^2\mathrm{Cos}(t_{ik})\, X^*_{i,k} = a_1\boldsymbol{\Sigma}t_i^2\mathrm{Cos}(t_{ik}) + a_2\, \boldsymbol{\Sigma}t_i^3 \mathrm{Sin}(t_{ik})\mathrm{Cos}(t_{ik}) + a_3\boldsymbol{\Sigma}t_i^4\mathrm{Cos}^2(t_{ik}) \tag{7}$$

Solving equation (5), (6), and (7), simultaneously, using Cramer's Rule, we obtained:

$$\begin{aligned}\Delta_0 = {} & m\{\boldsymbol{\Sigma}t_i^2\mathrm{Sin}^2(t_{ik})\, \boldsymbol{\Sigma}t_i^4\mathrm{Cos}^2(t_{ik}) - (\boldsymbol{\Sigma}t_i^3 \mathrm{Sin}(t_{ik})\mathrm{Cos}(t_{ik}))^2 \} \\ & - \boldsymbol{\Sigma}t_i\mathrm{Sin}(t_{ik})\, \{\boldsymbol{\Sigma}t_i\mathrm{Sin}(t_{ik})\, \boldsymbol{\Sigma}t_i^4\mathrm{Cos}^2(t_{ik}) - \boldsymbol{\Sigma}t_i^2\mathrm{Cos}(t_{ik})\, \boldsymbol{\Sigma}t_i^3 \mathrm{Sin}(t_{ik})\mathrm{Cos}(t_{ik})\} \\ & + \boldsymbol{\Sigma}t_i^2\mathrm{Cos}(t_{ik})\{\boldsymbol{\Sigma}t_i\mathrm{Sin}(t_{ik})\boldsymbol{\Sigma}t^3 \mathrm{Sin}(t_i)\mathrm{Cos}(t_{ik}) - \boldsymbol{\Sigma}t_i^2\mathrm{Cos}(t_i)\, \boldsymbol{\Sigma}t_i^2\mathrm{Sin}^2(t_{ik})\}\end{aligned} \tag{8}$$

Δ_1 $= \mathbf{\Sigma}\ X^*_{i,k}\ \{\mathbf{\Sigma} t_i^2 Sin^2(t_{ik})\mathbf{\Sigma} t_i^4 Cos^2(t_{ik})-(\mathbf{\Sigma} t_i^3 Sin(t_{ik})Cos(t_{ik}))^2\}-$
$\mathbf{\Sigma} t_i Sin(t_{ik})\{\mathbf{\Sigma} t_i Sin(t_{ik})\ X^*_{i,k}\ \mathbf{\Sigma} t^4 Cos^2(t_{ik})-\mathbf{\Sigma} t_i^2 Cos(t_{ik})\ X^*_{i,k}\ \mathbf{\Sigma} t_i^3 Sin(t_{ik})Cos(t_{ik})\}$
$+\mathbf{\Sigma} t_i^2 Cos(t_{ik})\{\mathbf{\Sigma} t_i Sin(t_{ik})\ X^*_{i,k}\ \mathbf{\Sigma} t_i^3 Sin(t_{ik})Cos(t_{ik})-\mathbf{\Sigma} t_i^2 Cos(t_{ik})\ X^*_{i,k}\ \mathbf{\Sigma} t_i^2 Sin^2(t_{ik})\}$ **(9)**

Δ_2 $=m\{\mathbf{\Sigma} t_i Sin(t_{ik})\ X^*_{i,k}\ \mathbf{\Sigma} t_i^4 Cos^2(t_{ik})-(\mathbf{\Sigma} t_i^2 Cos(t_{ik})\ X^*_{i,k}\ \mathbf{\Sigma} t_i^3\ Sin(t_{ik})Cos(t_{ik})\}$
$-\ \mathbf{\Sigma}\ X^*_{i,k}\ \{\mathbf{\Sigma} t_i Sin(t_{ik})\ \mathbf{\Sigma} t_i^4 Cos^2(t_{ik})-\ \mathbf{\Sigma} t_i^2 Cos(t_{ik})\ \mathbf{\Sigma} t_i^3\ Sin(t_{ik})Cos(t_{ik})\}$
$+\mathbf{\Sigma} t_i^2 Cos(t_{ik})\ \{\mathbf{\Sigma} t_i Sin(t_{ik})\ \mathbf{\Sigma} t_i^2 Cos(t_{ik})\ X^*_{i,k} - \mathbf{\Sigma} t_i^2 Cos(t_{ik})\ \mathbf{\Sigma} t_i Sin(t_{ik})\ X^*_{i,k}\ \}$ **(10)**

Δ_3 $=$ $= m\{\mathbf{\Sigma} t_i^2 Sin^2(t_{ik})\ \mathbf{\Sigma} t_i^2 Cos(t_{ik})\ X^*_{i,k}\ -(\mathbf{\Sigma} t_i^3\ Sin(t_{ik})Cos(t_i)\ \mathbf{\Sigma} t_i Sin(t_{ik})\ X^*_{i,k}\ \}$
$-\ \mathbf{\Sigma} t_i Sin(t_{ik})\ \{\mathbf{\Sigma} t_i Sin(t_{ik})\ \mathbf{\Sigma} t_i^2 Cos(t_{ik})\ X^*_{i,k} - \mathbf{\Sigma} t_i^2 Cos(t_{ik})\ \mathbf{\Sigma} t_i Sin(t_{ik})\ X^*_{i,k}\ \}$
$+\mathbf{\Sigma}\ X^*_{i,k}\ \{\mathbf{\Sigma} t_i Sin(t_{ik})\ \mathbf{\Sigma} t_i^3\ Sin(t_{ik})Cos(t_{ik})-\ \mathbf{\Sigma} t_i^2 Cos(t_{ik})\ \mathbf{\Sigma} t_i^2 Sin^2(t_{ik})$ **(11)**

Therefore, from equation 3.7, 3.8, 3.9 and 3.10, we obtained:

$$a_1 = \frac{\Delta_1}{\Delta_0} \tag{12}$$

$$a_2 = \frac{\Delta_2}{\Delta_0} \tag{13}$$

$$a_3 = \frac{\Delta_3}{\Delta_0} \tag{14}$$

Substitute (12), (13), and (14) into (2), we obtained:

$$\overset{\Lambda}{X}{}^*{}_{i,k} = \frac{\Delta_1}{\Delta_0} + \frac{\Delta_2}{\Delta_0} tSin(t_{ik}) + \frac{\Delta_3}{\Delta_0} t^2 Cos(t_{ik}) \,. \tag{15}$$

Since $X^*_{i,k}$ is the expected value of $X_{t,i,k}$, Equation (15) can be rewrite as:

$$\overset{\Lambda}{X}{}_{t,i,k} = \frac{\Delta_1}{\Delta_0} + \frac{\Delta_2}{\Delta_0} tSin(t_{ik}) + \frac{\Delta_3}{\Delta_0} t^2 Cos(t_{ik}) \tag{16}$$

The model equations (15) and (16) could only be visible provided that there is an occurrence within a month of any sampled year.

MODIFIED NONLINEAR TRIGONOMETRIC TRANSFORMATION TIME SERIES MODEL (MNTTTSM)

In situation where a whole monthly data is missing, the above model may be difficult to apply. Therefore the model for such occurrence was formulated below. If the data in (3) above are reformed such that the monthly mean are as follow;

k/i	1	2	3	4	5	6	7	8	9	10	11	12	**$\Sigma x_i/12$**
1	$X^*_{1,1}$	$X^*_{2,1}$	$X^*_{3,1}$	$X^*_{4,1}$	$X^*_{5,1}$	$X^*_{6,1}$	$X^*_{7,1}$	$X^*_{8,1}$	$X^*_{9,1}$	$X^*_{10,1}$	$X^*_{11,1}$	$X^*_{12,1}$	X^{*y}_1
2	$X^*_{1,2}$	$X^*_{2,2}$	$X^*_{3,2}$	$X^*_{4,2}$	$X^*_{5,2}$	$X^*_{6,2}$	$X^*_{7,2}$	$X^*_{8,2}$	$X^*_{9,2}$	$X^*_{10,2}$	$X^*_{11,2}$	$X^*_{12,2}$	X^{*y}_2
.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.
k	$X^*_{1,k}$	$X^*_{2,k}$	$X^*_{3,k}$	$X^*_{4,k}$	$X^*_{5,k}$	$X^*_{6,k}$	$X^*_{7,k}$	$X^*_{8,k}$	$X^*_{9,k}$	$X^*_{10,k}$	$X^*_{11,k}$	$X^*_{12,k}$	X^{*y}_k
	x^{*m}_1	x^{*m}_2	x^{*m}_3	x^{*m}_4	x^{*m}_5	x^{*m}_6	x^{*m}_7	x^{*m}_8	x^{*m}_9	x^{*m}_{10}	x^{*m}_{11}	x^{*m}	$\frac{\Sigma x^{*m}_{,i}}{12} = X_*$

$X^*_{i,k}$ = a + b sin(t_i*)+ ε_i where $1 \le i \le 12$, $1 \le t_i \le 365^1/_4$ **(17)**

If the expected value of $X^*_{i,k}$ is X^{*m}_i then the Equation (17) can take the form of:
X^{*m}_i = a + b sin(t_i*)+ ε_i where $1 \le i \le 12$, $1 \le t_i^* \le 365^1/_4$ … **(18)**

An Ordinary Least Square Method was used in estimating the parameters a and b as shown below:

If $S_m = \varepsilon_i^2 = \mathbf{\Sigma} (X^{*m}_i - (a+b\sin(t_i^*))^2$ then differentiate with respect to a and b

$$\frac{\partial S_m}{\partial a} = -2\mathbf{\Sigma} (X^{*m}_i - a + b\sin(t_i^*))$$

As $\frac{\partial S_m}{\partial a} \to 0$

⇨ $\mathbf{\Sigma} X^{*m}_i = 12a + b\mathbf{\Sigma} \sin(t_i^*)$ **(19)**

Also, $\frac{\partial S_m}{\partial b} = -2\mathbf{\Sigma} (\sin(t_i^*) (X^{*m}_i - (a+b\sin(t_i^*)))$

As $\frac{\partial S_m}{\partial b} \to 0$

⇨ $\mathbf{\Sigma}\sin(t_i^*) X^{*m}_i = a\mathbf{\Sigma}\sin(t_i^*) + b\mathbf{\Sigma}\sin^2(t_i^*)$ **(20)**

Solving Equations (19) and (20) simultaneously, using Cramer's Rule we obtained:

$\Delta_4 = 12\Sigma Sin^2(t_i^*) - \left(\Sigma Sin\ (t_i^*)\right)^2$

$\Delta_5 = \Sigma X^{*m}{}_i \Sigma Sin^2(t_i^*) - \Sigma Sin(t_i^*) X^{*m}{}_i \Sigma Sin(t_i^*)$

$\Delta_6 = 12\Sigma Sin(t_i^*) X^{*m}{}_i - \Sigma X^{*m}{}_i \Sigma Sin(t_i^*)$

where parameters $a = \frac{\Delta_5}{\Delta_4} = \frac{\Sigma X^{*m}{}_i \Sigma Sin^2(t_i^*) - \Sigma Sin(t_i^*) X^{*m}{}_i \Sigma Sin(t_i^*)}{12\Sigma Sin^2(t_i^*) - \left(\Sigma Sin\ (t_i^*)\right)^2}$ **(21)**

and b = $\frac{\Delta_6}{\Delta_4} = \frac{12\Sigma Sin(t_i^*)X^{*m}{}_i - \Sigma X^{*m}{}_i \Sigma Sin(t_i^*)}{12\Sigma Sin^2(t_i^*) - \left(\Sigma Sin\ \ (t_i^*)\right)^2}$ **(22)**

Therefore, the model for Monthly occurrence is:

$$X^{*m}{}_i = \frac{\Delta_5}{\Delta_4} + \frac{\Delta_6}{\Delta_4} Sin(t_i^*) \quad \textbf{(23)}$$

Since $X^{*m}{}_i$ is an expected value for $X^*_{i,k}$ then equation 3.22 can be written as below:

$$X^*{}_{i,k} = \frac{\Delta_5}{\Delta_4} + \frac{\Delta_6}{\Delta_4} Sin(t_i^*) \quad \textbf{(24)}$$

Similarly, along the sampled year; $X^{*y}{}_K$ = c + d Sin(λk) for -∞≤k≤∞, 15≤λ≤75,. The λ must be chosen such that Σ ε_i =0, **Σ**ε_i^2 is as minimum as possible.

If $S_y = \varepsilon_i^2 = \mathbf{\Sigma}\ (X^{*y}{}_K\ \text{-}(c+d\sin(\lambda k))^2$ then $\frac{\partial S_y}{\partial c} = -2\mathbf{\Sigma}\ (X^{*y}{}_K\ \text{-}(c+d\sin(\lambda k))$

As $\frac{\partial S_y}{\partial c} \to 0$

⇨ $\mathbf{\Sigma}\ X^{*y}{}_K$ = nc + d**Σ** sin(λk) **(25)**

Also, $\frac{\partial S_y}{\partial d} = -2\mathbf{\Sigma}\ (\sin(\lambda k)\ X^{*y}{}_K\ \text{-}(c+d\sin(\lambda k)))$

As $\frac{\partial S_y}{\partial d} \to 0$

⇨$\mathbf{\Sigma}\sin(\lambda k)\ X^{*y}{}_K = c\mathbf{\Sigma}\sin(\lambda k) + d\mathbf{\Sigma}\sin^2(\lambda k)$ **(26)**

Solving Equations (25) and (26) simultaneously, using Cramer's Rule. Then we obtained:

$\Delta_7 =$ $\quad n\Sigma Sin^2(\lambda k) - (\Sigma Sin(\lambda k))^2$

$\Delta_8 =$ $\quad \Sigma X^{*y}{}_k \Sigma Sin^2(\lambda k) - \Sigma Sin(\lambda k) X^{*y}{}_k \Sigma Sin(\lambda k)$

$\Delta_9 =$ $\quad n\Sigma Sin(\lambda k) X^{*y}{}_k - \Sigma X^{*y}{}_k \Sigma Sin(\lambda k)$

Where the parameters:

c = $\frac{\Delta_8}{\Delta_7} = \frac{\Sigma X^{*y}{}_k \Sigma Sin^2(\lambda k) - \Sigma Sin(\lambda k) X^{*y}{}_k \Sigma Sin(\lambda k)}{n\Sigma Sin^2(\lambda k) - (\Sigma Sin(\lambda k))^2}$ **(27)**

and d = $\frac{\Delta_9}{\Delta_7} = \frac{n\Sigma Sin(\lambda k) X^{*y}{}_k - \Sigma X^{*y}{}_k \Sigma Sin(\lambda k)}{n\Sigma Sin^2(\lambda k) - (\Sigma Sin(\lambda k))^2}$ **(28)**

$\therefore\ X^{*y}{}_k = \frac{\Delta_8}{\Delta_7} + \frac{\Delta_9}{\Delta_7} Sin(\lambda k)$ **(29)**

The method of getting expected occurrences in contingency table of a Chi-square was applied using equation 3.23 and 3.28 to obtain the model of finding the daily occurrences for a particular month of a particular year. Therefore, the model for expected daily occurrences is:

$$X_{t,i,k} = \frac{n(X^{*m}{}_i)(X^{*y}{}_k)}{\Sigma X^{*y}{}_k} \qquad \textbf{(30)}$$

Substituting (24) and (29) into (30), we obtained:

$$X_{t,i,k} = \frac{n\left(\frac{\Delta_5}{\Delta_4} + \frac{\Delta_6}{\Delta_4} Sin(t_i^*)\right)\left(\frac{\Delta_8}{\Delta_7} + \frac{\Delta_9}{\Delta_7} Sin(\lambda k)\right)}{\sum\left(\frac{\Delta_8}{\Delta_7} + \frac{\Delta_9}{\Delta_7} Sin(\lambda k)\right)} \qquad \textbf{(31)}$$

MODEL ANALYSIS, RESULTS AND DISCUSSIONS

The data on the daily mean temperature for Ikeja, Ibadan, Ilorin, Minna and Zaria collected from Meteorological Centre- Oshodi Lagos were used. The parameters of the models were estimated and the fitted Models for each zones are given below.

Table 1: The Fitted Models for ANPTSM.

ZONES	Augmented Nonlinear Parametric Time Series Model (ANPTSM)
IKEJA	$26.88642582 + 0.047971536 t Sin(t_{ik}) - 0.000143793 t^2 Cos(t_{ik})$
IBADAN	$26.36612286 + 0.054847742 t Sin(t_{ik}) - 0.0000344912 t^2 Cos\, t_{ik}$
ILORIN	$26.2476883 + 0.048115874 t Sin(t_{ik}) - 0.000833551 t^2 Cos(t_{ik})$
MINNA	$25.72428 + 0.062853 t Sin(t_{ik}) - 0.00073 t^2 Cos(t_{ik})$
ZARIA	----

Source : the result of Analysis

Table 1, is the fitted models for Ikeja, Ibadan, Ilorin, and Minna for ANPTSM. Data of daily mean temperature was used to estimate their parameters. We could see from the table that the fitted model for Zaria could not be formulated due to the fact that many months data were missed.

Table 2: The Fitted Models for MNTTTSM.

ZONES	Modified Nonlinear Trigonometric Transformation Time Series Model (MNTTTSM)
IKEJA	$\frac{10(26.87226+1.420072Sin\,t_i^*)(26.88996+0.13116Sin60k)}{\sum_{K=1}^{10}(26.88996+0.1311Sin60k)}$
IBADAN	$\frac{10(26.36749+1.591834Sin\,t_i^*)(26.36761+0.13535Sin90k)}{\sum_{K=1}^{10}(26.36761+0.1311Sin90k)}$
ILORIN	$\frac{10(26.45708+1.816182Sin\,t_i^*)(26.40106+0.409024Sin45k)}{\sum_{K=1}^{10}(26.40106+0.409024Sin45k)}$
MINNA	$\frac{10(27.56143+2.508736Sin\,t_i^*)(27.67047+0.112148Sin90k)}{\sum_{K=1}^{10}(27.67047+0.112148Sin90k)}$
ZARIA	$\frac{10(24.98532+1.210108Sin\,t_i^*)(25.00445+0.222282Sin90k)}{\sum_{K=1}^{10}(25.00445+0.222282Sin90k)}$

Source: result of the Analysis.

Table 2 is the fitted models for Ikeja, Ibadan,Ilorin, Minna, and Zaria for MNTTTSM, using the data of their daily mean temperature to estimate their parameters. The fitted model for Zaria was formulated because MNTTTSM has the strength of addressing the problem of more missing values. Even though many months' data were missed in Zaria's daily mean temperature, MNTTTSM parameters could still be estimated. This is one of the advantages of MNTTTSM over ANPTSM.

Table 3: Correlation Coefficients.

ZONES	TYPES	ANPTSM		MNTTTSM	
		COEFFICIENTS	SIG.	COEFFICIENTS	SIG.
IKEJA	PEARSON COR.	0.607	.000	0.607	.000
	SPEARMAN'S RHO	0.620	.000	0.620	.000
IBADAN	PEARSON COR.	0.594	.000	0.575	.000
	SPEARMAN'S RHO	0.622	.000	0.584	.000
ILORIN	PEARSON COR.	0.503	.000	0.589	.000
	SPEARMAN'S RHO	0.560	.000	0.612	.000
MINNA	PEARSON COR.	0.596	.000	0.676	.000
	SPEARMAN'S RHO	0.656	.000	0.686	.000
ZARIA	PEARSON COR.	-	-	0.419	.000
	SPEARMAN'S RHO	-	-	0.445	.000

Source: result of the Analysis

From Table 3, we see that the results of the Pearson Product Moment Correlation coefficients and the Spearman Brown's Rank Order Correlation Coefficients for Ikeja, Ibadan, Ilorin, and Minna are highly and positively correlated which indicate the strong relationship between the actual data and estimated data of the daily mean temperature, while in Zaria the correlation coefficient for MNTTTSM is positive but low, which indicates the weak relationship between the actual and estimated daily mean temperature.

Apart from Ibadan, in which the correlation coefficient in ANPTSM is greater than MNTTTSM, and Ikeja, which has an equal Correlation Coefficient, then in all other Zones, the correlation coefficient in MNTTTSM is greater than ANPTSM. This indicates that MNTTTSM has stronger relationship between their actual and estimated values than ANPTSM. Though the relationship between actual and estimated values of MNTTTSM in Zaria is weak but positive, while that of ANPTSM could not be estimated at all due to too much missing values in the data. Also, all the correlations are significant at the 0.01 level (2-tailed).

From Table 4, the mean of the actual and estimated values for each zones of each models are almost equal. The differences there are due to approximation (truncate error) during calculation. Also, the mean of actual and estimated values of MNTTTSM are closer than that of ANPTSM, which implies that MNTTTSM estimates better than ANPTSM. It was also discovered from the Table 4 that the more missing values we have in data, the weaker the ANPTSM in estimating, while in MNTTTSM, the model would still maintain its precision.

From Table 5, we discovered that the standard deviations for MNTTTSM are lesser than that of ANPTSM which indicate that MNTTTSM is better in estimating and forecasting than ANPTSM. Similarly, apart form the standard error of ANPTSM and MNTTTSM of Ikeja which are equal, it was discovered that the standard errors for MNTTTSM were also lesser than that of ANPTSM which indicate that MNTTTSM is better in estimating and forecasting than ANPTSM for a Time Series data with missing values.

From Table 6, we discovered that at Ikeja we are 95% sure that the differences between the actual and estimated daily mean temperature would lie between -0.00749 and 0.07119 in ANPTSM and -0.00748 and 0.07118 in MNTTTSM. Similarly, at Ibadan; -0.0462 and 0.04473 in ANPTSM and -0.0496 and 0.04114 in MNTTTSM, at Ilorin; 0.1505 and 0.2723 in ANPTSM and -0.0592 and 0.05218 in MNTTTSM, at Minna; 1.1155 and 1.2601 in model I and -0.0689 and 0.05482 in MNTTTSM while in Zaria is between -0.0546 and 0.12310.

It was also discovered that the range of the confidence interval for MNTTTSM is lesser than that of ANPTSM for Ikeja and Ibadan. In Ilorin and Minna, the lower confidence intervals of differences for ANPTSM are positive which indicate that we are 95% sure that the differences between their actual and estimated daily temperature (actual – estimate) are positive while those of MNTTTSM are not so. This implies that the estimated daily temperatures for ANPTSM at Ilorin and Minna were under-estimated. Hence MNTTTSM is better in estimating and forecasting than ANPTSM when there are missing values in the time series.

Table 4: Comparison of ANPTSM and MNTTSM Means.

ZONES	N	ANPTSM		MNTTTSM	
		ACTUAL	ESTIMATED	ACTUAL	ESTIMATED
IKEJA	3660	26.9077	26.8759	26.9077	26.8759
IBADAN	3601	26.3749	26.3756	26.3749	26.3791
ILORIN	3580	26.4558	26.2443	26.4558	26.4593
MINNA	3362	27.5489	26.3611	27.5489	27.5559
ZARIA	3588	-	-	25.0514	25.0172

Source: result of the analysis

Table 5: Comparison of ANPTSM and MNTTSM's Standard Deviation and Standard Error of Differences.

ZONES	ANPTSM		MNTTTSM	
	STD. DEV.	STD. ERROR MEAN	STD. DEV.	STD. ERROR MEAN
IKEJA	1.2138	0.02006	1.2137	0.02006
IBADAN	1.3913	0.02319	1.3882	0.02313
ILORIN	1.8585	0.03106	1.6996	0.02841
MINNA	2.1381	0.03688	1.8293	0.03155
ZARIA	-	-	2.7152	0.04533

Source: result of the analysis

Table 6: Comparison of ANPTSM and MNTTTSM'S 95 % Confidence Interval of the Difference.

ZONES	ANPTSM		MNTTTSM	
	LOWER	UPPER	LOWER	UPPER
IKEJA	-0.00749	0.07119	-0.00748	0.07118
IBADAN	-0.0462	0.04473	-0.0496	0.04114
ILORIN	0.1505	0.2723	-0.0592	0.05218
MINNA	1.1155	1.2601	-0.0689	0.05482
ZARIA	-	-	-0.0546	0.12310

CONCLUSION

The first model was reviewed as stated in the introduction while the other model was formulated. The models were tested using daily mean temperatures at Ikeja, Ibadan, Ilorin, Minna, and Zaria, and the results were analyzed. It was discovered that ANPTSM could be used in forecasting provided the data is having few missing values. However MNTTTSM estimates forecasts better than ANPTSM in estimating missing values and forecasting. Conclusively, MNTTTSM is more efficient in estimating missing values and forecast better than ANPTSM based on the analysis and discussions above.

RECOMMENDATION

The beauty of a good model developed for Nonlinear Time Series Modeling is the ability to forecast more accurate data. The new method MNTTTSM is therefore recommended for numerical solutions for nonlinear model with missing values because of higher capacity of addressing missing values. It was also noted that the mathematical derivative of MNTTTSM is simpler than ANPTSM which could not even forecast better. Further research work could still be carried out by putting a condition in which a data is having a year or more missing values into consideration.

REFERENCES

1. Barnett, W.A., J. Powell, and G.E. Tauchen (eds.). 1991. "Nonparametric and Semi Parametric Methods in Econometrics and Statistics". *Proceedings of the 5th International Symposium in Economic Theory and Econometrics*. Cambridge University Press: Cambridge, UK.

2. Bates, D.M. and D.G. Watts. 1988. *Nonlinear Regression Analysis and its Applications*. Wiley: New York, NY.

3. Box, G.E.P. and G.M. Jenkins. 1970. *Time Series Analysis, Forecasting and Control*. Holden-Day: San Francisco, CA.

4. Brock, W.A. and S.M. Potter. 1993. " Nonlinear Time Series and Macro-econometrics". In: G.S. Maddala, C.R. Rao, and H.R. Vinod (eds.). *Handbook of Statistics, Vol. 11*. North-Holland: Amsterdam, Netherlands. 195-229.

5. De Gooijer, J.G. and K. Kumar. 1992. "Some Recent Developments in Non-Linear Time Series Modeling, Testing, and Forecasting". *International Journal of Forecasting*. 8:135-156.

6. Friedman, J.H. and W. Stuetzle. 1981. "Projection Pursuit Regression". *Journal of the American Statistical Association*. 76: 817-823.

7. Gallant, A.R. 1987. *Nonlinear Statistical Models*. Wiley: New York, NY.

8. Gallant, A.R. and G. Tauchen. 1989. "Semi Nonparametric Estimation of Conditionally Constrained Heterogeneous Processes: Asset Pricing Application". *Econometrica*. 57: 1091-1120.

9. Granger, C.W.J. and J.J. Hallman. 1991a. "Nonlinear Transformations Integrated Time Series". *Journal of Time Series Analysis*. 12: 207-224.

10. Olowofeso, O.E. 2006. "Varying Co-efficient Regression Models for Non-Linear Time Series: Estimation and Testing". *13th meeting of the Forum for Interdisciplinary Mathematical and Statistical Techniques*.

11. Priestley, M. 1988. *Non-linear and Non-stationary Time Series Analysis*. Academic Press: London, UK.

12. Robinson, P.M. 1983. "Non-Parametric Estimation for Time Series Models". *Journal of Time Series Analysis*. 4:185-208.

13. Sugihara, G. and R.M. May. 1990. " Nonlinear Forecasting as a way of Distinguishing Chaos from Measurement Error in Time Series". *Nature*. 344:734-741.

14. Terasvirta, T. and H.M. Anderson. 1992. "Modeling Nonlinearities in Business Cycles using Smooth Transition Autoregressive Models". *Journal of Applied Econometrics*. 7:S119-S136.

15. Tsay, R.S. 1986. "Nonlinearity Tests for Time Series". *Biometrika*. 73:461-466.

SUGGESTED CITATION

Bashiru, K.A., O.E. Olowofeso, and S.A. Owabumoye. 2009. "Nonlinear Trigonometric Transformation Time Series Modeling". *Pacific Journal of Science and Technology*. 10(2): 217-227.

Pacific Journal of Science and Technology