A Generalized Rectified Sinusoidal PWM Technique for Harmonic Elimination.

C.U. Ogbuka, M.Eng. and M.U. Agu, Ph.D.

Department of Electrical Engineering, University of Nigeria, Nsukka, Nigeria.

E-mail: ucogbuka@yahoo.com
drmarcelagu@yahoo.co.uk

ABSTRACT

To reduce or eliminate harmonic distortions of inverter output signals, pulsewidth modulation strategies are employed. The pulsewidth modulation is presently one of the most popular methods of voltage and frequency control applied in motor drives. Many modulation strategies have been developed to reduce lower order harmonics in inverter outputs, among which is the sinusoidal pulsewidth modulation (SPWM) technique. A detailed Fourier analysis of the rectified sinusoidal pulsewidth modulation technique is carried–out and MATLAB® simulation using the resulting Fourier based analysis shows that the effects of the 5th and 7th harmonics are highly suppressed and insignificant when compared to the fundamental. This technique when applied in motor control, leads to reduction in the rate at which the motor heats-up and to reduction in electromagnetic interference (EMI).

(Keywords: sinusoidal pulsewideth modulation, harmonics, modulation index, frequency ratio, MATLAB®)

INTRODUCTION

The harmonic content in the output of the inverters can be reduced by employing pulse-width modulation (PWM). The PWM techniques and strategies have been the subject of intensive research since 1970's to fabricate a sinusoidal ac output voltage. The use of power electronic equipments in industrial and consumer applications has been increased in recent years. Such loads draw nonlinear sinusoidal current and voltage from the source resulting in the harmonics in the networks [1]. They occur frequently in variable frequency drives or any electronic devices using solid state switching to convert AC or DC [2].

When inverters are employed for power conversion, the integrated output voltage is almost close to sinusoidal waveform but contains several harmonics affecting the power quality inevitably [3,4]. Sinusoidal PWM (SPWM) is effective in reducing lower order harmonics while varying the output voltage and has gone through many revisions and has a history of three decades [5-9]. When operated as a SPWM Inverter, a three phase voltage V_a, V_b, and V_c of variable amplitude are compared in three separate comparators with a common isosceles triangular carrier wave of a fixed amplitude as shown in Figure 1 [10].

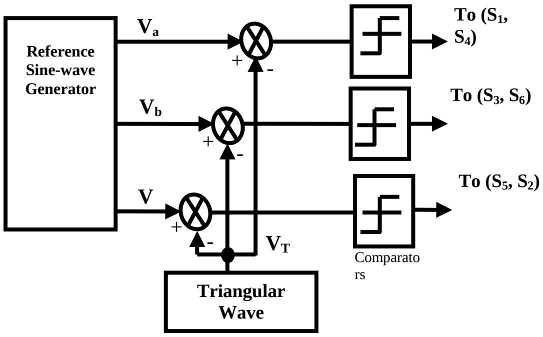


Figure 1: Control Signal Generator for Sinusoidal Pulsewidth Modulation (SPWM).

The output of the comparators 1, 2 and 3 form the control signals for the three legs of the inverter formed by switch pairs (S_1, S_4), (S_3, S_6), (S_5, S_2) respectively as shown in Figure 2 [11].

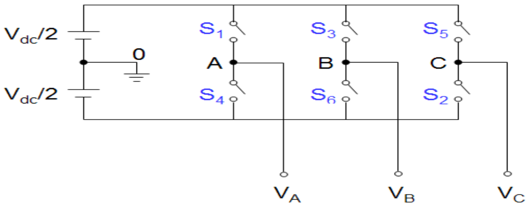


Figure 2: Three-Phase Sinusoidal PWM Inverter.

Considering the operation of the switch pair (S_1, S_4), which controls the voltage V_{AO} of the machine phase A with respect to the imaginary middle point of the dc source, O. The inverter operation is such that,

When $V_a > V_T$, $V_{A0} = V_{dc}/2$ **(1)**

When $V_a < V_T$, $V_{A0} = -V_{dc}/2$ **(2)**

The waveform of V_{B0} and V_{C0} with respect to switch pairs (S_3, S_6) and (S_5, S_2) respectively are similarly determined as shown in figure 3. Also shown in Figure 3 are the line to line voltages, V_{AB}, V_{BC}, V_{CA} where;

$V_{AB} = V_{A0} – V_{B0}$ **(3)**

$V_{BC} = V_{B0} – V_{C0}$ **(4)**

$V_{CA} = V_{C0} – V_{A0}$ **(5)**

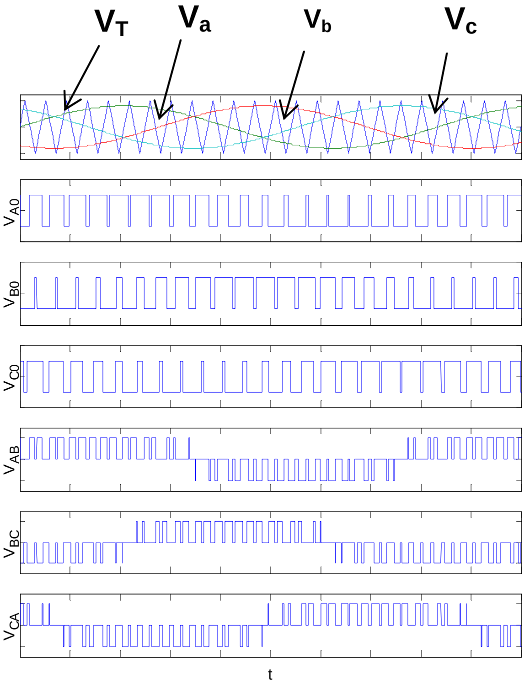


Figure 3: Waveforms of Three-Phase Sine PWM Inverter.

In most applications, PWM in addition to varying the instantaneous value of an output variable is carried-out to reduce the harmonic content of the variable.

Reduction of harmonic content helps to:

a) Reduce circuit loses

b) Reduce electromagnetic interference (EMI)

ANALYSIS OF THE RECTIFIED SINUSOIDAL PWM

To achieve the aim of reduced harmonic content, many methods of modulation have been proposed. The most popular is the sinusoidal modulation method where a reference sinusoid V_r of amplitude V_{rm} is used as a modulation waveform to interact with a higher frequency carrier signal V_c of amplitude V_{cm} (usually a triangular waveform) to generate the inverter switching signal to shape the inverter output voltage.

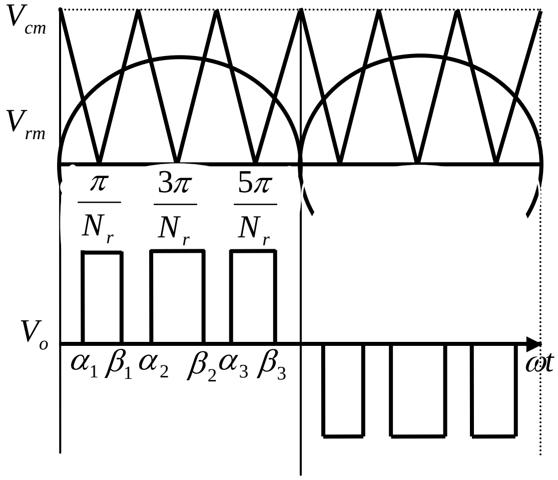
The control parameter is the modulation index M, which is the ratio of the amplitude of the modulating signal (V_{rm}) to the amplitude of the carrier signal (V_{cm}) [12].

$$M = \frac{V_{rm}}{V_{cm}} \qquad \textbf{(6)}$$

For meaningful and fruitful modulation to reduce the lower order harmonic, frequency ratio, which is the ratio of the frequency of the carrier signal (f_c) to the frequency of the reference or modulating signal (f_r) has to be at least 10.

$$\frac{f_c}{f_r} \geq 10 \qquad \textbf{(7)}$$

It is important to note that for the rectified SPWM, V_r and V_c accept both odd and even frequency ratio $\frac{f_c}{f_r}$ and still retain the output V_o as ac waveform as shown below.



The starting angular distance of the ac pulse 1 (α_1 is given by the intersection of the negative sloped path of V_c and V_r.

$$V_r = V_{rm} \sin \omega t \qquad \textbf{(8)}$$

Angular distance covered by each V_c cycle is $\frac{2\pi}{6} = \frac{\pi}{3}$.

Similarly, if $\frac{f_c}{f_r} = N_r$, then angular speed covered by each cycle of the carrier is $\frac{2\pi}{N_r}$. Half of this cycle period is $\frac{\pi}{N_r}$ as shown in figure above. Equation of the 1st negative sloped section of V_c is:

$$V_{cp1} = m\omega t + c \qquad \textbf{(9)}$$

Where m is the slope and c is the intersection. For $V_{cp1} = 0$, $c = V_{cm}$ therefore,

$$m = \frac{-V_{cm} N_r}{\pi} \text{ (ie negative slope)} \qquad \textbf{(10)}$$

Therefore,

$$V_{cp1} = \frac{-V_{cm} N_r}{\pi} \omega t + V_{cm} \qquad \textbf{(11)}$$

At $\omega t = \alpha_1$, Equation 8 and 11 must be equal.

$$V_{rm} \sin \alpha_1 = \frac{-V_{cm} N_r}{\pi} \alpha_1 + V_{cm} \qquad \textbf{(12)}$$

Dividing thorough by V_{cm},

$$\frac{V_{rm}}{V_{cm}} \sin \alpha_1 = \frac{-N_r}{\pi} \alpha_1 + 1 \qquad \textbf{(13)}$$

$$M \sin \alpha_1 = \frac{-N_r}{\pi} \alpha_1 + 1 \qquad \textbf{(14)}$$

Where M is the modulating index
Generally, the k^{th} sloped section of V_c has,

$$c = [1 + 2(k-1)]V_{cm} = (2k-1)V_{cm} \qquad \textbf{(15)}$$

α_k is the beginning of the k^{th} pulse. So for the k^{th} pulse:

$$V_{rm} \sin \alpha_k = \frac{-V_{cm} N_r}{\pi} \alpha_k + (2k-1)V_{cm} \qquad \textbf{(16)}$$

Dividing through by V_{cm},

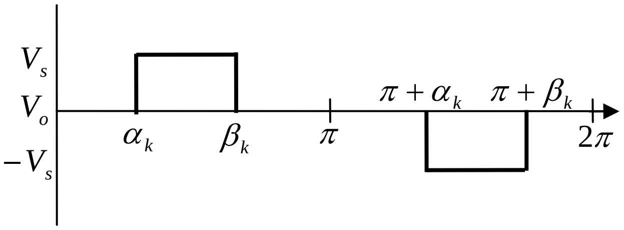
$$M \sin \alpha_k = \frac{-N_r}{\pi} \alpha_k + 2k - 1 \qquad \textbf{(17)}$$

In a similar manner, the slope m for the positive-sloped section is:

$m = \frac{V_{cm} N_r}{\pi}$ and the intersection for the k^{th} pulse is $c_k = [-2k+1]V_{cm}$ therefore,

$$M \sin \beta_k = \frac{N_r}{\pi} \beta_k + 1 - 2k \qquad \textbf{(18)}$$

For this rectified method, N_r has to be even. Analyzing the harmonic spectra, we can consider two symmetrical pulses, one positive and the other negative.



$$V_{ok} = \sum_{n=1,3,5}^{\infty} a_{nk}\cos n\omega t + b_{nk}\sin n\omega t \qquad \textbf{(19)}$$

Where a_{nk} and b_{nk} are the cosine and sine amplitudes of the n^{th} harmonic component.

$$a_{nk} = \frac{1}{\pi}[\int_0^{\pi} V_{ok}\cos n\omega t\,\delta(\omega t) \qquad \textbf{(20)}$$

$$a_{nk} = \frac{1}{\pi}[\int_{\alpha_k}^{\beta_k} V_s\cos n\omega t\,\delta(\omega t) - \int_{\pi+\alpha_k}^{\pi+\beta_k} V_s\cos n\omega t\,\delta(\omega t)] \qquad \textbf{(21)}$$

$$a_{nk} = \frac{V_s}{n\pi}[\sin n\omega t\ \Big|_{\alpha_k}^{\beta_k} - \sin n\omega t\ \Big|_{\pi+\alpha_k}^{\pi+\beta_k}] \qquad \textbf{(22)}$$

$$a_{nk} = \frac{2V_s}{n\pi}(\sin n\beta_k - \sin n\alpha_k) \qquad \textbf{(23)}$$

Similarly,

$$b_{nk} = \frac{1}{\pi}[\int_0^{\pi} V_{ok}\sin n\omega t\,\delta(\omega t) \qquad \textbf{(24)}$$

$$b_{nk} = \frac{1}{\pi}[\int_{\alpha_k}^{\beta_k} V_s\sin n\omega t\,\delta(\omega t) - \int_{\pi+\alpha_k}^{\pi+\beta_k} V_s\sin n\omega t\,\delta(\omega t)] \qquad \textbf{(25)}$$

$$b_{nk} = \frac{V_s}{n\pi}[-\cos n\omega t\ \Big|_{\alpha_k}^{\beta_k} + \cos n\omega t\ \Big|_{\pi+\alpha_k}^{\pi+\beta_k}] \qquad \textbf{(26)}$$

$$b_{nk} = \frac{2V_s}{n\pi}(\cos n\alpha_k - \cos n\beta_k) \qquad \textbf{(27)}$$

Where the n^{th} harmonic amplitude of the k^{th} pulse pair is:

$$C_{nk} = \sqrt{a_{nk}^2 + b_{nk}^2} \qquad \textbf{(28)}$$

Its rms value is:

$$c_{nk} = \frac{C_{nk}}{\sqrt{2}} = \sqrt{\frac{a_{nk}^2 + b_{nk}^2}{2}} \qquad \textbf{(29)}$$

Hence the n^{th} harmonic amplitude of V_0 made-up of $\frac{N_r}{2}$ pulse pairs is:

$$c_n = \sum_{k=1}^{N_r/2} c_{nk} \qquad \textbf{(30)}$$

SAMPLE PROJECT: FULLBRIDGE SINGLE PHASE INVERTER FOR THE RECTIFIED SPWM METHOD

This project is to determine c_n up to c_{13} as the modulation index, M, varies from 0.1 to 1 in steps of 0.01 for an inverter with $V_s = 1$, $\frac{f_c}{f_r} = 12$ for a Fullbridge Single Phase Inverter for the rectified V_r and V_c PWM method.

Aim- To determine the range of harmonic order, n, over which harmonic content is essentially zero. MATLAB program is written to determine the nth harmonic amplitude c_n of the output V_0. The program sequence is to evaluate the cosine amplitude a_{nk} and the sine amplitude b_{nk} of the n_{th} harmonic component. These are used to determine the n^{th} harmonic amplitude of the k^{th} pulse pair, C_{nk} and in rms form c_{nk} and finally the nth harmonic amplitude c_n of the output V_0, the result is as show in Figure 4.

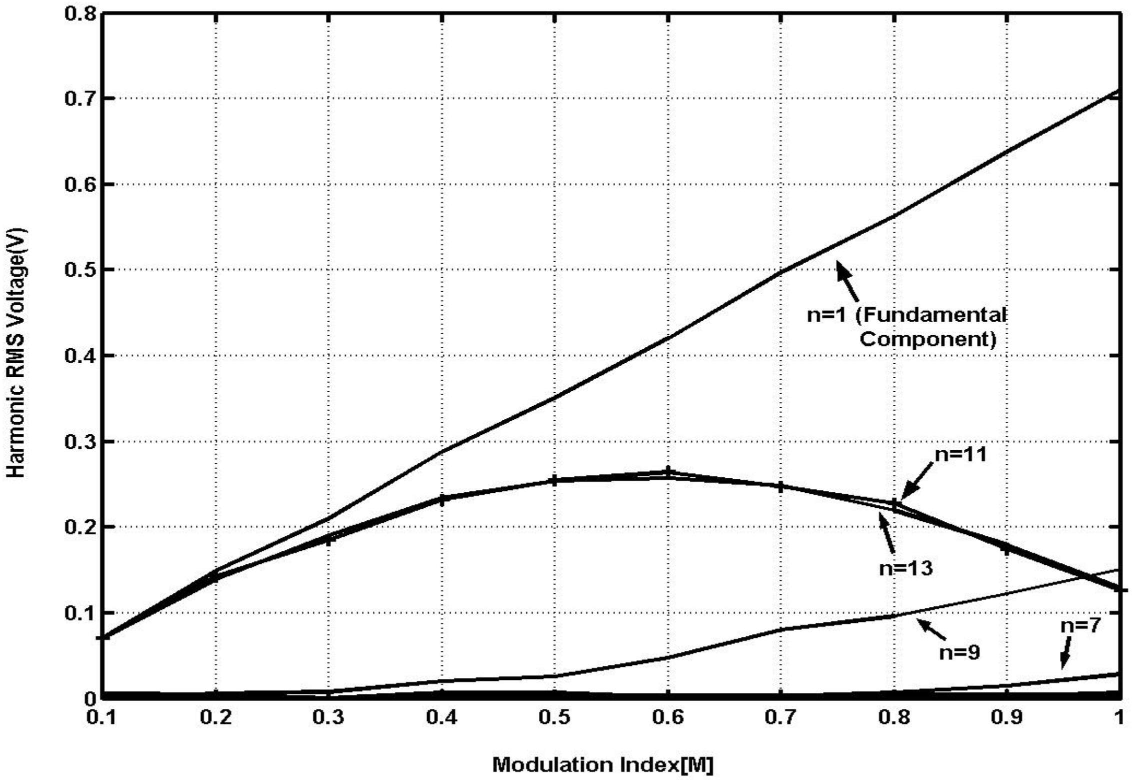


Figure 4: Plot of Harmonic RMS Voltages against Modulation Index.

CONCLUSION

The result obtained shows that the harmonic RMS voltages for harmonic orders less than 9 are highly suppressed and insignificant when compared to the fundamental RMS voltage and other higher order harmonics voltages. This modulation method, therefore, when applied to motor control reduces the heating-up of the motor and the intensity of electromagnetic interference (EMI) and other forms of circuit losses since the lower order harmonics which are predominantly responsible for these effects have been sufficiently eliminated. MATLAB® also proved to be a very good tool for the purpose of programming this result. It is also worth noting that for the modulation index, $M > 1$, the characteristics of the pulsewidth modulation discussed becomes invalid. Thus useful range of M is $0 < M < 1$. The pulse amplitude has also been assumed to be unity throughout this discussion.

REFERENCES

1. Wagner, V.E. 1993. ‘‘Effect of Harmonics on Equipment’’. *IEEE Trans. Power Delivery*. 8(2):672-680.

2. Jegathesan, V. and J. Jerome. 2008. ‘‘Non-Traditional Method-Based Solution for Elimination of Lower Order Harmonics in Voltage Source Inverter Feeding an Induction Motor Drive’’. *Serbian Journal of Electrical Engineering*. 5(2): 273-283.

3. Rasid, M.H. 1993. *Power Electronics: Circuits, Devices, and Applications*. Prentice Hall: New Jersey.

4. Briickner, M.T. and D.G. Holmes. 2003. “Optimal Pulsewidth Modulation for Three-Level Inverters”. *IEEE Trans.* 33:165-170.

5. Boost, M.A. and P.D. Ziogas. 1998. ‘‘State-of-the-Art–Carrier PWM Techniques: A Critical Evaluation’’. *IEEE Transactions Industry Applications*. 24(2):271-280.

6. Holtz, J. 1992. ''Pulsewidth Modulation-A Survey'', *IEEE Transaction Industrial Electronics*. 39(5): 410-420.

7. Ziogas, P.D. 1981. ''The Delta Modulation Technique in Static PWM Inverters''. *IEEE Transactions on Industry Application*. 1 (A-17): 199-203.

8. Jeevananthan, S., P. Dananjayan, and A.M. Asif-Fisal. 2004. ''A HPWM Method for Thermal Management in a Full-Bridge Inverter with Loss Estimation and Electro-Thermal Simulation". *AMSE Periodicals of Modeling, Measurement and Control – Series B*: 73(6):1-20.

9. Enjeti, P., P.D. Ziogas, and J.F. Lindsay. 1988. ''Programmed PWM Techniques to Eliminate Harmonics-A Critical Evaluation''. *IEEE IAS Conference Record*. 418-430.

10. Bose, B.K. 1997. *Power Electronics and Variable Frequency Drives*. IEEE Press: New York, NY.

11. Keyhani, A. 2008. Pulse-Width Modulation (PWM) Techniques. Unpublished Lecture Notes.

12. Agu, M.U. 2007/2008. "Electric Machine Drives". Unpublished Course Notes for EE 614, Department of Electrical Engineering, University of Nigeria, Nsukka, 2007/2008 Session.

ABOUT THE AUTHORS

Engr. Ogbuka, **Cosmas Uchenna** received his B.Eng. Degree (First Class Honors) and M.Eng Degree (Distinction) in 2004 and 2009, respectively in the Department of Electrical Engineering University of Nigeria, Nsukka where he presently works as a Lecturer/Research Student. His research interests are in Adjustable Speed Drives of Electrical Machines: (DC and AC Electric Machine Torque/Speed Control with Converters and Inverters) and Power Electronics.

Engr. Prof. Agu, Marcel U. obtained his B.Sc in Electrical Engineering in 1974 at the University of Nigeria, Nsukka. He also received his M.A.Sc. and Ph.D. in 1978 and 1982, respectively, in Power Electronics from the University of Toronto Canada. He is a Professor of Power Electronic in the Department of Electrical Engineering University of Nigeria, Nsukka. His research interests are, but not limited to, Power Electronic Circuits (Solid State AC/DC and DC/DC Converters, Inverter Circuits and Cyclo-converter/inverter circuits), Static Electric Motor Drives (DC and AC Electric Machine Torque/Speed Control with Converters and Inverters), Analogue, digital and microprocessor-based electronic control circuits, Static Induction Heating Power Supplies for heat treatment of metals (melting, casting, forging, forming, annealing, hardening), Renewable Energy Sources (especially in the area of working with solar photo voltaic cells and panels to realize solar lighting, heating, battery charging and water pumping), Power System Distribution for Residential, Commercial and Industrial Areas.

SUGGESTED CITATION

Ogbuka, C.U. and M.U. Agu. 2009. "A Generalized Rectified Sinusoidal PWM Technique for Harmonic Elimination". *Pacific Journal of Science and Technology*. 10(2): 21-26.

Pacific Journal of Science and Technology