A Generalized Vector Method of Induction Motor Transient and Steady State Analysis.

C.U. Ogbuka, M.Eng. and C.A. Nwosu, M.Eng.

Department of Electrical Engineering, University of Nigeria, Nsukka, Enugu State.

E-mail: ucogbuka@yahoo.com
cajethannwosu@yahoo.com

ABSTRACT

The qd transformation theory has been used by many authors for the study of both the steady state and the transient behaviors of induction machines. It has, however, been observed that the complexity of the approaches adopted often reduce the ease of comprehension of this concept, especially by young researchers. This paper, therefore, sets out to utilize the elegant vector approach in arriving at the machine models. The steady state and transient behaviors of a sample motor is examined in the synchronously rotating reference, as a case study, and presented in MATLAB/SIMULINK®.

(Keywords: induction motor, steady state, transient state, reference frame, MATLAB/SIMULINK)

INTRODUCTION

The induction machine is the most used machine in industry because of its robustness, reliability, low cost, high efficiency, and good self-starting capability [1, 2, 3]. The induction motor, particularly with a squirrel cage rotor, is the most widely used source of mechanical power fed from an AC power system. The recent improvements in power semiconductors devices and fast digital signal processing hardware have further accelerated this progress [4]. Its low sensitivity to disturbances during operation make the squirrel cage motor the first choice when selecting a motor for a particular application.

During start-up and other severe motoring operations, the induction motor draws large currents, produces voltage dips, oscillatory torques, and can even generate harmonics in the power system [5, 6]. It is, therefore, important to be able to predict these phenomena. Various models have been developed and the qd or two axis model for the study of transient behaviors has been tested and proved to be very reliable and accurate [7]. This method of modeling has been applied by several authors in their analysis [5]. This method, using the vector approach, is adopted in this paper for the purpose of transient and steady state analysis.

GENERALIZED EQUIVALENT CIRCUIT OF THREE PHASE INDUCTION MACHINE

The generalized equivalent circuit for transient and steady state analysis [8] is shown in Figure 1.

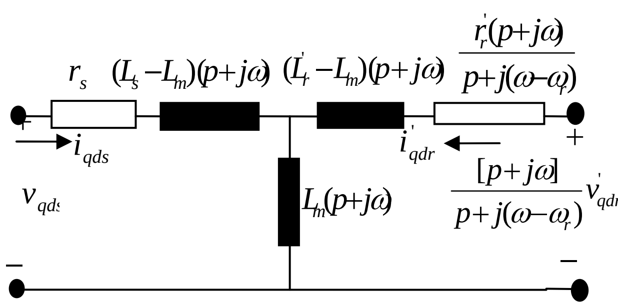

Figure 1: Generalized Equivalent Circuit of a Three-Phase Induction Motor.

Where

$L_s = L_{ls} + L_m$ and $L'_r = L'_{lr} + L_m$

Where p is the operator d/dt, r_s is the stator resistance, r'_r is the rotor resistance referred to the stator side, L_{ls} leakage inductance, L'_{lr} is the rotor leakage inductance, and L_m is the magnetizing inductance. Figure 2 shows the relationship between abc and arbitrary qd quantities.

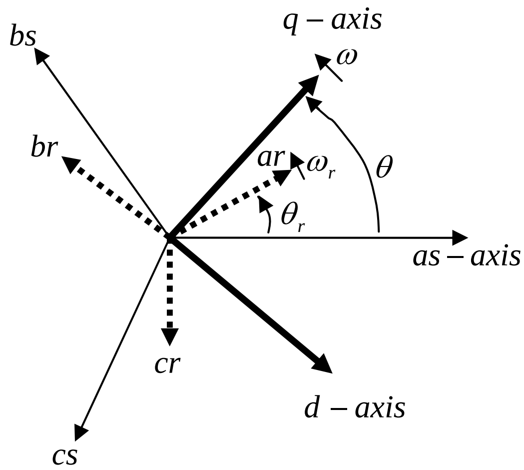

Figure 2: Relationship Between abc and Arbitrary qd Quantities.

ω is the rotating angular speed of the q or d-axis while ω_r is the rotor angular speed both in electrical radians/sec. θ and θ_r are the respective angular positions of the rotating q-axis and a chosen point on the rotor with respect to the ‘‘a’’ phase mmf axis.

OPEN LOOP TRANSIENT AND STEADY STATE EQUATIONS

From the equivalent circuit of Figure 1, the dynamic equations of the three-phase induction motor is derived using the Kirchorff's circuit laws as follows:

From the stator terminal,

$$v_{qds} = \left[r_s + (L_s - L_m)(p + j\omega)\right] i_{qds} + L_m(p + j\omega)(i_{qds} + i'_{qdr}) \quad \textbf{(1)}$$

From the referred rotor terminal,

$$\frac{(p+j\omega)}{p+j(\omega-\omega_r)} v'_{qdr} = \left[\frac{r'_r(p+j\omega)}{p+j(\omega-\omega_r)} + (L'_r - L_m)(p+j\omega)\right] i'_{qdr} + L_m(p+j\omega)(i_{qds} + i'_{qdr}) \quad \textbf{(2)}$$

Simplifying Equations 1 and 2, it is obtained that;

$$\begin{bmatrix} v_{qds} \\ v'_{qdr} \end{bmatrix} = \begin{bmatrix} r_s + L_s(p+j\omega) & L_m(p+j\omega) \\ r'_r + L'_r(p+j(\omega-\omega_r)) & L_m(p+j(\omega-\omega_r)) \end{bmatrix} \begin{bmatrix} i_{qds} \\ i'_{qdr} \end{bmatrix} \quad \textbf{(3)}$$

Equation 3 can be expanded to get the quadrature and direct axis voltage and current by applying the vector relation that;

$$F_{qds} = F_{qs} - jF_{ds} \quad \textbf{(4)}$$

Where F can represent the motor voltage, current, and flux linkage λ. Applying the above relation to the first part of Equation 3, then:

$$v_{qs} - jv_{ds} = [r_s + r_s(p + j\omega)][i_{qs} - ji_{ds}] + L_m(p + j\omega)[i'_{qr} - ji'_{dr}] \quad \textbf{(5)}$$

The real part of the right hand side (RHS) of Equation 5 is v_{qs} while the negative of the imaginary part is v_{ds}. The second part of Equation 3 can similarly be broken down into two parts, v'_{qr} and v'_{dr}. The resulting equation is shown below:

$$\begin{bmatrix} v_{qs} \\ v_{ds} \\ v'_{qr} \\ v'_{dr} \end{bmatrix} = \begin{bmatrix} r_s + L_s p & \omega L_s & L_m p & \omega L_m \\ -\omega L_s & r_s + L_s p & -\omega L_m & L_m p \\ L_m p & (\omega-\omega_r)L_m & r'_r + L'_r p & (\omega-\omega_r)L'_r \\ -(\omega-\omega_r)L_m & L_m p & -(\omega-\omega_r)L'_r & r'_r + L'_r p \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \\ i'_{qr} \\ i'_{dr} \end{bmatrix} \quad \textbf{(6)}$$

The electromechanical torque developed by the machine is derived as [9]:

$$T_e = \frac{3P}{4} L_m [i_{qs} i'_{dr} - i_{ds} i'_{qr}] \quad \textbf{(7)}$$

Where P is the number of motor poles.

The analysis done above recognizes that in a vast majority of cases, the machine is connected in delta or wye such that the neutral current does not flow. In this case, the neutral axis voltages and currents are identically zero.

The steady state equivalent circuit of the induction machine is found by replacing $p = \frac{d}{dt}$ by $j(\omega_e - \omega)$, where ω_e is the stator supply frequency. Slip, s, is defined as the per unit speed difference between the stator supply and the rotor as:

$$s = \frac{\omega_e - \omega_r}{\omega_e} \quad \textbf{(8)}$$

The steady state equivalent circuit is shown in Figure 3 below.

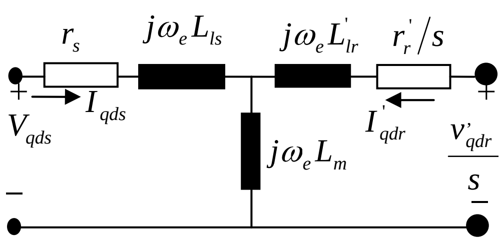

Figure 3: Steady State Equivalent Circuit of a Three-Phase Induction Motor.

For shorted winding or squirrel cage induction motor, $v'_{qdr} = 0$. The equivalent circuits for the transient and steady state conditions are shown in Figures 4 and 5, respectively.

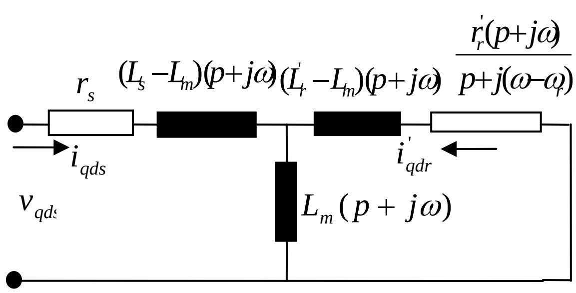

Figure 4: Transient Equivalent Circuit of a Squirrel Cage Induction Motor.

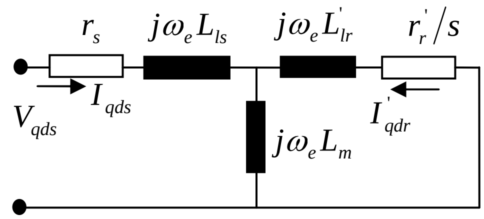

Figure 5: Steady State Equivalent Circuit of a Squirrel Cage Induction Motor.

For the squirrel cage induction motor, v_{qs} and v_{ds} are related to the abc applied voltage. Suppose the applied voltages are:

$$v_{as} = \sqrt{2}V_s \sin(\omega_e t + \theta_e(0)) \quad \textbf{(9a)}$$

$$v_{bs} = \sqrt{2}V_s \sin(\omega_e t + \theta_e(0) - \frac{2\pi}{3}) \quad \textbf{(9b)}$$

$$v_{cs} = \sqrt{2}V_s \sin(\omega_e t + \theta_e(0) - \frac{4\pi}{3}) \quad \textbf{(9c)}$$

v_{qds} is defined, vectorially, as:

$$v_{qds} = \frac{2}{3}e^{-j\theta}[v_{as} + a v_{bs} + a^2 v_{cs}] \quad \textbf{(10)}$$

Where,

$$\theta = \omega t + \theta(0) \quad \textbf{(11)}$$

and $\theta(0)$ is the value of θ at $t = 0$

$$a = e^{\frac{j2\pi}{3}} = -\frac{1}{2} + j\frac{\sqrt{3}}{2} \quad \textbf{(12)}$$

$$a^2 = e^{\frac{j4\pi}{3}} = -\frac{1}{2} - j\frac{\sqrt{3}}{2} \quad \textbf{(13)}$$

Substituting Equations 11, 12, and 13 into Equation 10 and expanding yields:

$$v_{qds} = \sqrt{2}V_s[\cos\{(\omega - \omega_e)t + \theta(0) - \theta_e(0)\} - j\sin\{(\omega - \omega_e)t + \theta(0) - \theta_e(0)\}] \quad \textbf{(14)}$$

The real part of the right hand side (RHS) of Equation 14 is v_{qs} while the negative of the imaginary part is v_{ds}. The resulting equations are as shown below.

$$v_{qs} = \sqrt{2}V_s \cos[(\omega - \omega_e)t + \theta(0) - \theta_e(0)] \quad \textbf{(15)}$$

$$v_{ds} = \sqrt{2}V_s \sin[(\omega - \omega_e)t + \theta(0) - \theta_e(0)] \quad \textbf{(16)}$$

If it is assumed that $\theta_e(0) = 0$ and that at $t = 0$, the d-axis is aligned to the as-axis.

Then, $\theta(0) = \theta_e(0) = 0$. Under this simplifying assumption,

$$v_{qs} = \sqrt{2}V_s \cos(\omega - \omega_e)t \quad \textbf{(17)}$$

$$v_{ds} = \sqrt{2}V_s \sin(\omega - \omega_e)t \quad \textbf{(18)}$$

THE MECHANICAL MODEL OF INDUCTION MACHINE

The mechanical model of an induction motor is comprised of equation of motion of the motor and the driven load as shown in Figure 6 and is usually represented as a second order differential equation [10].

$$J_m p^2 \theta_m = T_e - T_l \quad \textbf{(19)}$$

Decomposing Equation 19 into two first order differential equations gives:

$$p = \theta_m \omega_m \quad \textbf{(20)}$$

$$J_m (p\omega_m) = T_e - T_l \quad \textbf{(21)}$$

But,

$$\omega_r = \frac{P}{2}\omega_m \quad \textbf{(22)}$$

$$\theta_r = \frac{P}{2}\theta_m \quad \textbf{(23)}$$

Where P is the number of motor poles.

Where, ω_m, θ_m, ω_r, θ_r, J_m, T_l represent angular velocity of the rotor, rotor angular position, electrical rotor angular velocity, electrical rotor angular position, combined rotor and load inertia coefficient, and applied load torque, respectively.

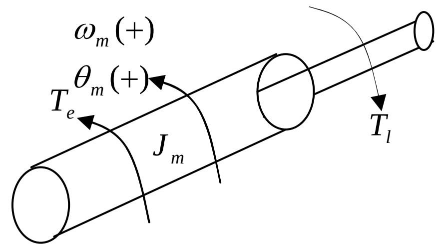

Figure 6: Schematic Diagram of Motor Mechanical Model

COMMONLY USED REFERENCE FRAMES

Although the behavior of a symmetrical induction machine may be described in any frame of reference, there are three which are commonly employed: namely, the stationary reference frame first employed by H.C. Stanly, the rotor reference frame which is park's transformation applied to induction machines, and the synchronously rotating reference frame. The voltage equations for each of these reference frames may be obtained from the voltage equations in the arbitrary reference frame by assigning the appropriate speed to ω. That is $\omega = 0$ for the stationary, $\omega = \omega_r$ for the rotor, and $\omega = \omega_e$ for the synchronously rotating reference frame [7, 9, 11].

Stationary Reference Frame

When $\omega = 0$, this is called the stationary reference frame because the qd axis does not rotate. In this case, the reference frame is said to be fixed in the stator. In addition, the q-axis in Figure 2 is chosen to coincide with the stator as-axis. Equations 17 and 18 now modify to:

$$v_{qs} = \sqrt{2}V_s \cos\omega_e t \quad \textbf{(24)}$$

$$v_{ds} = -\sqrt{2}V_s \sin\omega_e t \quad \textbf{(25)}$$

Synchronously Rotating Reference Frame

When $\omega = \omega_e$, this is called the synchronously rotating reference frame because the qd axes rotate at synchronous speed. So the qd axis is synchronous with the supply mmf. From Figure 2, it is seen that $\omega = p\theta = \omega_e$. Equations 17 and 18 are modified as:

$$v_{qs} = \sqrt{2}V_s \quad \textbf{(26)}$$

$$v_{ds} = 0 \quad \textbf{(27)}$$

It is evident from Equations 26 and 27 that the stator excitation voltages when simulating on synchronously rotating reference frame are DC quantities.

Rotor Reference Frame

When $\omega = \omega_r$, this is called the rotor reference frame because the qd axes rotate with the rotor. In addition, the q-axis coincides with the rotor ar-axis. It can, as well, be inferred from Figure 2 that $\omega = p\theta = \omega_r$. Hence Equations 17 and 18 now modify to:

$$v_{qs} = \sqrt{2}V_s \cos(\omega_r - \omega_e)t \quad \textbf{(28)}$$

$$v_{ds} = \sqrt{2}V_s \sin(\omega_r - \omega_e)t \quad \textbf{(29)}$$

A CASE STUDY OF SYNCHRONOUSLY ROTATING REFERENCE FRAME

Taking the synchronously rotating reference frame as a case study, since $\omega = p\theta = \omega_e$ and $v_{ds} = v'_{qr} = v'_{dr} = 0$, the transient Equation 6 now becomes,

$$\begin{bmatrix} \sqrt{2}V_s \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} r_s + L_s p & \omega_e L_s & L_m p & \omega_e L_m \\ -\omega_e L_s & r_s + L_s p & -\omega_e L_m & L_m p \\ L_m p & (\omega_e - \omega_r) L_m & r'_r + L'_r p & (\omega_e - \omega_r) L'_r \\ -(\omega_e - \omega_r) L_m & L_m p & -(\omega_e - \omega_r) L'_r & r'_r + L'_r p \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \\ i'_{qr} \\ i'_{dr} \end{bmatrix}$$

(30)

Eliminating $p = d/dt$ from Equation 30, the steady state equations on the synchronously rotating reference frame is realized as shown in Equation 31.

$$\begin{bmatrix} \sqrt{2}V_s \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} r_s & \omega_e L_s & 0 & \omega_e L_m \\ -\omega_e L_s & r_s & -\omega_e L_m & 0 \\ 0 & (\omega_e - \omega_r) L_m & r'_r & (\omega_e - \omega_r) L'_r \\ -(\omega_e - \omega_r) L_m & 0 & -(\omega_e - \omega_r) L'_r & r'_r \end{bmatrix} \begin{bmatrix} I_{qs} \\ I_{ds} \\ I'_{qr} \\ I'_{dr} \end{bmatrix}$$

(31)

OPEN LOOP STEADY STATE AND TRANSIENT PERFORMANCES

Open Loop Steady State Performances

Using Equation 31, the steady state performances of the sample motor with the parameters shown in Table One below are obtained using MATLAB® program.

Table 1: Sample Machine Data.

Rated Voltage	400V
Winding Connection	Star
Rated Frequency	50H$_z$
Number of Poles	6
Rated Speed	960rpm
Stator Resistance	0.4Ω
Rotor Referred Resistance	0.2Ω
Stator Reactance	1.5Ω
Rotor Referred Reactance	1.5Ω
Magnetizing Reactance	30Ω
Moment of Inertia	2.1kg.m^2

The steady state currents, I_{qs} , I_{ds} , I'_{qr} , I'_{dr} and electromechanical torque, T_e are determined in steps of 5 rpm for motor speed range of zero to synchronous speed. The response curves are shown below.

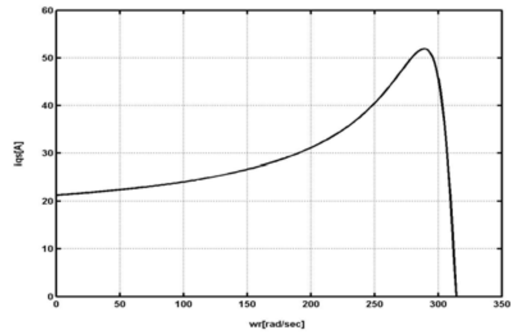

Figure 7: Plot of I_{qs} against Motor Speed, ω_r .

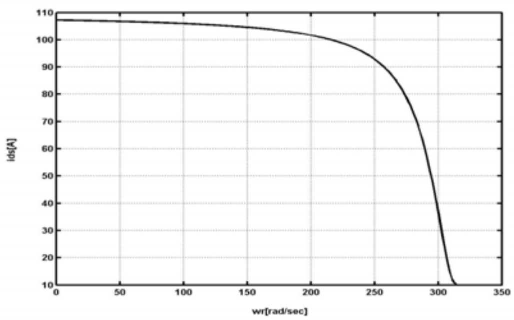

Figure 8: Plot of I_{ds} against Motor Speed, ω_r .

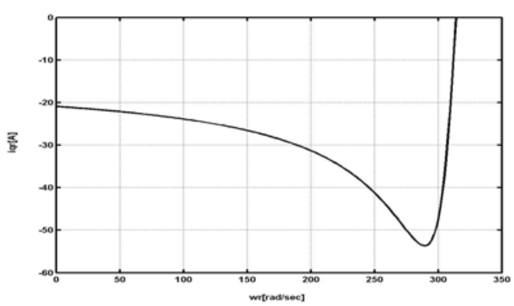

Figure 9: Plot of $I_{qr}^{'}$ against Motor Speed, ω_r .

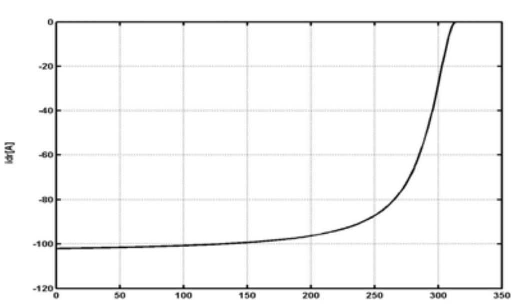

Figure 10: Plot of $I_{dr}^{'}$ against Motor Speed, ω_r .

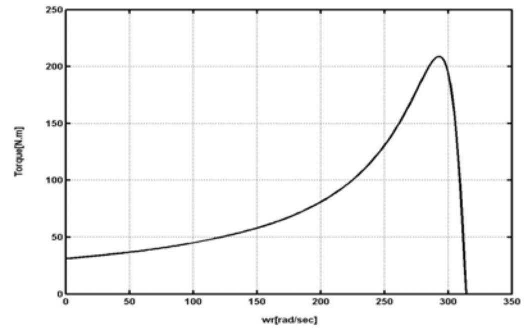

Figure 11: Plot of T_e against Motor Speed, ω_r .

Open Loop Transient Performances

Using Equations 7, 21, and 30 and the intermediate analysis equations as described in [12], the transient performance of the unloaded motor of Table One are simulated using MATLAB/SIMULINK®. The currents, i_{qs} , i_{ds} , $i_{qr}^{'}$, $i_{dr}^{'}$, electromechanical torque T_e , and the motor speed, ω_r , are modeled and the response curves shown below.

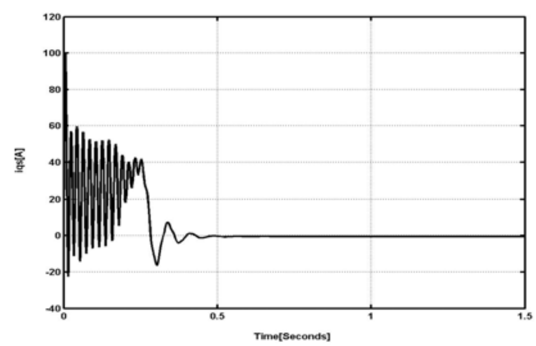

Figure 12: Plot of i_{qs} against Time.

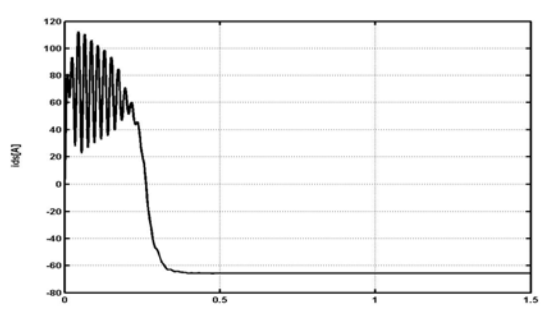

Figure 13: Plot of i_{ds} against Time.

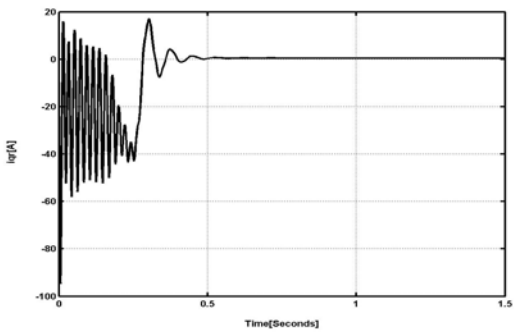

Figure 14: Plot of i'_{qr} against Time.

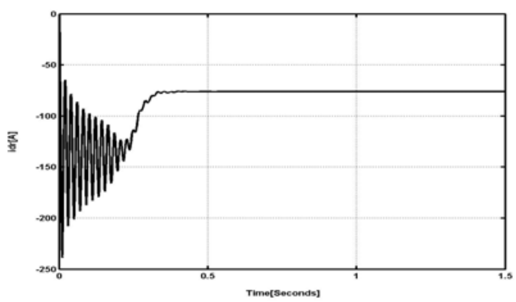

Figure 15: Plot of i'_{dr} against Time.

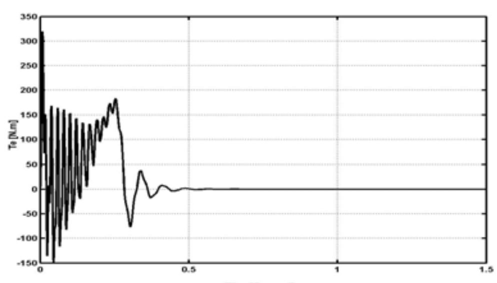

Figure 16: Plot of Electromechanical Torque T_e against Time.

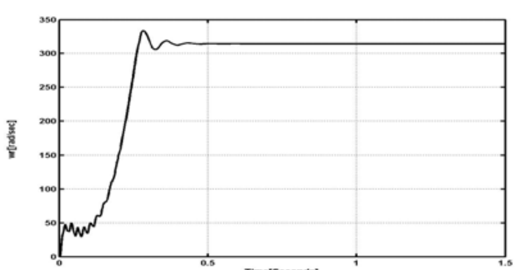

Figure 17: Plot of Speed ω_r against Time.

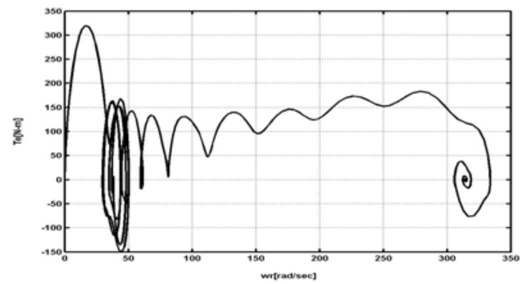

Figure 18: Plot of Electromechanical Torque T_e against Speed, ω_r .

OBSERVATIONS AND CONCLUSIONS

Figures 7 to 11 show the steady state response of the motor as it accelerates from rest to rated speed. It is observed in Figure 11 that the maximum torque is obtained just before the rated speed which therefore implies that to obtain the advantage of a high torque to current ratio, high efficiency, and a good power factor, the motor should always be operated on the portion of the torque speed curve with negative slope.

For the transient analysis, Figures 12 to 18 represent the free acceleration (no-load) characteristics of the sample motor in the synchronously rotating reference frame. Here, the zero position of the reference frame is selected so that v_{qs} is the amplitude of the stator applied phase voltage and $v_{ds} = 0$.

The rotor accelerates from stall with zero load torque and, since friction and windage losses are not taken into account, the machine accelerates to synchronous speed. It is important to note that since the machine operates in the steady state mode upon reaching synchronous speed, the examined variables become constants corresponding to their instantaneous values at the time the rotor speed becomes equal to synchronous speed.

The instantaneous electromagnetic torque, immediately following the application of the stator voltage, varies about an average positive value while the steady state torque-speed curve averages to zero. In the vicinity of synchronism, the rotor has enough kinetic energy and mobility to briefly enter the generator mode of operation, and then it returns to below the synchronous speed. This mode of operation can be repeated several times as represented by the curl around the synchronous speed in Figure 18. The analysis and results presented have shown how elegant the vector method is in the steady state and transient analysis of three phase induction motor.

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ABOUT THE AUTHORS

Engr. Ogbuka, **Cosmas Uchenna** received his B.Eng. (First Class Honors) and M.Eng. degrees in 2004 and 2008, respectively, in the Department of Electrical Engineering University of Nigeria, Nsukka where he presently works as a Lecturer/Research Student. His research interests are in Adjustable Speed Drives of Electrical Machines: (DC and AC Electric Machine Torque/Speed Control with Converters and Inverters) and Power Electronics.

Engr. Nwosu, Cajethan A. obtained his B.Eng. and M.Eng. degrees in Electrical Engineering from the University of Nigeria, Nsukka in 1994 and 2004, respectively. He is presently working towards his Ph.D. degree in Electrical Engineering. He is currently a Lecturer in the Department of Electrical Engineering, University of Nigeria, Nsukka. His research interests include Power Electronic Converters and Renewable Energy Technologies.

SUGGESTED CITATION

Ogbuka, C.U. and C.A. Nwosu. 2009. "A Generalized Vector Method of Induction Motor Transient and Steady State Analysis". *Pacific Journal of Science and Technology*. 10(1):59-66.