# Some New Type of Generalized Difference Paranormed Sequence Spaces Defined by Sequence of Orlicz Functions Associated with Multiplier Sequences.

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## ABSTRACT

In this article we introduce some new type of sequence spaces

$c\{\mathbf{M}, \Delta^m, \Lambda, p\}$,
$c_0\{\mathbf{M}, \Delta^m, \Lambda, p\}$,
and $\ell_\infty\{\mathbf{M}, \Delta^m, \Lambda, p\}$.

Also studied some different properties and established some inclusion relations.

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## INTRODUCTION

Throughout $w$, $c$, $c_0$, $\ell_\infty$ denote the spaces of *all*, *convergent*, *null,* and *bounded* sequences, respectively.

The notion of difference sequence space was introduced by Kizmaz [5] as follows:

$X(\Delta)=\{(x_k) \in w: (\Delta x_k) \in X\}$, for $X = c, c_0, \ell_\infty$, where $\Delta x_k = x_k - x_{k+1}$ , for all $k \in N$.

Later on this subject was generalized by Et and Colak [1] as follows:

$Z(\Delta^n)=\{ (x_k) \in w: (\Delta^n x_k) \in Z\}$

for $X = \ell_\infty$, $c$, $c_0$ ,where $n \in N$ ; $\Delta^0 x_k = x_k$, for all $k \in N$, and

$$\Delta^n x_k = \Delta^{n-1} x_k - \Delta^{n-1} x_{k+1} = \sum_{\nu=0}^{n} (-1)^\nu \binom{n}{\nu} x_{k+\nu},$$

for all $k \in N$.

The notion of paranormed sequence spaces was introduced by Nakano [11] and Simons [14]. It was further investigated by Maddox [10], Lascarides [7] and many others.

We write $r = (r_k) =( p_k^{-1})$.

An *Orlicz function* is a function $M$: $[0,\infty) \to [0, \infty)$, which is continuous , non-decreasing and convex with $M(0) = 0$, $M(x)>0$ for $x>0$ and $M(x) \to \infty$, as $x \to \infty$.

The study of Orlicz sequence spaces was initiated with certain specific purpose in Banach space theory. Indeed Lindberg [9] got interested in Orlicz spaces in connection with finding Banach spaces with symmetric. Subsequently Lindenstrauss and Tzafriri [8] investigated these Orlicz sequence spaces in detail. Later on, different classes of sequence spaces defined by Orlicz function were studied by Nung and Lee [12], Woo [16], Parashar and Choudhary [13] and many others. The Orlicz sequence spaces are the special case of Orlicz spaces studied by [6].

The scope for the studies on sequence spaces was extended by using the notion of an associated multiplier sequence. Goes and Goes [3], defined the differentiated sequence space $dE$ and integrated sequence space $\int E$ for a given sequence space $E$, by using the multiplier sequences ($k^{-1}$) and ($k$) respectively. Tripathy [15] used a general multiplier sequence $\Lambda = (\lambda_k)$ of non-zero scalars for his studies on sequence spaces, associated with multiplier sequences.

In this paper we shall consider a general multiplier sequence $\Lambda = (\lambda_k)$ of non–zero scalars.Then for a sequence space $E$, the multiplier sequence space $E(\Lambda)$ ,associated with the multiplier sequence $\Lambda$ is defined as:

$$E(\Lambda) = \left\{(x_k) \in w: (\lambda_k x_k) \in E\right\}.$$

## DEFINITIONS AND PRELIMINARIES

<u>**DEFINITION**</u>. A sequence space $E$ is said to be *solid* (or *normal*) if $(x_k) \in E$ and $(\alpha_k)$ a sequence of scalars with $|\alpha_k| \le 1$, for all $k \in N$, then $(\alpha_k x_k) \in E$.

<u>**DEFINITION**</u>. A sequence space $E$ is said to be *symmetric* if $(x_{\pi(k)}) \in E$ whenever $(x_k) \in E$, where $\pi$ is a permutation of $N$.

Let $\mathbf{M} = (M_k)$ be a sequence of Orlicz functions. We introduce the following sequence spaces associated with a multiplier sequence $\Lambda = (\lambda_k)$ is a sequence of non-zero scalars:

$c\{\mathbf{M}, \Delta^m, \Lambda, p\}=$

$$\left\{ (x_k) \in w : \lim_{k\to\infty}\left\{ \left[M_k\left(\frac{|\lambda_k(\Delta^m x_k)|}{\rho}\right)\right]^{p_k} r_k \right\} = 0,\ for\ some\ \rho > 0\right\}$$

;

$c_0\{\mathbf{M}, \Delta^m, \Lambda, p\}=$

$$\left\{ (x_k) \in w : \lim_{k\to\infty}\left\{ \left[M_k\left(\frac{|\lambda_k(\Delta^m x_k - L)|}{\rho}\right)\right]^{p_k} r_k \right\} = 0,\ for\ some\ L \in C\ and\ \rho > 0\right\}$$

;

$\ell_\infty\{\mathbf{M}, \Delta^m, \Lambda, p\}=$

$$\left\{ (x_k) \in w : \sup_k\left\{ \left[M_k\left(\frac{|\lambda_k(\Delta^m x_k)|}{\rho}\right)\right]^{p_k} r_k \right\} < \infty,\ for\ some\ \rho > 0\right\}$$

## DISCUSSION

($i$) Let $M_k(x) = x$, for all $k$ in $N$ and $m = 1$, then the spaces $c\{\Delta, \Lambda, p\}$, $c_0\{\Delta, \Lambda, p\}$ and $\ell_\infty\{\Delta, \Lambda, p\}$ introduced and studied by Tripathy [15].

($ii$) Let $M_k(x) = x$ and $\lambda_k = 1$, for all $k$ in $N$ and $m = 0$, then the spaces $c_0\{p\}$, $c\{p\}$ and $\ell_\infty\{p\}$ are studied by Lascarides [7].

($iii$) Let $M_k(x) = x$ , $\lambda_k = 1$, $p_k = 1$, for all $k$ in $N$ and $m = 0$, then $c$, $c_0$ and $\ell_\infty$ are the spaces of *convergent*, *null* and *bounded* sequences respectively.

The following results will be used for establishing some results of this article.

<u>**LEMMA 1 (Lascarides [7],Proposition 1)**</u>.
*Let* $h = \inf_k p_k$, $H = \sup_k p_k$. *Then the following conditions are equivalent:*

($i$) $H < \infty$ *and* $h > 0$;
($ii$) $c_0(p) = c_0$ or $\ell_\infty(p) = \ell_\infty$;
($iii$) $\ell_\infty\{p\} = \ell_\infty(p)$;
($iv$) $c_0\{p\} = c_0(p)$;
($v$) $\ell\{p\} = \ell(p)$.

<u>**LEMMA 2 (Lascarides[7], Corollary 1)**</u>.
*Let $p$, $q$ be two sequences of strictly positive numbers. Then* $c_0\{p\} \cong c_0\{q\}$ *if and only if there exists a sequence $u = (u_k)$ of strictly positive numbers such that*

$$\lim_\eta \lim\text{–}\sup_k \left\{ \left( u_k\, p_k^{p_k^{-1}} \eta^{-\left(1+\frac{1}{p_k}\right)^{q_k}} \right) q_k^{-1} \right\} = 0 \quad \textbf{(1)}$$

*and*

$$\lim_\eta \lim\text{–}\sup_k \left\{ \left( u_k\, q_k^{q_k^{-1}} \eta^{-\left(1+\frac{1}{q_k}\right)^{p_k}} \right) p_k^{-1} \right\} = 0 \quad \textbf{(2)}$$

<u>**LEMMA 3 (Lascarides[7], Corollary 2)**</u>.
*Let the sequence $a = (a_k)=$* $\left(q_k^{q_k^{-1}}\, p_k^{-p_k^{-1}}\right)$. *Then* $c_0\{p\} \cong c_0\{q\}(a)$ *if and only if the following conditions hold*:

$$\lim_\eta \lim\text{–}\sup_k \eta^{-q_k\left(1+p_k^{-1}\right)} = 0 \quad \textbf{(3)}$$

and

$\lim_{\eta} \lim - \sup_k \eta^{-p_k\left(1+q_k^{-1}\right)}=0$ **(4)**

**LEMMA 4 (Lascarides[7], Corollary 3).**
*Let the sequence a =* $(a_k)= \left(q_k^{q_k^{-1}} p_k^{-p_k^{-1}}\right)$. *Then* $\lim_{k\to\infty}\left(\frac{1}{p_k}-\frac{1}{q_k}\right)=0$ *implies* $c_0\{p\}\cong c_0\{q\}(a)$.

**LEMMA 5(Lascarides [7],Proposition 3).**
*Let* $f_k = \frac{p_k}{q_k}$, *for every* $k\in N$. *Let* $(f_k)$ *and* ( $f_k^{-1}$ ) both be in $\ell_\infty$. Then $\ell_\infty\{p\}\cong \ell_\infty\{q\}(f)$.

**LEMMA 6 (Lascarides[7], Theorem 3).**
*Let* $q\in \ell_\infty$. *Then* $\ell_\infty\{p\}\subseteq \ell_\infty\{q\}$ *if and only if* $\liminf_k q_k (Jp_k)^{-q_k p_k^{-1}}>0$, *for every integer J* >1. **(5)**

**LEMMA 7 (Lascarides[7], Theorem 4).**
*Let* $q\in \ell_\infty$ *and* $c_0\{p\}\cong c_0\{q\}$, *then* $c_0(p)\cong c_0(q)$.

**MAIN RESULTS**

**THEOREM 1**. *For any sequence p =* $(p_k)$ *of strictly positive real numbers, the classes* $c\{\mathbf{M}, \Delta^m, \Lambda, p\}$, $c_0\{\mathbf{M}, \Delta^m, \Lambda, p\}$ *and* $\ell_\infty\{\mathbf{M}, \Delta^m, \Lambda, p\}$ *are linear spaces.*

**PROOF.** We prove the result for the case $c\{\mathbf{M}, \Delta^m, \Lambda, p\}$. The rest of the results follow similarly. Let $(x_k)$, $(y_k) \in c\{\mathbf{M}, \Delta^m, \Lambda, p\}$ and $\alpha,\beta$ be two scalars in *C*. Then there exist $\rho_1>0$ and $\rho_2>0$ such that:

$$\lim_{k\to\infty}\left\{\left[M_k\left(\frac{|\lambda_k(\Delta^m x_k - L_1)|}{\rho_1}\right)\right]^{p_k} r_k\right\}=0, \text{ for some } L_1\in C,$$

and $$\lim_{k\to\infty}\left\{\left[M_k\left(\frac{|\lambda_k(\Delta^m x_k - L_2)|}{\rho_2}\right)\right]^{p_k} r_k\right\}=0, \text{ for some } L_2\in C.$$

Let $\rho = \max\{2|\alpha|\rho_1, 2|\beta|\rho_2\}$. Then we have:

$$\left\{\left[M_k\left(\frac{|\lambda_k\{(\alpha\Delta^m x_k+\beta\Delta^m y_k)-(\alpha L_1+\beta L_2)\}|}{\rho}\right)\right]^{p_k} r_k\right\}\le D\left\{\left[M_k\left(\frac{|\lambda_k(\Delta^m x_k - L_1)|}{\rho_1}\right)\right]^{p_k} r_k\right\}+D\left\{\left[M_k\left(\frac{|\lambda_k(\Delta^m x_k - L_2)|}{\rho_2}\right)\right]^{p_k} r_k\right\},$$ where $D = \max(1, 2^{H-1})$ and $H = \sup_k p_k <\infty$.

Taking limit $k\to\infty$ on both sides of the above inequality, we have:

$(\alpha(x_k)+\beta(y_k))\in c\{\mathbf{M}, \Delta^m, \Lambda, p\}$.

Therefore $c\{\mathbf{M}, \Delta^m, \Lambda, p\}$ is a linear space.

**THEOREM 2**. *Let* $0< \inf p_k \le \sup p_k < \infty$, *then the space* $\ell_\infty\{\mathbf{M}, \Delta^m, \Lambda, p\}$ *is a paranormed space, paranormed by*

$$f_\Delta(x) = \sum_{s=1}^{m}|x_s| + \inf\left\{\rho^{\frac{p_k}{K}} : \sup_{k\ge1}\left\{\left[M_k\left(\frac{|\lambda_k(\Delta^m x_k)|}{\rho}\right)\right] r_k^{\frac{1}{p_k}}\right\}\le 1, \text{ for some } \rho>0\right\},$$

where $K = \max(1,H)$.

**PROOF.** Clearly $f_\Delta(x) = f_\Delta(-x)$. Since $M_k(0) = 0$, for all $k\in N$. We get $f_\Delta\{\bar{\theta}\} = 0$, for $x = \bar{\theta}$.

Let $x = (x_k)$, $y = (y_k)$ be two elements of $\ell_\infty\{\mathbf{M}, \Delta^m, \Lambda, p\}$ and let us choose $\rho_1 > 0$ and $\rho_2 > 0$ such that:

$$\sup_{k\ge1}\left\{\left[M_k\left(\frac{|\lambda_k(\Delta^m x_k)|}{\rho_1}\right)\right] r_k^{\frac{1}{p_k}}\right\} \le 1$$

and

$$\sup_{k\ge1}\left\{\left[M_k\left(\frac{|\lambda_k(\Delta^m y_k)|}{\rho_2}\right)\right] r_k^{\frac{1}{p_k}}\right\} \le 1$$

Let $\rho = \rho_1 + \rho_2$. Then we get:

$$\sup_{k\ge1}\left\{\left[M_k\left(\frac{|\lambda_k(\Delta^m x_k + \Delta^m y_k)|}{\rho}\right)\right] r_k^{\frac{1}{p_k}}\right\}$$

$$\le \left(\frac{\rho_1}{\rho_1+\rho_2}\right)\sup_{k\ge1}\left\{\left[M_k\left(\frac{|\lambda_k(\Delta^m x_k)|}{\rho_1}\right)\right] r_k^{\frac{1}{p_k}}\right\} + \left(\frac{\rho_2}{\rho_1+\rho_2}\right)\sup_{k\ge1}\left\{\left[M_k\left(\frac{|\lambda_k(\Delta^m y_k)|}{\rho_2}\right)\right] r_k^{\frac{1}{p_k}}\right\} \le 1$$

Now,

$$f_\Delta(x+y) = \sum_{s=1}^{m}|x_s + y_s| + \inf\left\{(\rho_1+\rho_2)^{\frac{p_k}{K}} : \sup_{k\ge1}\left\{\left[M_k\left(\frac{|\lambda_k(\Delta^m x_k + \Delta^m y_k)|}{\rho_1+\rho_2}\right)\right] r_k^{\frac{1}{p_k}}\right\} \le 1\right\}$$

$$\le \sum_{s=1}^{m}|x_s| + \inf\left\{(\rho_1)^{\frac{p_k}{K}} : \sup_{k\ge1}\left\{\left[M_k\left(\frac{|\lambda_k(\Delta^m x_k)|}{\rho_1}\right)\right] r_k^{\frac{1}{p_k}}\right\} \le 1\right\}$$

$$+ \sum_{s=1}^{m}|y_s| + \inf\left\{(\rho_2)^{\frac{p_k}{K}} : \sup_{k\ge1}\left\{\left[M_k\left(\frac{|\lambda_k(\Delta^m y_k)|}{\rho_2}\right)\right] r_k^{\frac{1}{p_k}}\right\} \le 1\right\}$$

$$\le f_\Delta(x) + f_\Delta(y)$$

Finally let $\gamma$ be a given scalar in *C*, then the continuity of the scalar multiplication follows from the following equality:

$$f_\Delta(\gamma x) = \sum_{s=1}^{m}|\gamma x_s| + \inf\left\{(\rho)^{\frac{p_k}{K}} : \sup_{k\ge1}\left\{\left[M_k\left(\frac{|\lambda_k(\Delta^m(\gamma x_k))|}{\rho}\right)\right] r_k^{\frac{1}{p_k}}\right\} \le 1,\ for\ some\ \rho > 0\right\}$$

$$= |\gamma|\sum_{s=1}^{m}|x_s| + \inf\left\{(|\gamma|t)^{\frac{p_k}{K}} : \sup_{k\ge1}\left\{\left[M_k\left(\frac{|\lambda_k(\Delta^m x_k)|}{t}\right)\right] r_k^{\frac{1}{p_k}}\right\} \le 1,\ for\ t > 0\right\},$$

where $t = \frac{\rho}{|\gamma|}$. This completes the proof.

**COROLLARY 1**. *Let* $p \in \ell_\infty$, *then the spaces* $c\{\mathbf{M}, \Delta^m, \Lambda, p\}$ *and* $c_0\{\mathbf{M}, \Delta^m, \Lambda, p\}$ *are paranormed spaces, paranormed by* $f_\Delta$ *defined above*.

**THEOREM 3**. *Let* $p \in \ell_\infty$, *then the spaces* $c\{\mathbf{M}, \Delta^m, \Lambda, p\}$, $c_0\{\mathbf{M}, \Delta^m, \Lambda, p\}$ *and* $\ell_\infty\{\mathbf{M}, \Delta^m, \Lambda, p\}$( *with* inf $p_k > 0$) *are complete paranormed spaces, paranormed by* $f_\Delta$.

**PROOF**. Let $(x^n) \in \ell_\infty\{\mathbf{M}, \Delta^m, \Lambda, p\}$ be a Cauchy sequence, where $x^n = (x_k^n)_{k=1}^{\infty}$, for all $n \in N$. Then, we have,

$$f_\Delta(x^i - x^j) \to 0, \text{ as } i, j \to \infty.$$

For a given ε>0, let $u$ and $v_0$ be such that $\frac{\varepsilon}{u v_0}$ >0 and $M_k\left(\frac{u v_0}{2}\right) \geq \sup_{k\geq 1}(p_k)^{r_k}$ .

Then $f_\Delta(x^i - x^j) \to 0$, as $i, j \to \infty$, implies that there exists $n_0 \in N$ such that

$$f_\Delta(x^i - x^j) < \frac{\varepsilon}{u v_0}, \text{ for all } i,j \geq n_0$$

$$\Rightarrow \sum_{s=1}^{m} | x_s^i - x_s^j | < \frac{\varepsilon}{u v_0}$$

and
$$\inf\left\{(\rho)^{\frac{p_k}{K}} : \sup_{k\geq 1}\left\{\left[M_k\left(\frac{|\lambda_k(\Delta^m x_k^i - \Delta^m x_k^j)|}{\rho}\right)\right] r_k^{\frac{1}{p_k}}\right\} \leq 1, \text{ for } \rho > 0\right\} < \frac{\varepsilon}{u v_0} \qquad \textbf{(6)}$$

$\Rightarrow$ ( $x_s^i$ ) is a Cauchy sequence in $C$, so ( $x_s^i$ ) is convergent in $C$. Let $\lim_{i\to\infty} x_s^i$ = $x_s$(say).

Then

$$\lim_{i\to\infty} \sum_{s=1}^{m} | x_s^i - x_s^j | < \frac{\varepsilon}{u v_0}$$

$$\Rightarrow \sum_{s=1}^{m} | x_s^i - x_s^j | < \frac{\varepsilon}{u v_0} .$$

From (6), we have,

$$\left\{\left[M_k\left(\frac{|\lambda_k(\Delta^m x_k^i - \Delta^m x_k^j)|}{f_\Delta(x^i - x^j)}\right)\right] r_k^{\frac{1}{p_k}}\right\} \leq 1$$

$$\Rightarrow M_k\left(\frac{|\lambda_k(\Delta^m x_k^i - \Delta^m x_k^j)|}{f_\Delta(x^i - x^j)}\right) \leq (p_k)^{r_k} \leq M_k\left(\frac{u v_0}{2}\right)$$

$$\Rightarrow |\lambda_k(\Delta^m x_k^i - \Delta^m x_k^j)| < \frac{u v_0}{2} \cdot \frac{\varepsilon}{u v_0} < \frac{\varepsilon}{2}$$

$$\Rightarrow \left(\lambda_k\left(\Delta^m x_k^i\right)\right) \text{ is a Cauchy sequence in } C, \text{ for all } k \in N.$$

Hence $\left(\lambda_k\left(\Delta^m x_k^i\right)\right)$ converges in $C$. Thus $\lim_{i\to\infty}\left(\lambda_k\left(\Delta^m x_k^i\right)\right) = (\lambda_k(\Delta^m x_k))$, for all $k \in N$ .

By the continuity of $M_k$ , we have:

$$\lim_{j\to\infty} \sup_{k\geq 1}\left\{\left[M_k\left(\frac{|\lambda_k(\Delta^m x_k^i - \Delta^m x_k^j)|}{\rho}\right)\right] r_k^{\frac{1}{p_k}}\right\} \leq 1$$

$$\Rightarrow \sup_{k\geq 1}\left\{\left[M_k\left(\frac{|\lambda_k(\Delta^m x_k^i - \Delta^m x_k)|}{\rho}\right)\right] r_k^{\frac{1}{p_k}}\right\} \leq 1.$$

Let $i \geq n_0$ and taking infimum over $\rho$ , we have,

$$f_\Delta(x^i - x) < \varepsilon.$$

Then $(x^i - x) \in \ell_\infty\{\mathbf{M}, \Delta^m, \Lambda, p\}$.

Therefore $x = x^i - (x^i - x) \in \ell_\infty\{\mathbf{M}, \Delta^m, \Lambda, p\}$, since $\ell_\infty\{\mathbf{M}, \Delta^m, \Lambda, p\}$ is a linear space.

Hence $\ell_\infty\{\mathbf{M}, \Delta^m, \Lambda, p\}$ is complete. The other results follow in similar way.

**PROPOSITION 1**. *The spaces* $c\{\mathbf{M}, \Delta^m, \Lambda, p\}$, $c_0\{\mathbf{M}, \Delta^m, \Lambda, p\}$ *and* $\ell_\infty\{\mathbf{M}, \Delta^m, \Lambda, p\}$ *are K-spaces.*
Proof follows by the Theorem 3.

**THEOREM 4**. *Let* $\mathbf{M} = (M_k)$ *be a sequence of Orlicz functions. Then* $c_0\{\mathbf{M}, \Delta^m, \Lambda, p\} \subset c\{\mathbf{M}, \Delta^m, \Lambda, p\} \subset \ell_\infty\{\mathbf{M}, \Delta^m, \Lambda, p\}$ *and the inclusion is proper.*

**PROOF.** The inclusion $c_0\{\mathbf{M}, \Delta^m, \Lambda, p\} \subset c\{\mathbf{M}, \Delta^m, \Lambda, p\}$ is obvious. We need to show $c\{\mathbf{M}, \Delta^m, \Lambda, p\} \subset \ell_\infty\{\mathbf{M}, \Delta^m, \Lambda, p\}$.

Let $x = (x_k) \in c\{\mathbf{M}, \Delta^m, \Lambda, p\}$. Then there exists some positive number $\rho > 0$ such that,

$$\lim_{k\to\infty} \left\{ \left[ M_k \left( \frac{|\lambda_k(\Delta^m x_k - L)|}{\rho} \right) \right]^{p_k} r_k \right\} = 0.$$

Since $M_k$ is non-decreasing and convex, we have

$$\left[ M_k \left( \frac{|\lambda_k(\Delta^m x_k)|}{\pi} \right) \right]^{p_k} r_k \le D \left\{ \left[ M_k \left( \frac{|\lambda_k(\Delta^m x_k - L)|}{\rho} \right) \right]^{p_k} r_k \right\} + D \max \left[ 1, M_k \left( \frac{|\lambda_k(L)|}{\rho} \right) \right]^K$$

where $H = \sup_k p_k$ and $D = \max(1, 2^{H-1})$.

Thus $x \in \ell_\infty\{\mathbf{M}, \Delta^m, \Lambda, p\}$. Next to show that the inclusions are strict consider the following example.

**Example 1**. Let $p_k = 3$, for $k$ even and $p_k = 4$, for $k$ odd. Let $m \ge 0$ be given.

Let $M_k(x) = x^2$, for all $x \in [0,\infty)$ and $\lambda_k = 1$, for all $k \in N$.
Consider the sequence $x = (k^m, k^m, k^m, -,-,-)$.
Then $x \in c\{\mathbf{M}, \Delta^m, \Lambda, p\}$, but $x$ does not belongs to $c_0\{\mathbf{M}, \Delta^m, \Lambda, p\}$.

**COROLLARY 2**. *The spaces* $c_0\{\mathbf{M}, \Delta^m, \Lambda, p\}$ *and* $c\{\mathbf{M}, \Delta^m, \Lambda, p\}$ *are nowhere dense subsets of* $\ell_\infty\{\mathbf{M}, \Delta^m, \Lambda, p\}$. The proof of the result follows from theorem 1. The proofs of the following results are easy, so omitted.

**THEOREM 5**. *Let* $\mathbf{M} = (M_k)$ *and* $\mathbf{U} = (U_k)$ *be sequences of Orlicz functions those satisfy the* $\Delta_2$*-condition, then:*

(*a*) *if* $(p_k) \in \ell_\infty$, *then* $Z\{\mathbf{M}, \Delta^m, \Lambda, p\} \subseteq Z\{\mathbf{M}.\mathbf{U}, \Delta^m, \Lambda, p\}$;

(*b*) $Z\{\mathbf{M}, \Delta^m, \Lambda, p\} \cap Z\{\mathbf{U}, \Delta^m, \Lambda, p\}\} \subseteq Z\{\mathbf{M}+\mathbf{U}, \Delta^m, \Lambda, p\}$, *and*

(*c*) $Z\{\mathbf{M}, \Lambda, p\} \subseteq Z\{\mathbf{M}, \Delta^m, \Lambda, p\}$; *for* $Z = c_0, c, \ell_\infty$.

**RESULT 1**. *The spaces* $c_0\{\mathbf{M}, \Delta^m, \Lambda, p\}$, $c\{\mathbf{M}, \Delta^m, \Lambda, p\}$ *and* $\ell_\infty\{\mathbf{M}, \Delta^m, \Lambda, p\}$ *are not solid, for* $m > 0$.
Proof of the result follows from the following example.

**Example 2**. Let $\lambda_k$ =1, for all $k \in N$ ; $M_k(x) = x^2$, for all $x \in [0,\infty)$ and $p_k$ = 1, for $k$ even and $p_k$ = 2, for $k$ odd. Consider a sequence $(x_k)$ defined by $x_k =( x_k^i )$, where $x_k^i = ( k^{m-1}, k^{m-1}, k^{m-1}$-----),for all $k \in N$, then $(x_k) \in c_0\{\mathbf{M}, \Delta^m, \Lambda, p\}$.

Now consider $(\alpha_k)$ defined as $\alpha_k = (-1)^k$, for all $k \in N$.

Thus $(\alpha_k x_k)$ does not belong to $c_0\{\mathbf{M}, \Delta^m, \Lambda, p\}$, for $m$>0.

To show that $c\{\mathbf{M}, \Delta^m, \Lambda, p\}$and $\ell_\infty\{\mathbf{M}, \Delta^m, \Lambda, p\}$are not solid, for $m$>0.

Consider the sequence $x = (x_k) = ( k^m )$ and $\alpha_k = (-1)^k$, for all $k \in N$.

**RESULT 2**. *The spaces* $c_0\{\mathbf{M}, \Delta^m, \Lambda, p\}$, $c\{\mathbf{M}, \Delta^m, \Lambda, p\}$*and* $\ell_\infty\{\mathbf{M}, \Delta^m, \Lambda, p\}$*are not symmetric*, *for* $m$>0.

**PROOF.** The spaces are not symmetric follows from the following example.

**Example 2**. Let $\lambda_k = k$, for all $k \in N$ ; $M_k(x) = x^2$, for all $x \in [0,\infty)$ and $p_k$ = 1, for $k$ odd and $p_k$ = 2, for $k$ even. Consider a sequence $x = (x_k)$ defined by:

$(x_k) = ( k^2, k^2, k^2$,-,-,-), for all $k \in N$.

Then $x \in c_0\{\mathbf{M}, \Delta^m, \Lambda, p\}$,for $m$ =1.

Let the rearrangement $(y_k)$ of $(x_k)$ be,

$(y_k) = ( x_1, x_2, x_4, x_3, x_9, x_5, x_{16}, x_6, x_{25}, x_7$,-,-,-).

Then $(y_k)$ does not belong to $Z\{\mathbf{M}, \Delta^m, \Lambda, p\}$, for $m$ >0, where $Z = c_0, c, \ell_\infty$.

Hence the result follows. Proofs of the following results follows from the lemmas listed in the section 2.

**PROPOSITION 2**. *Let* $h = \inf_k p_k$, $H = \sup_k p_k$. *Then the following conditions are equivalent:*

(*i*) $H < \infty$ *and* $h > 0$;

(*ii*) $c_0\{\mathbf{M}, \Delta^m, \Lambda, p\} = c_0(\mathbf{M}, \Delta^m, \Lambda, p)$;

(*iii*) $\ell_\infty\{\mathbf{M}, \Delta^m, \Lambda, p\} = \ell_\infty(\mathbf{M}, \Delta^m, \Lambda, p)$.

**PROPOSITION 3**. *Let* $p$, $q$ *be two sequences of strictly positive numbers*. *Then* $c_0\{\mathbf{M}, \Delta^m, \Lambda, p\} \cong c_0\{\mathbf{M}, \Delta^m, \Lambda, q\}$ *if and only if there exists a sequence* $u = (u_k)$ *of strictly positive numbers such that the equations* (1) *and* (2) *holds*.

**PROPOSITION 4**. *Let the sequence* $v = (v_k) = \left(q_k^{q_k^{-1}} p_k^{-p_k^{-1}}\right)$. *Then* $c_0\{\mathbf{M}, \Delta^m, \Lambda, p\} \cong c_0\{\mathbf{M}, \Delta^m, \Lambda, q\}(v)$ *if and only if the equations* (3) *and* (4) *holds*.

**PROPOSITION 5**. *Let the sequence* $v$ = $(v_k)$= $\left(q_k^{q_k^{-1}} p_k^{-p_k^{-1}}\right)$. *Then* $\lim_{k\to\infty}\left(\frac{1}{p_k}-\frac{1}{q_k}\right)$=0 *implies* $c_0${**M**, $\Delta^m$ ,$\Lambda$, *p*}≅ $c_0${**M**, $\Delta^m$ ,$\Lambda$, *q*}(*v*).

**PROPOSITION 6**. *Let* $g_k = \frac{p_k}{q_k}$, *for every k*∈*N. Let* ( $g_k$ ) *and* ( $g_k^{-1}$ ) both be in $\ell_\infty$. Then $\ell_\infty${**M**, $\Delta^m$ ,$\Lambda$ , *p*}≅ $\ell_\infty${**M**, $\Delta^m$ ,$\Lambda$ , *q*} (*g*).

**PROPOSITION 7**. *Let q be a bounded sequence of strictly positive numbers*. *Then* $\ell_\infty${**M**, $\Delta^m$ ,$\Lambda$, *p*}⊆ $\ell_\infty${**M**, $\Delta^m$ ,$\Lambda$ , *q*}*if and only if the equation* (5) *hold*.

**PROPOSITION 8**. *Let q be a bounded sequence of strictly positive numbers and* $c_0${**M**, $\Delta^m$ ,$\Lambda$, *p*}≅ $c_0${**M**, $\Delta^m$ ,$\Lambda$, *q*}, *then* $c_0$ (**M**, $\Delta^m$ ,$\Lambda$, *p*) ≅ $c_0$ (**M**, $\Delta^m$ ,$\Lambda$, *q*).

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